T C 15 b

# TRANSACTIONS

OF THE

# ROYAL SOCIETY OF EDINBURGH.

VOL. XXI. PART IV., FOR THE SESSION 1856–57.

## CONTENTS.

Page

ART. XXXI.—*On New Forms of Marine Diatomaceæ, found in the Firth of Clyde and in Loch Fine.* By WILLIAM GREGORY, M.D., F.R.S.E., Professor of Chemistry. *Illustrated by numerous Figures, drawn by* R. K. GREVILLE, LL.D., F.R.S.E., . . 473

XXXII.—*On the Urinary Secretion of Fishes, with some Remarks on this Secretion in other Classes of Animals.* By JOHN DAVY, M.D., F.R.SS. Lond. and Edin. . . 543

XXXIII.—*On the Minute Structure of Involuntary Muscular Fibre.* By JOSEPH LISTER, Esq., F.R.C.S. Eng. and Edin., Assistant-Surgeon to the Royal Infirmary, Edinburgh. Communicated by Dr CHRISTISON, . . . . . . 549

XXXIV.—*On a Dynamical Top, for exhibiting the Phenomena of the Motion of a System of Invariable Form about a Fixed Point, with some Suggestions as to the Earth's Motion.* By J. C. MAXWELL, B.A., Professor of Natural Philosophy in Marischal College, Aberdeen, . . . . . . . . . 559

XXXV.—*On the Products of the Destructive Distillation of Animal Matters.* Part IV. By THOMAS ANDERSON, Professor of Chemistry, University of Glasgow, . . 571

XXXVI.—*On the Application of the Theory of Probabilities to the Question of the Combination of Testimonies or Judgments.* By GEORGE BOOLE, LL.D., Professor of Mathematics in Queen's College, Cork, . . . . . . . 597

[*For remainder of Contents see last page of Cover.*]

# XXXI.—*On New Forms of Marine Diatomaceæ, found in the Firth of Clyde and in Loch Fine.* By WILLIAM GREGORY, M.D., F.R.S.E., Professor of Chemistry. *Illustrated by numerous Figures, drawn by* R. K. GREVILLE, LL.D., F.R.S.E.

(Read 19th January 1857.)

In two papers read before this Society, I have very fully described the Diatomaceæ of the Glenshira Sand, which is very remarkable both for the large number of species found in it, which is certainly more than 320, and for the circumstances in which it must have been deposited. There can be no doubt, from the nature of the locality, which I have lately visited, that this bed was formed in the bottom of the Dhu Loch, a shallow fresh-water lake, at that time extending about two miles farther up the valley than it now does, and being at a higher level. In consequence of a rise in the level of the land, or a fall in that of the sea (from which—that is, from Loch Fine, the lower end of the lake is separated by a narrow and low barrier, through which the waters of the lake pass to Loch Fine), the lake has long ago been drained, till its upper end is nearly two miles from the point it must have reached when the bed of sand was formed. The present level of the lake is considerably lower than it was then; the precise difference I had no means of ascertaining, but I believe it is about 30 feet. Now, the most interesting fact about this lake is, that its actual level is that of half-tide, so that at low water the lake is discharged into the sea, while at high water the tide flows upward into the lake. Hence marine plants and animals are found in the Dhu Loch; herring, for example, are often caught in it, and were taken while I was in the neighbourhood. Hence also the present deposit in the lake exhibits a mixture of fresh-water and marine Diatomaceous forms. Now, the older sand, the subject of my paper, deposited at a considerably higher level, also contains both marine and fresh-water Diatoms; and while the individuals of the two classes are both abundant, the marine species are at least twice, perhaps thrice, as numerous as those of fresh water.

The natural, and, I have no doubt, the true explanation of the occurrence of so many marine forms in an inland deposit, formed in a fresh-water lake, is this: that at the period when the sand was formed the relative levels of the Dhu Loch and of Loch Fine were the same as now, when similar results ensue.

But as the lake was then at a higher level than now, so also must the sea have been at a level as much above its present one. This conclusion is in accordance with those derived from the observations made on raised beaches on the

banks of the Firth of Clyde, the level of which must always have regulated that of Loch Fine since the present form of the coast has existed.

There was, however, a circumstance which at first tended to throw some doubt on this conclusion, according to which the marine forms of the Glenshira sand must have come from Loch Fine. For although the known and described marine Diatoms found in the sand occur on our coasts, yet I was struck with the fact that out of upwards of fifty new or undescribed forms, there seemed to be no trace in deposits from the Firth of Clyde, examined by more than one naturalist during the progress of my investigation. The fact of these forms being undescribed was *prima facie* evidence that they had not yet occurred on the British coasts.

Yet it was evident that the formation of the Glenshira sand was, geologically speaking, very recent; so recent, indeed, that we could not suppose any number of species to have since become extinct. I came, accordingly, to the conclusion, that these undescribed forms must still exist in the waters of Loch Fine, or, what is the same thing, of the Firth of Clyde. I was therefore desirous to examine with care deposits from these waters, and this, during the past six months, I have been enabled fully to do.

The materials which I have examined are the following:—

1. A small quantity of dirt or sand washed from some nests of *Lima hians*, dredged in Lamlash Bay on the 19th of July last, in 4 fathoms, by Professor ALLMAN. This material, though, when cleaned, very scanty, proved the richest of all.

2. Four dredgings, made by myself, with the kind assistance of the DUKE of ARGYLL, in Loch Fine, at different points within two or three miles of Inveraray. These were all different, and three of them were interesting. They were taken at depths of from 14 to 18 fathoms, early in October last.

3. Three dredgings made at the same time by the Rev. Dr BARCLAY, in Loch Fine, off Strachur, at depths of 15, 20, and 60 fathoms, also in October last.

4. Three materials forwarded to me in October by the Rev. Mr MILES of Glasgow, who was for some time on the Holy Island, in Lamlash Bay.

One of these was washed from the nests of *Lima hians*, as I had reported the richness of the former. These last were from 7 fathoms in Lamlash Bay. This material, dredged, I think, in June, was not so rich in Diatoms as Professor ALLMAN's, but yet contained many interesting forms.

The second was a coarse red sand, dredged off Invercloy, Arran, which was rather poor.

The third was a mass of *Corallina officinalis*, taken with the hand, in rocky pools, at Corregills, Arran, when the tide was low. The Corallina proved to have been a good Diatom trap, and yielded a material, not remarkable for the number

of species, but rich in individuals, and these nearly all of interesting, rare, or new species.

I had thus eleven different materials, no two of which were exactly alike, although in all certain prevalent forms occurred. In each, on the other hand, some forms, few or many, were peculiar, and their presence gave a distinct character. A careful study of the whole has yielded interesting results; and these it is the object of the present paper to state as briefly as may be consistent with accuracy.

The first observation is, that these waters contain a very large proportion of all the known and described marine forms belonging to Britain, including a good many which have hitherto been very rare; so scarce, indeed, in some instances, that few observers have seen them. I may specify the following as being by no means rare, several, indeed, being abundant in these materials:—

Coscinodiscus concinnus.
Eupodiscus crassus.
... Ralfsii.
... sculptus.
Campylodiscus Ralfsii.
... Horologium.
Navicula Hennedyi.
... granulata, Bréb.
... Lyra, Ehr.
Pleurosigma rigidum.
... obscurum.
Pleurosigma delicatulum.
... transversale.
Surirella lata.
Hunantidium (?) Williamsoni.
Amphiprora elegans.
Podosira Montagnei.
Orthosira marina.
Grammatophora macilenta.
Biddulphia Baileyi.
... turgida.

The second observation which I made was, that, as I had anticipated, nearly the whole of the new forms figured by me from the Glenshira sand are found living, and generally abundant, in these waters. The following list contains the names of such of the marine species, figured in my former papers, as I have found in the new materials:—

Cocconeis distans.
... costata.
Eupodiscus Ralfsii; also var. $\beta$, sparsus.
Campylodiscus simulans.
Surirella fastuosa, very large.
Amphiprora recta.
... lepidoptera.
Navicula rhombica.
... maxima.
... angulosa, and var. $\beta$.
... humerosa.
... latissima.
... clavata.
... splendida.
... incurvata.
... didyma, var. $\gamma$, costate
Navicula didyma $\delta$.
... crassa.
Pinnularia Pandura.
... longa.
... inflexa.
Amphora Arcus.
... crassa.
... elegans.
... plicata.
... obtusa.
... Grevilliana.
... rectangularis.
... lineata.
Synedra undulata.
Tryblionella constricta.
... apiculata.

I think we can hardly doubt that all the new Glenshira marine forms will ultimately be found in the neighbouring waters.

Before going farther, I have to remark, that two of the forms in the first list above given, namely, *Campylodiscus Horologium* and *Himantidium Williamsoni*, which had only been found by Professor WILLIAMSON, who detected them both in a dredging made by Mr BARLEE on the coast of Skye, in which they were very scarce indeed, have occurred abundantly, the former in one of the Loch Fine dredgings, and sparingly in some of the others, the latter in another of them, and, though less abundantly, yet frequent in nearly all the Clyde materials. We shall see that *Himantidium Williamsoni*, which Professor SMITH had referred doubtfully to that genus, not having been able to see more than the front view of it, is really no Himantidium; the side view, which is very abundant in one of my dredgings, having characters quite incompatible with the genus Himantidium. On this account, I shall refer to it among the new forms which I have to mention. I have found it a matter of very great difficulty, if not impossible, to refer it to any of the genera in SMITH's Synopsis. I may here add, that *Synedra undulata*, which I had recognised in the Glenshira sand, but which had never occurred entire in that deposit, is frequent in the first material from Lamlash Bay (Professor ALLMAN's), where it occurs quite entire in more than half of those I have seen, and, as I had concluded, from the imperfect specimens I had seen, attains a length of from 0·015 to about 0·02, which, for a Diatom, is gigantic. I had previously noticed a fragment of it in a recent gathering made by Professor SMITH, and he had himself subsequently found it frequent in Cork harbour. The first observer, however, was Professor BAILEY, of West Point, New York, who had found it still larger on the American coast, which I was not aware of till long after my observations on the Glenshira sand were made.

The third observation I shall here record is, that in these dredgings I found, in sufficient abundance, several very curious forms which had occurred in the Glenshira sand; but the description and figuring of which I had postponed, because either they were so scarce that I could not obtain good specimens, or, being only found in a fragmentary, detached, or imperfect state, I was quite at a loss to determine their true nature and position. I think I may say that in every such case I have been enabled, by the study of the new materials, to understand the nature and structure of these obscure or doubtful forms, and to establish them as new and distinct species. I have also been enabled to understand better several of the forms which were figured in my former papers, and to correct some errors which had crept into these.

I need not here give a list of the forms just alluded to, as they will be included in that of the new forms to be described. In that list, I shall mark them with a G, to indicate that they were first noticed in the Glenshira sand.

Lastly, in the new materials I have found a large number of entirely new and undescribed species, which I shall now proceed to enumerate. I may here mention, that although a good many fresh-water forms do occur in these dredgings,

as must, indeed, be the case, since the Clyde and all its tributaries bring down such forms, yet the new forms in question appear to be all of marine origin. They are, in general, much too abundant to have been derived from any other quarter, whereas the fresh-water forms among them are much scattered. It is proper also to state, that although all these forms are, to the best of my belief, new to Britain, yet a few of them have been described by EHRENBERG in some of his numerous works, and also by DE BRÉBISSON. The great majority, however, have not anywhere been figured; not, at least, in any works accessible to me.

As the new forms belong to a very few genera, it will be convenient to arrange them in groups. Those I shall adopt are as follow:—

I. Naviculoid Forms.
II. Cocconeides.
III. Filamentous Forms.
IV. Discs, including Campylodisci.
V. Amphiproræ.
VI. Amphoræ. {A. Simple. B. Complex.
VII. Miscellaneous.

## GROUP I.

### NAVICULOID FORMS.

These, as is usual in all gatherings, are numerous. Including two or three varieties of species already known, those which I have recognised as new are the following:—

| | | | |
|---|---|---|---|
| 1. Navicula | minor, n. sp. | 10. Navicula | spectabilis, n. sp. |
| 2. ... | Cluthensis, n. sp. | 11. ... | prætexta, Ehr. |
| 3. ... | inconspicua, n. sp. | 12. ... | Bombus, Ehr. |
| 4. ... | brevis, n. sp. | 13. ... | Lyra, Ehr. |
| 5. ... | Claviculus, n. sp. | 14. ... | Lyra, Ehr. var. β, abrupta. |
| 6. ... | Musca, n. sp. | 15. ... | Smithii, var. β, fusca. |
| 7. ... | rectangulata, n. sp. | 16. ... | Smithii, var. γ, nitescens. |
| 8. ... | nebulosa, n. sp. | 17. ... | Smithii, var. δ, suborbicularis. |
| 9. ... | Barclayana, n. sp. | 18. ... | maxima, Greg. |

1. *Navicula minor*, n. sp. Pl. IX., fig. 1. Form rectangular in the middle, acuminate at the ends, which are acute. Length from 0·0012″ to 0·0025″; breadth 0·0004″ to 0·0008″. Striæ fine, inclined near the ends, not reaching the median line, 36 to 40 in 0·001″. The whole form has a delicate aspect.

This little form, represented in fig. 1, occurs in two or three of the Loch Fine dredgings, in one of which it is sufficiently frequent.

2. *Navicula Cluthensis*,* n. sp. Pl. IX., fig. 2. Form oval, rather broad. Median line broader at the centre, narrower at the apices. Central nodule definite, large; terminal nodules smaller. Length 0·0013″ to 0·0016″; breadth about 0·001″. Striæ conspicuous, clear, and sharp, inclined slightly in the middle, strongly near the ends; about 20 in 0·001″.

Fig. 2 represents this form, which occurs in Professor ALLMAN's dredging from Lamlash Bay, and though not abundant, is yet frequent enough for all practical purposes. It is very uniform in its characters, and though the description above given may not appear very characteristic, yet I know of no form with which this one can be confounded. Its aspect is so peculiar that it is instantly recognised.

3. *Navicula* (?) *inconspicua*, n. sp. Pl. IX., fig. 3. Form linear, rather narrow, with rounded ends. Median line strong, complex, interrupted in the middle. Nodule definite. Aspect of valve hyaline. Striation so fine that it has not yet been resolved; and at all events it cannot be visible under a power of 400. Length 0·002″ to 0·0032; breadth about 0·00035.″

This little form occurs both in Lamlash Bay and in Loch Fine. I do not feel quite sure that it is a Navicula, as it may possibly belong to a filamentous species; it may be, for example, a Diadesmis; or it may prove to be a Schizonema. This can only be ascertained by observations on examples in the living state.

4. *Navicula brevis*, n. sp. Pl. IX., fig. 4. Form nearly elliptical in the middle; broad, short; contracted to short, produced, obtuse extremities. Length about 0·0023″; greatest breadth 0·0013″. Striæ fine, about 35 in 0·001″; very slightly inclined, not reaching the median line, and at the centre leaving a large, round, blank spot, within which the two halves of the median line end in small rounded expansions. Central nodule indefinite; terminal nodules definite.

This form is easily recognised by its short, squat shape, and is distinguished from *N. semen*, which it resembles in form, by its much finer striation. It occurs in Lamlash Bay, and is not very scarce in Professor ALLMAN's dredging from that locality.

5. *Navicula Claviculus*, n. sp. Pl. IX., figs. 5, 5 *b*, and 5 *c*. Form of valve linear, narrow, with one central and two terminal expansions, separated only by two constrictions. The terminal expansions are much elongated, oval at the extremities, and rather broader than the central expansion. Central nodule definite. Length 0·0015″ to 0·002″; greatest breadth 0·0002″ to 0·0003″. On the S.V., figs. 5 and 5 *b*, the central expansion is unstriated. Striæ on the rest of the valve transverse, sharp, not quite reaching the median line; about 32 in 0·001″. The F.V., fig. 5 *c*, is rectangular, with slightly expanded and bevelled angles, and exhibits the same absence of striæ from the middle part.† The striæ are seen to a

* From Clutha, the Clyde.

† Figs. 5 *b* and 5 *c* are magnified 800 diameters, to bring out the details. Fig. 5 is magnified only 400 times, and is below the average size.

certain extent on the F.V., but most towards the extremities, indicating that the S.V. is more convex near the ends than in the middle. I have named this form from its resemblance to a small two-headed club.

It occurs only in one of the Loch Fine dredgings, in which, though far from frequent, I have been able to examine many more specimens than are required to ascertain the characters of the dead form. I observe that it often occurs in pairs, as well as solitary, so that it may perhaps belong to a filamentous genus, such as Diadesmis. But as I cannot be sure of this, without seeing the living or growing form, I refer it, for the present, to Navicula.

6. *Navicula Musca*, n. sp. Pl. IX., fig. 6. Form of valve deeply constricted in the middle, broadest at a point near the middle on each side of it, and almost triangular thence to the acute apices. Length 0·002″; greatest breadth 0·0011″; breadth at middle 0·00075″. Striation confined to a marginal band, which is rather broad, and nearly of uniform width, except at the apices. Median line sharp; central nodule definite. Striæ coarse, 18 in 0·001″; distant, moniliform. Aspect of the valve transparent.

The form of this very pretty species is allied to that of *N. didyma* and the other panduriform Naviculæ, which are so frequent in marine gatherings. Even its form, however, is peculiar, and it is at once distinguished from all the others by its marginal striation. It so much resembles in shape the body of a bee or wasp, that I should have named it Apis or Vespa, had not these names been already appropriated to other species by Ehrenberg. I have chosen, therefore, the specific name Musca, as the form is also that of various large flies. It occurs in the same Loch Fine gathering, as Nos. 1, 3, and 5; a gathering which, though very scanty and very stony, has proved singularly rich in undescribed forms, especially of Amphoræ, as we shall see farther on. This dredging was a very coarse sand, which, after boiling with acid, I was on the point of rejecting as useless, when I observed a very trifling cloud of finer matter. This, though full of mica, supplied a remarkable proportion of new species; so much so, that I believe it contained as many of these as of known species; and of the undescribed forms found in it, a majority have occurred in it alone. I mention these facts, in order to show that every dredging, however unpromising, in such localities as Loch Fine and the Clyde, ought to be closely examined. This one was most unpromising; yet it turned out not only rich in new species, but very different from the other dredgings made in the same waters.

7. *Navicula rectangulata*, n. sp. Pl. IX., fig. 7. Form of S.V. rectangular, the extremities being rounded; rather narrow. Length from 0·003″ to 0·004″; breadth about 0·0006″ to 0·0008″. Striation highly radiate, there being three centres of radiation on each side—one in the middle, and one at each end. Striæ soft, not very fine, subdistant; about 22 in 0·001″, not quite reaching the median

line, and leaving a very small, round, blank space in the centre. Central nodule indefinite. The F.V. has not yet been recognised.

This form is remarkable for the shape of the S.V., which is that usually found in the F.V. It occurs rather sparingly in Professor ALLMAN'S dredging from Lamlash Bay, which, of all the dredgings, is the richest in species, whether known or undescribed.

8. *Navicula nebulosa*, n. sp. Pl. IX., fig. 8. Form oval, broad; generally with a slight tendency to angularity in the middle, and also a tendency to acumination at the apices. Length from 0·0025″ to 0·0035″; breadth 0·0013″ to 0·0016″. Median line sharp, ending in two elongated expansions at the centre, which do not meet. Nodlue indefinite. On each side of, and close to, the median line, is a narrow rectangular band of striation, interrupted at the nodule. At the margin is a somewhat broader, but still narrow, striated band, almost exactly of uniform width throughout. Striæ 34 to 36 in 0·001″. Aspect of valve hazy and indistinct. Striated portions pale blue under the half-inch objective. This form is allied to *N. Hennedyi*, figured in my second paper on the Glenshira sand (see *Trans. Mic. Soc.*, vol. iv., pl. v., fig. 3.) But I have found it necessary to separate it from that species, in consequence of its very different aspect. *N. nebulosa* is a smaller form than *N. Hennedyi*, the one here figured being an unusually large one. It is also much more finely striated; and, above all, it has invariably that peculiar indistinctness of aspect from which I have named it; whereas *N. Hennedyi*, even when of a smaller size, as we sometimes find it, is always remarkable for the sharpness of its markings. The tendency to angularity generally seen in *N. nebulosa* is never found in *N. Hennedyi*. Lastly, the striation of the former is so much finer, that the striated parts, seen under a low power, have a very pale bluish tinge never seen in the latter. When the two forms are seen in the same field of view, as often happens in Professor ALLMAN'S dredging from Lamlash Bay, and even when *N. Hennedyi* is the smaller, though it is generally much larger, the difference between them is very striking. I might have considered *N. nebulosa* as a variety of *N. Hennedyi*, but that I have found both forms exceedingly uniform in their characters, and have not been able to observe any tendency to transition from one to the other. *N. nebulosa* is frequent in the Lamlash Bay dredging just mentioned, in which *N. Hennedyi* also occurs; but elsewhere I have hardly ever seen the present species.

9. *Navicula Barclayana*, n. sp. Pl. IX., fig. 9. Form an elongated oval, somewhat suddenly contracted to acute extremities, terminated by small round apiculi. Median line narrow, ending at the middle in two small expansions. Nodule indefinite. Length 0·004″ to 0·0045″; breadth 0·001″ to 0·0012″. Striæ about 38 in 0·001″, somewhat inclined, sharp, minutely moniliform, confined to a marginal band, which is rather narrow, and of uniform width except near the apices, where it becomes narrower.

This is a fine conspicuous form, and occurs not unfrequently in the same Loch

Fine dredging, with *N. Claviculus*, *N. Musca*, &c. I have also seen it, though much more sparingly, in some of the other dredgings, both from Loch Fine and Lamlash Bay.

10. *Navicula spectabilis*, n. sp. Pl. IX., fig. 10. Form elliptic-lanceolate, very broad; ends subacute or acute. Length from 0·003″ to 0·005″; breadth from 0·0023″ to 0·0032″. Median line sharp, having close to it on each side a narrow striated band, interrupted at the middle. Central nodule large, indefinite. There is a marginal band of striation, which is very broad in the middle, where it projects inwards to an obtuse point, and very narrow at the apices. Striæ coarse, moniliform, about 22 in 0·001″. The blank spaces between the marginal and central bands are very broad; and this part of the valve is so thick and strong, that in fractured specimens we never find it broken across, but we often see the entire blanks, united by the central nodule, which is elongated laterally, separated from all the striated parts, forming a singular object.

This form is allied to *N. Lyra*, Ehr., and also to *N. Hennedyi*. I consider it, however, quite distinct from either. It occurs frequently in Lamlash Bay; but I have not yet seen it elsewhere. The form and width of the marginal band distinguish it from *N. Hennedyi*, while the broad blank spaces distinguish it from *N. Lyra*, in which, as we shall see, these spaces are linear. Besides this, *N. Lyra* very often occurs with produced ends, and never has the peculiar form of *N. spectabilis*. The latter never occurs with produced ends. Moreover, under a low power, *N. spectabilis* has a bright brown colour, in the striated parts, not observed in *N. Lyra*. Lastly, I find this form remarkably uniform and constant in its characters. It is very conspicuous, and generally larger than *N. Lyra*.

11. *Navicula prætexta*=*Pinnularia prætexta*, Ehr. Pl. IX., fig. 11. Form a pure and broad oval. Length from 0·004″ to 0·005″; breadth 0·0025″ to 0·003″. Median line sharp, the central extremities ending in large, rounded expansions, which are bent to the same side. Central nodule indefinite, extending transversely. On each side of, and close to the median line, a narrow linear band of very coarse and coarsely moniliform striæ. At the margin of the valve is a rather broad band of striæ, exactly like those of the central bands. This marginal band is of uniform width till near the apices, where it gradually becomes narrow. Striæ 8 to 10 in 0·001″. The broad intermediate space between the marginal and central bands is not blank, as in *N. Hennedyi*, but is irregularly dotted or stippled with round granules, precisely the same as those of the striæ Towards the centre, and near the ends, the median striated bands pass gradually into the sparsely dotted space. Between these points the median bands end more abruptly. The scattered granules are consequently most thickly set round the nodule and near the apices. The granules are so large, that there are not more than five in each of the longest of the marginal striæ.

This conspicuous and beautiful species has been figured by EHRENBERG, as

occurring in the Clay Marl of Ægina, a bed belonging either to the Chalk formation or to the oldest Eocene strata. It seems to be very scarce there, for EHRENBERG has figured an imperfect specimen. I found it first rather sparingly in Professor ALLMAN'S Lamlash Bay dredging; and, since then, still more sparingly in Mr MILES'S, from the same locality, as well as in several of my Loch Fine dredgings. It is obviously a member of the same group as *N. Hennedyi*, *N. nebulosa*, and others, with marginal and central striated bands. It is distinguished by its size, by the remarkable coarseness of its striation, and by the peculiarity that granules, such as form the striæ, are scattered over the unstriated space, without regularity. I have been informed that a form of *N. Hennedyi* occurs, with a similar character, but this I have not seen. I presume it will be easily known by its much finer striation, and its smaller size. Though this species is hitherto scarce, I have been able to examine a large number of examples, and also to supply various correspondents with specimens.

I avail myself of this opportunity to point out, that we have here an excellent example of the occurrence, in the recent state, in our seas, of a species hitherto known only as a fossil one. But as the Clay Marl of Ægina is the oldest deposit in which Diatoms have been detected with certainty, we have evidence that a species which is among the oldest of known Diatoms still exists. Nor is this by any means an unusual occurrence. In EHRENBERG'S plate of the microscopic forms of this Eocene clay marl (Eocene at least, if not Cretaceous), he figures many other forms, all of marine origin; and all, or nearly all, of which are still living species. Indeed, I have seen upwards of three-fourths of these Diatoms in the dredgings described in this paper. Among these are *Actinocyclus undulatus*, *Coscinodiscus radiatus*, *Pyxidicula cruciata*, *Navicula prætexta*, *N. Bombus*, and many other frequent forms. I feel assured that every form of Diatom found in that Clay Marl, still lives in the present seas. And if this be the case with the oldest Diatomaceous deposit, it is no less likely to hold good of such as are of later date. In the great bed of Richmond, Virginia, which is marine, and said to be of the Miocene period, perhaps the most frequent form is *Orthosira marina*, Sm. (*olim Melosira sulcata*, Kütz.); a form which I find, as already mentioned, very abundant in Lamlash Bay. In the same deposit occurs *Coscinodiscus centralis*, Ehr., a splendid disc, to be described farther on, as occurring in the Clyde; and I might multiply similar examples almost *ad infinitum*.

Here the question naturally presents itself, Are there any extinct species of Diatoms? Strange as it may seem, when compared with what is found to occur in organisms of other and higher classes, I believe that this question ought to be answered in the negative.

In the earlier works of EHRENBERG, we frequently meet with species, and even with large groups of species, or almost genera, which are stated to be "fossil only," and which were believed to be extinct. Such forms are *Campylodiscus*

*clypeus*, found in the polishing slates of Bohemia, and the whole series of *dentate Eunotiæ*, found so abundantly in the Lapland Bergmehls.

But the progress of observation has shown that these forms are still in existence. *C. clypeus* has recently been found in British waters; and in America, and elsewhere, the *dentate Eunotiæ*, such as *E. Diadema*, *E. heptodon*, *E. octodon*, *E. Serra*, and others, have been found recent. I have myself often found, during the last two years, *E. triodon*, a form long regarded as extinct, in many of our streams, although scattered. But last summer I detected it as the predominant form in a gathering made by Professor BALFOUR, in a small stream on a hill in Arran, not far from Lamlash.

I conclude, therefore, that our knowledge of the existing species of Diatoms is yet far too limited to allow us to say that any fossil species no longer exists. In this very paper, I make known the actual existence of several species, hitherto supposed to be exclusively fossil, and every day adds to the number of existing forms, while it diminishes that of those conjectured to be extinct, few of which are now left. Surely, when one or two localities yield so many undescribed forms as I have here the honour to lay before the Society, we are not entitled to conclude that any form is extinct, because hitherto it has only been met with in the fossil state. In the present state of our knowledge, it is far more probable, that we shall ultimately find, as I have done in the case of *N. prætexta*, that the supposed extinct species are all still in existence.

But, it may be asked, How is it that you suppose no species of Diatoms to have become extinct, when, in almost every other class, the extinct species far outnumber the existing ones? In answer, I would observe, first, that we have no undoubted evidence of the existence of Diatoms earlier than the Clay Marl above named, which is either Eocene, or a member of the latest Chalk deposits. Now, if it be Eocene, then we know that that formation contains, even among fishes, a certain proportion of existing species. This proves that the condition of the Eocene period did not differ nearly so much from the present conditions as those of earlier deposits must have done; those, for example of the Carboniferous series, of the Old Red Sandstone, or of the Silurian strata.

*Secondly*, the size of Diatoms is so very minute, and their structure so exceedingly simple, that they must be little, if at all, affected, even by very considerable climatic variations. Of this, indeed, we have ample evidence, so far at least as concerns existing differences of climate. If we consult the plates of EHRENBERG'S Microgeology, we shall see that the existing species of Diatoms found in the most distant and different parts of the world, in the Arctic and Antarctic Seas, in the tropical zone, and in our own temperate regions, are, for the most part, absolutely identical. There are, no doubt, local differences; but these, as is shown in this paper, may be very great in almost contiguous localities. On the other hand, having examined the diatoms in a large number of American and other exotic

soils, I have always found a very great majority of common British species. As an example, I may specify two soils particularly rich in Diatoms; one from the Sandwich Islands, the other from Lebanon. The former was quite like an ordinary fresh-water gathering, the latter resembled a poorer material. In both, I have great doubts whether all the numerous species are not identical with our own. Some few of the species, indeed, are not to be found in SMITH's Synopsis; but most of these have been described by others, or by myself, as British forms, since that work appeared.

So far, therefore, as the greatest actual differences of climate are concerned, Diatoms are apparently not affected; as in the cases just mentioned, it is impossible to distinguish the exotic specimens from British ones.

If, therefore, Diatoms did not exist earlier than the Eocene period, it is quite conceivable that none of them may have become extinct.

I have already stated that the Clay Marl of Ægina is supposed by some to belong to the formation next below the Eocene, that is, to the latest Cretaceous beds; but that there is no satisfactory evidence of Diatoms in any earlier formation. If we admit the Xanthidia to be Diatoms, these forms are known to occur in chalk flints. But the Xanthidia are not usually regarded as Diatoms, and I have not seen, either in flint or in chalk, any admitted or recognised Diatoms.

EHRENBERG figures many microscopic forms from the Chalk and older strata, some even from the Silurian Greensand. But these older forms, at least so far as are shown in the Microgeology, are not Diatoms, but either the siliceous Polycystineæ, or the calcareous Polythalamia, or, finally, sponge spicules.

I admit that Diatoms may have existed in the Chalk or earlier, and that, by a slow chemical change, they may have been destroyed, so that their form is lost, the siliceous material alone remaining, whether alone or in combination. We may even suppose that flint has been formed in part from the shells of Diatoms which lived along with the Foraminifera or Polythalamia of that period. But these are mere conjectures, and till Diatoms are found in the older strata, it must remain doubtful whether they existed previous to the Eocene period.

The Chalk or Marl of Meudon, near Paris, and that of Caltanisetta, in Sicily, exhibit a mixture of microscopic forms, calcareous and siliceous, including Diatoms. Here Diatomaceous shells, in contact with excess of calcareous matter, have remained unaltered; and if the Chalk of the true Cretaceous period had originally contained Diatoms, it seems probable that they would have been found as little altered as those of the newer beds just alluded to.

On the whole, then, it is probable that the continued existence of all, or nearly all, the known fossil species of Diatoms is the result, first, of their comparatively late introduction, and secondly, of their small susceptibility to climatic changes, arising from their minute size and very simple structure.

12. *Navicula Bombus*, Ehr. Pl. IX., fig. 12. Form much constricted in the

middle, the two halves broad and rounded, with subacute extremities. Median line broad; central nodule square, definite. Length about 0·0045″; greatest breadth 0·0018″. Striation coarse, strongly moniliform, not reaching the median line, but leaving a narrow blank space on each side of it. Striæ about 18 in 0·001, much inclined near the apices.

This form, which I had found frequent in the Glenshira sand, is also frequent in the new dredgings. It is regarded by many as a variety of *N. didyma*, and by others as a variety of *N. Crabro*. I am disposed to consider it a distinct species, on account of its peculiar and very constant form, and also because it has a decided light-brown colour in balsam, under a low power, which *N. didyma* has not. It is much larger than *N. didyma*. As to *N. Crabro*, the moniliform structure in it is always obscure, and the form is also different. I cannot perceive that *N. Bombus* passes into either of these species by intermediate forms. But whatever be the ultimate decision on this point, I give it here as the form called by EHRENBERG *N. Bombus*; which, be it species or variety, is at all events conspicuous, and very constant in its characters.

In my last paper on the Glenshira sand, I have figured several Naviculæ and Pinnulariæ of the panduriform group, and I have pointed out that this remarkable group requires thorough investigation. In order to contribute towards this end, I have figured the present form, as well as *N. Musca*, a new member of the same group; and I shall describe, farther on, another, namely a remarkable form of *Pinnularia Pandura*, Bréb.

13. *Navicula Lyra*, Ehr. Pl. IX., fig. 13 and 13 *b*. Form oblong-elliptic, broad; often contracted to short produced extremities. Length from 0·002″ to 0·0045″; breadth 0·0007″ to 0·0018″. Median line fine, interrupted by a large indefinite nodule, extending transversely. On each side of, and in contact with, the median line, is a linear, somewhat broad, striated band, and this is separated from the very broad, marginal, striated band by a narrow linear blank space. These linear blank spaces are, in each half, united by their base to the extremities of the nodular blank. They bend outwards from this point, then inwards, and finally again outwards at their extremities, thus forming, in the entire valve, two lyrate shapes united by their bases. Hence the name. The lyrate character is often much more decided than in the specimens figured. The extremities of these lyrate blanks generally reach the margin of the valve near its apices, but sometimes fall short of this, as in the figure. Striæ about 22 or 24 in 0·001″, somewhat inclined near the apices.

This species, which occurs in the Glenshira sand, and is scattered through all the dredgings here mentioned, has been described, though not as I have described it above, in the 2d volume of Professor SMITH'S Synopsis. Professor SMITH seems to have seen only a variety, to be presently mentioned, which does not possess the lyrate character, and has therefore omitted that character. He refers to a figure

in vol. i. (fig. 152 *a**) given as *N. elliptica*, which is not lyrate; but the name given by EHRENBERG proves that he regarded the lyrate character as a principal one. I have therefore figured it, in order to show that it occurs in Britain as EHRENBERG described it.

14. *Navicula Lyra*, Ehr., var. β, *abrupta*. Pl. IX., figs. 14 and 14 *b*. Form usually oval, more or less elongated; sometimes linear in the middle, broad, with parallel sides, and obtusely acuminate at the extremities. I have hardly ever seen it with contracted and produced ends, as is so often observed in *N. Lyra*. Size and striation as in *N. Lyra*, but the blank spaces, which are linear, as in that species, instead of being recurved at the ends, or lyrate, bend inwards at the ends, so as to form two narrow ellipses meeting in the central nodule. These linear blanks in this variety stop abruptly at some distance from the terminal margin of the valve, which, in *N. Lyra*, they often, though not always, reach.

This form, which is frequent in the Glenshira sand, as well as in the dredgings, is that already referred to as having been figured in vol. i. of the Synopsis (fig. 152 *a**), as *N. elliptica*, and since referred to as *N. Lyra*, in vol. ii.

I figure it here, both that it may be compared with the *N. Lyra* of EHRENBERG, and that it may be contrasted with *N. spectabilis* (fig. 10), which is supposed by some to be a form of *N. Lyra*. The form of the latter, and the fact that the blank spaces in it are not linear, but broad, and reach the margin, all which characters are very constant; to which may be added the rich brown colour of *N. spectabilis* in balsam, under a low power, seem to me to be sufficient to distinguish it from *N. Lyra*. The reader is requested to compare fig. 10 with figs. 13, 13 *b*, 14, and 14 *b*. The latter forms are colourless in balsam.

15. *Navicula Smithii*, var. β, *fusca*. Pl. IX., fig. 15. Form an elongated oval, broad, with rounded ends. Length from 0·003″ to 0·0063″, and even more; greatest breadth from 0·0014″ to 0·0028″. Median line narrow at the terminal nodules, which are a little within the apices; broad, and formed of three parts, all ending in expansions, on each side of the central nodule. Nodule large, broad, indefinite. Striation very coarse, and coarsely moniliform, not reaching the median line, but leaving on each side of it a narrow blank line, terminating in the angles of the nodular blank. The whole spaces, taken together, form two very acute long triangles, base to base Striæ about 10 in 0·001″. At about one-third of the distance from the median blank lines to the margin, the striæ are traversed by a strong, dark line, which is often, as in the figure, nearly rhombic, but is generally curved, though very slightly. This line is caused by a ridge or elevation of the valve, and is very conspicuous. Valve thick, and highly convex, of a strong brown colour, in balsam, under the ½ or ⅔ of an inch objectives.

This form, which is very conspicuous, occurs, like the two preceding, both in the Glenshira sand and in the dredgings; and in that of Professor ALLMAN from Lamlash Bay and one from Loch Fine, it is frequent. I give it as a variety of *N.*

*Smithii* (olim *N. elliptica* Sm.), because I have always understood the typical *N. Smithii* to be a form which is very frequent in the Glenshira sand, and occurs also in the new materials. It is of a short, broad, inelegant, oval shape, flat, colourless, and much less coarsely striated. Neither does it exhibit the longitudinal ridge so distinctly. It may be, that the present form, *N. fusca*, is the typical one, and the other a variety of it; but in my experience I have only seen *N. fusca* in the gatherings above named, while I observe *N. Smithii* in every marine gathering.

16. *Navicula Smithii*, var. γ, *nitescens*. Pl. I., fig. 16. Form lanceolate, tending to rhombic, with obtuse ends. Median line straight, nodule definite. Length from 0·002″ to 0·0035″; breadth from 0·0009″ to 0·0014″. Striæ about 16 in 0·001″, considerably inclined, obscurely moniliform, and of a shining aspect. They are traversed by a ridge, which is about half-way from the margin to the median line, and has an outline more rhombic than that of the valve.

This form occurs both in Lamlash Bay and in Loch Fine, and is not at all rare in some of the dredgings. It is conspicuous, from its elegant form and shining aspect. It is quite colourless under a low power. I have given it as a variety of *N. Smithii*, from a desire to avoid unnecessary multiplication of species. But I am inclined to regard it as distinct from that species, from its peculiar form, its smaller size, the character of the nodule and median line, and its bright white aspect; all of which characters are very constant.

17. *Navicula Smithii*, var. ͽ, *suborbicularis*. Pl. IX., fig. 17. Form a short, broad oval, or suborbicular. Length 0·002″ to 0·0026″; breadth 0·0013″ to 0·0018″. Median line bounded by white lines, curving inwards both to the apices and to the indefinite nodule. Striation conspicuous, much inclined. Striæ 16 or 18 in 0·001″, moniliform. There is a ridge, as in the two preceding forms, traversing the striæ, and when the striæ near the margin are in focus, those between the ridge and the median line are very faint.

This form occurs in Lamlash Bay, and is also tolerably frequent in one Loch Fine gathering, in which the preceding form is not found. Its small size, nearly round form, and peculiar median line, with the slightly-marked ridge, compared to that in the two preceding forms, seem to point it out as distinct; especially as it is very constant in its characters. But, for the reasons already stated, I give it as a variety. It is at least a form to be noticed, and to be considered with the others with a view to a more accurate determination of species than has yet been possible, but which, in the progress of observation, we may hope to attain.

18. *Navicula maxima*, Greg. Pl. IX., figs. 18, and 18 *b*. Form of S.V. linear, rather narrow, with obtuse ends. Length from 0002″ to 0·008″; breadth of S.V. from 00·0025″ to 0·00011″; of F.V. in the larger specimens, 0·0009″ in the middle, 0·00115″ at the ends. Striæ fine, but distinct, about 52 in 0·091″, parallel, not quite reaching the median line, from which, at the centre, they retire, leaving a pretty large round space. F.V. rectangular but narrowest at the middle, and

slightly expanded at the ends, the angles being bevelled. From the opposite ends the margin inclines very slightly, but visibly, to the middle. Nodules very conspicuous on the F.V., in which the striation also extends, on each side, to rather more than ¼th of the width of the frustule, which arises from the convexity of the S.V.

I first described this species in my first paper on the Glenshira sand, in which the figure was not characteristic. I figured it again in the second paper; giving, however, a shorter, broader, and constricted form as the type, and the present one as a variety. I have since found it frequent in all the dredgings, but especially abundant in one from Loch Fine, and am now satisfied that the linear form is typical and the broad constricted form a variety. I give the peculiar and characteristic F.V. for the first time. The S.V. in fig. 18 *b*, is that of a broad individual of the linear type. It is generally narrower, and often even no more than half this width. The shortest specimens are often still narrower. The broad, incurved form, at first regarded as the type, is very scarce in the dredgings, compared to the linear form.

This form has been supposed to be identical with *N. firma*, Kütz, var. $\beta$; but its marine habitat at once negatives this supposition; and, besides, its aspect and colour are quite different. *N. firma* is brown, while *N. maxima* is of a pale straw-colour. The striation in *N. firma* is coarser and more conspicuous; and, lastly, *N. firma* is broader, has acute extremities, and yields several marked varieties, such as EHRENBERG'S *N. dilatata* and *N. Amphigomphus;* while the only observable variety of *N. maxima* is the shorter, broader, incurved one, represented in fig. 2 of my second paper on the Glenshira sand.

19. *Pinnularia* (?) *subtilis*, n. sp. Pl. IX., fig. 19. Form linear rhombic, very narrow, with elongated apices. Length about 0·0035″; greatest breadth about 0·00025″. Nodule definite. Costæ about 28 or 30 in 0·001″; transverse, slightly inclined towards the apices.

This form occurs in Lamlash Bay. I do not feel quite sure about its genus. It may be a Navicula. The whole form is delicate and translucent, and it is far from conspicuous.

20. *Pinnularia rostellata*, n. sp. Pl. IX., fig. 20. Form linear, broad, with acuminate ends, terminating in short, acute apiculi. Length from 0·002″ to 0·0027″; breadth about 0·0007″. Central nodule definite. Costæ strong, subdistant, inclined near the ends, reaching the median line, about 14 in 0·001″.

This pretty form occurs both in Lamlash Bay and in Loch Fine. It is not frequent, but I have been able to examine a considerable number of specimens, which are quite constant in their character.

21. *Pinnularia Allmaniana*. Pl. IX., fig. 21. Form elliptic-lanceolate, broad, extremities subacute. Valve highly convex on one side, concave on the other. Length from 0·0016″ to 0·0026″; breadth from 0·001″ to 0·0014″. Costæ appa-

rently marginal, strong, about 20 in 0·001″, giving the appearance of a narrow marginal band of very strong costæ. Within this band, however, the valve, on close inspection, is found to be marked with similar but much fainter costæ nearly to the median line. The valve appears to be thicker near the margin than in the middle, and this perhaps is the reason why the costæ are so strong and conspicuous there.

This form is frequent in Professor ALLMAN's dredging from Lamlash Bay, and it occurs also in Loch Fine. I have named it after Professor ALLMAN, to whom I am indebted for this dredging, the richest of all those here described.

22. *Pinnularia Pandura* = *Navicula Pandura*, Bréb., var. *β*, *elongata*. Pl. IX., fig. 22. Form deeply constricted in the middle, with elongated sub-triangular ends, and obtuse apices. Length 0·0075″ or more; greatest breadth 0·002″, breadth at constriction 0·0014″. Median line sharply defined, broader at the centre than at the ends; nodule square, definite. On each side of the median line, and a little way from it, there is on each side a line or ridge, apparently formed of large granules, but probably only apparently so, from the sudden and sharp elevation of the ends of the costæ. Costæ, from this point to the margin, perfectly entire and glassy, like those of *P. alpina*. Valve thick, costæ 10 or 11 in 0·001″, somewhat inclined near the apices.

This seems to be a variety of DE BREBISSON's *Navicula Pandura*, which I have represented in the second paper on the Glenshira sand, in figs. 11, 12, and 12*. But as DE BREBISSON himself describes the costæ as being entire, and represents them distinctly so in his figure of the species, I have changed the generic name to *Pinnularia*. I consider it as quite distinct from *N. Crabro*, Ehr., as described by Professor SMITH, in vol. ii., of the Synopsis; for the latter has obscurely moniliform striæ, as is shown in Dr GREVILLE's figure of it from Trinidad, in the Microscopical Journal for January 1857.

The forms represented in figs. 11, 12, and 12* of my second paper on Glenshira (*Micr. Trans.* iv., pl. v.), and that here figured (fig. 22), are abundant in several of the dredgings; but of all the numerous examples I have examined, not one exhibits the slightest trace of moniliform structure; and I have had the satisfaction of having this observation confirmed by Dr GREVILLE, who is acquainted with the form in which that structure exists obscurely. Unless, therefore, we are prepared to abolish the distinction between entire and moniliform striation, on which Professor SMITH founds the distinction between his genera *Navicula* and *Pinnularia*, we cannot regard this form as a *Navicula*.

## GROUP II.

### Cocconeides.

The new forms belonging to this group are not numerous, but they are, in every case, interesting. In addition to one species, already figured, though imperfectly, as occurring in the Glenshira sand, I have detected in the new materials six additional species, all of them beautiful and well-marked forms. These constitute a largeaddition to a genus, which, in Britain at least, has hitherto been a very small one. The species to be described are:—

| | |
|---|---|
| 23. Cocconeis distans, Greg. | 27. Cocconeis pseudomarginata, n. sp. |
| 24. ... ornata, n. sp. | 28. ... major, n. sp. |
| 25. ... dirupta, n. sp. G. | 29. ... splendida, n. sp. |
| 26. ... nitida, n. sp. | |

23. *Cocconeis distans*, Greg. Pl. IX., fig. 23. Form oval, broad; ends subacute. Length from 0·0014″ to 0·0026″; breadth from 0·001″ to 0·002″. Median line delicate. The valve is marked by distant lines, much inclined near the apices, not reaching the median line. These lines are about 10 in 0·001″, and consist of white hyaline faint bars, on which are set small and distant granules. The number of granules is only 4 or 5 in the longest of these lines, so that the granules are very distant. They are, as nearly as possible, of equal size, and from their distance, give to the valve a spotted rather than a striated aspect. In the figure, the granules appear larger than they really are; but this, as I have since ascertained, depends on the focus, and is an effect of shadow. By careful focussing, the real size of the granules is easily seen. The valve is hyaline; but it is always easy, by focussing, to see the faint bars on which the granules are set, a character which at once distinguishes this form from *C. Scutellum*, to which some are disposed to refer it. Another character, besides that of its having much fewer lines and much fewer granules than the coarsest varieties of *C. Scutellum*, is, that in *C. distans* the granules are of equal size, while in *C. Scutellum* they diminish in size as they approach the median line. In *C. distans*, if there be any difference, it is that the marginal granules are somewhat smaller than the others. I may here also allude to the fact, that while *C. Scutellum* is a most variable form, *C. distans*, so far as I have seen, exhibits only one variety, and that quite different from any variety of *C. Scutellum*. This form, *C. distans*, var. *β*, *præmorsa*, is considerably larger than the type, having a length of 0·003″ to 0·0038″, and being a little narrower in proportion than the type. There is always, at one point of the margin, a notch or solution of continuity, as if a portion had been cut out, and then smoothed over. The only other difference is, that the granules are somewhat smaller, but the faint bars are exactly as in the type.

I figured this species, both in my first and in my third paper on the Glenshira

sand. But in the former, a variety of *C. Scutellum* was figured by mistake: and the latter figure was imperfect, because I had not then seen the faint white bars. This species, along with *C. costata*, also figured in the third paper alluded to, is so frequent in Lamlash Bay, that I have had ample means of studying it, and am quite satisfied of its being a good species. I may say the same of *C. costata*, with the remark, that I cannot ascertain from EHRENBERG's figures, whether his *Raphoneis fasciolata* may not be the same form. EHRENBERG's form seems to be much larger, and the markings much coarser and more conspicuous. I must leave this point undecided till I can compare the two forms.

24. *Cocconeis ornata*, n. sp. Pl. IX., fig. 24. Form a pure and elegant oval. Length about 0·0022″; breadth about 0·0014″. There is a broad marginal band, marked by strong distant costæ, the ends of which are rounded. Within this band the surface appears concave to the median line, which is delicate, with a large, definite, central nodule. The middle part is marked by fainter costæ, corresponding to those on the marginal band, and, like them, so much inclined near the apices, as to be nearly vertical. There is a narrow blank line between the marginal and central costæ, and the latter do not reach the median line, leaving a long lanceolate blank space in the middle. The whole valve has a rich ornate aspect.

This beautiful species occurs in Lamlash Bay; and although scarce as yet, I have been able to examine a sufficient number of specimens to ascertain its characters. I have also observed a few in Loch Fine.

25. *Cocconeis dirupta*, n. sp. Pl. IX., fig. 25. Form a broad, short oval, sometimes all but orbicular. Length from 0·001″ to 0·0024″; breadth from 0·0007″ to 0·0021″. Valve thick, and under the half-inch objective of a strong brown colour. Median line irregular, like a slit or tear down the middle of the external surface. The whole valve is marked, except the slit, with coarse, wavy, longitudinal striæ; but, when carefully focussed, fine transverse striæ are seen over the whole surface to the median line. Under the half-inch, there is an appearance of a long stauros, which, under a higher power, disappears as such, and can only be seen as a transverse gleam of light from below. The striated surface seems to be an outer one, torn asunder in the middle, and from this I have named it. Vertical striæ about 26, transverse striæ about 60 in 0·001″.

I had observed this form in the Glenshira sand, where, however, it was very scarce, and hardly ever entire, so that I postponed its investigation. It occurs very abundantly in Mr MILES's Corallina gathering, and less frequently in several of the dredgings. There is but one known form which is in any degree allied to it. This is *C. diaphana*, Sm., which I find to occur along with it. After many comparisons, I am disposed to conclude, although these two forms are not the same thing, since *C. dirupta* is by no means diaphanous, while its striæ are conspicuous and its colour brown, the striæ of *C. diaphana* being very obscure, and the valve colourless, that *C. diaphana* may perhaps be an imperfect form of

*C. dirupta*, possibly the surface which lies under the one here figured, or possibly also the lower valve, which in *Cocconeis* is often different from the upper. It is, however, at least equally probable that these two forms belong to different species. In the Corallina gathering, *C. dirupta* is infinitely more frequent than *C. diaphana*.

26. *Cocconeis nitida*, n. sp. Pl. IX., fig. 26. Form a very broad oval, suddenly contracted, above and below, to very short, subacute, produced apices. Length from 0·001″ to 0·0038″; breadth from 0·0008″ to 0·0035″. Valve very thick, aspect glassy. It is marked by lines of very large nitid granules, these lines forming, longitudinally, concentric striæ, the two inner ones of which bend slightly outwards from the median line, leaving a narrow lanceolate blank space; the others becoming more and more curved as they approach the margin. In large specimens, there are five such lines on each side, or from 3 to 4 in 0·001″. But the granules also form transverse lines, much inclined near the apices, of which there are, in large specimens, 28 or 30 in the length of the valve, or from 6 to 8 in 0·001″. The margin is marked by a series of finer striæ. Median line obscure. In the middle of the valve, the transverse lines contain on each side five granules, corresponding to the five vertical lines. The granules are generally of equal size or nearly so, except a few near the apices, which are smaller. The whole form is very conspicuous, from its glassy aspect, and the size and brilliancy of the granules.

This striking form occurs not unfrequently in Lamlash Bay, and sparingly in the Loch Fine dredgings. It is very uniform in its characters.

27. *Cocconeis pseudomarginata*, n. sp. Pl. IX., fig. 27. Form a very broad elliptic, or elliptic-lanceolate. Length 0·0016″ to 0·0033″. Breadth 0·0011″ to 0·003″. Valve thin and transparent. Within the margin is a line or shade parallel to it; and within this, half-way from the margin to the centre, is a very strong line, forming a broad lanceolate figure. At first sight it seems as if the latter were the inner boundary of a marginal striated band; but on close inspection the striæ, which are very fine, are seen to extend from the margin almost to the median line, where they leave a very narrow rhombic blank space, extending, in the median line, only to the inner of the two marginal lines. The third, or interior line, counting from the margin, is a very strong and raised ridge, the ends of which are almost in contact with the second line. Median line delicate, central nodule definite; terminal nodules placed within the ends of the third line. Striæ very delicate, but sharp, about 62 in 0·001″, transverse in the middle, nearly vertical at the ends. The first or outer margin is formed of two lines very close to each other.

This remarkable form occurs in Professor ALLMAN's dredging from Lamlash Bay, where it is rather scarce, and also in that of Mr MILES from the same locality. It requires a very good glass to resolve the markings perfectly.

28. *Cocconeis major*, n. sp. Pl. IX., fig. 28. Form a very broad oval. Length to 0·0015″ to 0·0038″; breadth from 0·001″ to 0·00315″. Median line distinct;

central nodule indefinite. Terminal nodules considerably within the margin, small. The two parts of the median line terminate in the middle in small rounded expansions, but do not meet. Striæ delicate, but sharp, transverse in the middle, and gradually more and more curved towards the apices, where they become nearly vertical. They are not so close together as in some forms in which they are equally delicate, and there are about 54 in 0·001". Valve thin, flat, nyaline.

This remarkable form occurs in Professor ALLMAN'S Lamlash Bay dredging, where, however, it is rather scarce; also in that of Mr MILES.

29. *Cocconeis splendida*, n. sp. Pl. IX., fig. 29. Form, a pure broad oval. Length about 0·0044"; breadth about 0·0039". Valve strong, and very richly marked with striæ, which are highly inclined and curved near the apices. These striæ are coarse, about 14 in 0·001", and are formed of granules, which gradually diminish in size towards the median line. The four or five outer granules of each of the striæ are set as closely together as possible, while the rest are separate. This gives the appearance of a broad marginal band. There is a small, nearly square blank at the centre, which is no doubt the indefinite central nodule. The two halves of the median line are strong, somewhat bent at the terminal ends, where they form elongated expansions, lying just within the dense marginal band. The central ends terminate in small expansions, which lie at the upper and under edges of the central blank.

This beautiful form occurs in Lamlash Bay, but it is hitherto scarce. No doubt it will some day be found more abundantly. It is, with the two preceding forms, remarkable for the size it attains, being the largest Cocconeis yet described, while *C. major* and *C. pseudomarginata* are but little below it in this respect, and even *C. nitida* is unusually large for this genus.

## GROUP III.

### FILAMENTOUS FORMS.

Of this class of forms, the number is considerable. It is worthy of remark, that most of them belong, so far as I am able to judge, to the genus Denticula, which hitherto has yielded only fresh-water species. But there are six of which the genus is doubtful; partly because the F.V. alone is as yet known, which is the case in four of them; partly because, if not Denticulæ, they cannot well be referred to any of the genera in SMITH'S Synopsis. One remarkable species, the genus of which is still uncertain, I have been compelled to remove from Himantidium, in which genus Professor SMITH had provisionally arranged it. The forms of this group are 14; viz.:—

30. Denticula (?) interrupta, n. sp.
31. ... (?) capitata, n. sp.
32. ... (?) ornata, n. sp.
33. ... (?) lævis, n. sp.
34. Denticula nana, n. sp.
35. ... minor, n. sp.
36. ... distans, n. sp.
37. ... staurophora, n. sp.

38. Denticula fulva, n. sp.
39. ... marina, n. sp.
40. Diadesmis (?) Williamsoni = Himantidium Williamsoni, Sm.
41. Meridion (?) marinum, n. sp. (or Gomphonema lineare ?).
42. Pyxidicula (Dictyopyxis) cruciata, Ehr.
43. Orthosira angulata, n. sp.

30. *Denticula* (?) *interrupta*. Pl. X., fig. 30. Form of F.V. nearly rectangular, the middle part very slightly convex, and the ends a little expanded. There is an apparent interruption of the margin at the middle point on each side, and on each side of this opening is a round punctum. Length about 0·0015″; breadth about 0·0004″. It occurs in chains of 2, and the two dots of each margin form a square with those of the margin of the adjacent frustule. The S.V. is not yet known, and the species is only provisionally referred to Denticula.

This species occurs in Lamlash Bay, but is scarce.

31. *Denticula* (?) *capitata*, n. sp. Pl. X., fig. 31. Form of F.V. generally rectangular, but with the middle part considerably convex, and the apices expanded and rounded. Length about 0·0018″; breadth about 0·0005″. Occurs, like the last species, in chains of two. S.V. as yet unknown; and it is only doubtfully and provisionally referred to Denticula.

This species occurs only in one of the Loch Fine dredgings, and it is as scarce as the preceding.

32. *Denticula* (?) *ornata*, n. sp. Pl. X., fig. 32. Form of F.V. rectangular, but somewhat expanded in the middle, and also, after a slight contraction, at the apices, which are finally truncate. The margin, at the expansions, is beautifully moulded, having a moulding or notch on each side of the middle point.—Occurs in chains of two. Length 0·0015″ to 0·0017″; breadth, in the middle, 0·0005″. S.V. not yet known, so that the species is only provisionally referred to Denticula.

This very pretty species occurs both in Lamlash Bay and in Loch Fine, and is much less scarce than the two preceding ones. Notwithstanding this, however, I have not been able to find the S.V.

33. *Denticula* (?) *lævis*, n. sp. Pl. X., Figs. 33, 33 *b*, and 33 *c*. Form of F.V. linear, rectangular, with a small sharp prominence in the middle of each margin, and the apices slightly, but sharply, expanded. Occurs in chains of two or three, and also solitary. Length from 0·0016″ to 0·0027″; breadth 0·0006″. It is striated on each side to one-third of the width, indicating that the S.V. is convex. Striæ delicate, but distinct under a high power, about 48 in 0·001″. The general aspect is smooth, and the striation is only seen on very careful adjustment. There are two terminal nodules visible on the F.V. at each end, which are joined by lines bounding the striæ. The middle prominence of the margin of the F.V. seems to indicate a central nodule on the S.V., which view has not yet been observed.

This species is by no means rare in Lamlash Bay, and it occurs also in Loch Fine; but I have hitherto been unable to detect the S.V. I have referred it provisionally to Denticula, but with many doubts. It is probable that if we had the

S.V. we might find it to be a Diadesmis, that is, a catenated species, having naviculoid frustules. But I do not venture to name it on conjecture, and I only refer it, with the three preceding forms, to Denticula provisionally, in order that some name may be used in speaking of them. Indeed it is probable that the three preceding species may also prove to belong to Diadesmis. My present object is, not to determine their genus, for which I do not possess the necessary data, but only to point them out as well-marked species, for the researches of other naturalists.

34. *Denticula nana*, n. sp. Pl. X., fig. 34. Form of the F.V., which occurs in chains of two, three, four, and occasionally more, rectangular, expanding a little in the middle, and also at the apices, which are truncate. Length from 0·0005″ to 0·001″; breadth, in the shorter examples, 0·0003″ to 0·0004″, and less in the longer ones. Margin of F.V. faintly denticulate, from the ends of the striæ. S.V. obtusely rhombic, broad, with a raphe in the median line. Striæ rather fine, inclined.

This little form is tolerably frequent, both in Lamlash Bay and in Loch Fine. I think it is properly referred to *Denticula*, although it has some resemblance to some of the forms figured by foreign authors under the name of *Zygoceros*.

35. *Denticula minor*, n. sp. Pl. X., figs. 35, 35 *b*, 35 *c*, and 35 *d*. Form of F.V., which occurs in chains of from two to seven or eight, on the whole rectangular; sometimes exactly so, more frequently with an angular expansion at the apices, which become capitate and subtruncate, while the margin is convex in the middle. Length from 0·0005″ to 0·002″; breadth from 0·0002″ to 0·0006″. Margins of F.V. strongly denticulate. S.V. rhombic or rhombic-lanceolate, very narrow, with srong marginal costæ. Costæ 18 or 20 in 0·001″.

This form, which is very frequent in the Lamlash Bay dredgings, and also in one of those from Loch Fine, varies much both in size and shape, the F.V. being sometimes as short as the shortest *D. nana*, and very broad in proportion, bulging in the middle, and capitate, sometimes longer, rectangular, and broad; and most frequently longer, much narrower, and capitate. The S.V. is so narrow, that the frustule seldom lies on that side, so as to present it to the eye. It appears to belong distinctly to Denticula.

36. *Denticula distans*, n. sp. Pl. X., figs. 36 and 36 *b*. Form of F.V., which occurs in chains of from two to five or six, and also solitary, rectangular, rather broad; often convex on the sides, and with the ends a little expanded. Margin strongly denticulate. Length 0·0017″ to 0·0026″; breadth 0·0006″ to 0·0008″. S.V. rhombic or rhombic-lanceolate, broad; marked with very strong, distant, sharp, and marginal costæ. Terminal nodules large and conspicuous. No central nodule. Costæ about 10 in 0·001″. Valve thick and glassy.

This fine species is tolerably frequent, both in Lamlash Bay and in Loch Fine. There is a considerable resemblance between this and the preceding species, so

that *D. minor* almost looks like a miniature of *D. distans*; but on a close comparison, they are found to be totally distinct. *D. distans* often occurs shorter than the average length of *D. minor*, but it never loses its own characters, the strong, distant, glassy costæ, and the broad S.V. But the two forms are evidently allied species, and both seem to be true *Denticulæ*.

37. *Denticula staurophora*, n. sp. Pl. X., figs. 37, 37 *b*, and 37 *c*. Form of F.V., which occurs in chains of two, three, and sometimes more, rectangular, with coarse marginal striæ, which in the middle on each side, are interrupted by a blank space, bounded by diverging lines. Length from 0·001″ to 0·0038″; breadth 0·0005″ to 0·0008″, the shorter examples being the broadest. S.V. lanceolate, rather narrow, marked with coarse moniliform striæ, except in the middle, where there is a broad stauros, on each side of which is a line, curved and concave towards the extremities. Striæ 14 to 16 in 0·001″.

This striking form is not unfrequent either in Lamlash Bay or in Loch Fine. I have referred it to Denticula, but perhaps it ought to be referred to *Diadesmis*, or, if the stauros be considered an objection, to a new genus allied to *Diadesmis* as *Stauroneis* is to Navicula. But this point must be left for farther investigation.

38. *Denticula fulva*, n. sp. Pl. X., figs. 38 and 38 *b*. Form of F.V., which occurs in chains of two, three, and sometimes four, linear, rectangular, and slightly expanded at the apices, the margin marked with the ends of somewhat coarse striæ. Length from 0·0018″ to 0·004″; breadth 0·0005″. S.V. linear, narrow, broadest in the middle, and gradually contracting to long, narrow extremities, which are ultimately subcapitate and rounded. Striæ moniliform, somewhat coarse, leaving a raphe in the middle, and the two terminal knobs unstriated. No central nodule in the S.V.; but the two nodules seen at each end of the F.V. appear to form the unstriated knobs at each end of the S.V.

This well-marked species occurs with the three last, and is even more frequent than they are. The genus cannot be considered as determined with certainty.

39. *Denticula marina*, n. sp. Pl. X., figs. 39 and 39 *b*. Form of F.V. linear, rectangular, with the angles very slightly expanded, and the margin strongly denticulate. It occurs in chains of from 2 to 18 or 20, so that the filament seems to be tenacious. Length from 0·002″, to 0·008″ or 0·009″; breadth from 0·0003″ to 0·0005″. S.V. linear, expanded at the middle, and obtusely acuminate at the ends. Striæ very coarse, and very coarsely moniliform, about 10 in 0·001″. On each side of the median line each of the striæ is formed of only two granules, which are distant, and half of a third, which seems to be on the margin. The two or three central striæ on each side, having only one and a half granules, the inner granule of each being absent, there is a blank space round the centre; and there is a smaller blank at each apex. The F.V. here figured is of about the usual

length, the S.V. is one of the longest. The whole form has a pale, whitish aspect on the S.V., and the single valves are hyaline.

This very fine and conspicuous form is frequent in Lamlash Bay, and even abundant in one of the Loch Fine dredgings. It occurs scattered in all the dredgings without exception. It seems to be a true *Denticula*.

40. *Diadesmis* (?) *Williamsoni*, n. sp. Pl. X., figs. 40 and 40 *b*. Form of F.V. rectangular, narrow, with three angular expansions, one in the middle, and one at each end; so that there are two long, narrow elliptical spaces between the adjacent frustules, which occur, as in the last species, in chains of from two to twenty, and upwards. Margin of F.V. strongly denticulate. Length from 0·0016″ to 0·008″; breadth from 0·0004″ to 0·0005″. S.V. linear, narrow, straight, very slightly incurved in the middle, and acuminate at the ends. Striæ coarse, coarsely moniliform, but closely set, giving to the valve a black aspect; about 16 or 18 in 0·001″. Central and terminal nodules large, white, conspicuous.

This remarkable and conspicuous form occurs with the last, and is even more abundant, especially in two of the Loch Fine dredgings. It is found in all the materials. The F.V. has been described and figured by Professor SMITH, as having occurred sparingly in a dredging made by Mr G. BARLEE, off the coast of Skye, and detected by Professor WILLIAMSON. It was referred by Professor SMITH, but doubtfully, from the absence of the S.V., to *Himantidium*. The S.V. is particularly frequent in one of my Loch Fine dredgings, and certainly cannot belong to *Himantidium*. The nearest genus is *Diadesmis;* but I do not feel at all sure that it is the true one. The conspicuous nodules agree with it; but the aspect of the species is unlike that of any known *Diadesmis*. I therefore give it as such with a doubt, and am satisfied with having, in the meantime, ascertained that it is not a *Himantidium*. I would indicate one curious character, that the striæ, on the F.V. seem to pass across the intervals between the adjacent frustules. This seems also to have been observed by Professor SMITH, as it is represented in his figures of the F.V., which are accurate. I have also observed, that the shortest examples, which are sometimes so short as to be nearly square on the F.V. are not only broader on both views than the larger ones, but also devoid of the central expansion or undulation, seen on the F.V. In this state, they approach in form to the rectangular F.V. of *D. distans*, but are at once recognised by the closeness of the striæ, as well as by their moniliform character. In one dredging I find many long filaments, especially of the shorter frustules; while in another, treated precisely in the same way, the detached frustules, generally exhibiting the S.V., and much longer, are much more common than the chains.

41. *Meridion* (?) *marinum* (or *Gomphonema lineare* (?)), n. sp. Pl. X., figs. 41 and 41 *b*. Form of F.V., which occurs in chains of two, three, or four, both in entire apposition, or attached only by an angle at the broader end, cuneate and truncate at both ends, and narrow. It is marked with coarse marginal denticu-

lations. S.V. linear, narrow, broadest at a point above the middle, from which it becomes narrower both ways; the shorter half being rather broader than the other, and rounded at the apex. The longer and narrower half is also rounded, and very slightly expanded at the end. Striæ coarse, not reaching the median line, but leaving a somewhat broad raphe in the middle. Striæ about 16 in 0·001″; Length about 0·0015″; breadth of F.V., at larger end, 0·0004″, at smaller end, 0·0003″. Breadth of S.V. 0·0002″ at the broadest point.

This form resembles both a *Meridion* and a *Gomphonema*. The absence of a central nodule prevents me from referring it to the latter genus; and with reference to the former, the mode of attachment, as well as the form of the frustules, agree pretty well with it. But I have not seen more than four attached; so that it is still doubtful whether it forms a spiral filament, as the slightly cuneate frustules must tend to do. It occurs, by no means sparingly, both in Lamlash Bay and in Loch Fine. Future observations on the living form will decide the question of its generic position, but in the meantime it is a well-marked species.

42. *Pyxidicula cruciata*, Ehr. Pl. X., fig. 42. Form of V. cup-shaped, hemispherical, with hexagonal cells over the whole surface, and in one direction, a crest composed of square or irregular cells running round the hemisphere in a plane at right angles to that of the junction of the two valves. Cells large, and of uniform size. Diameter 0·0019″.

The form here figured is only a detached valve, the entire frustule not having yet occurred in these dredgings. But it agrees precisely with EHRENBERG's figure of the valve. The specific name is given on account of the arrangement of some of the cells, as seen in a view obtained by looking down on the frustule, at right angles to the junction, in the form of a broad rectangular cross. This cannot be seen in the view here figured. The species is very scarce as yet, having occurred very sparingly in Lamlash Bay. But it is interesting, as being one of the forms which EHRENBERG has figured from the Ægina Clay Marl already mentioned at p. 482, and from the deposit of Richmond, Virginia. I have placed it in this group, because I have reason to think that Pyxidicula is a catenate form, though we cannot expect to see this in fossil deposits. I would here refer to the beautiful new form, detected by Professor WALKER ARNOTT, and figured in the Appendix to this paper, which, in the frustules, is so closely allied to the present species, that it may prove to be actually *P. appendiculate* of EHRENBERG, a form which occurs along with *P. cruciata*. Professor W. ARNOTT's form is decidedly catenate.

43. *Orthosira angulata*, n. sp. Pl. X., figs. 43 and 43 *b*. Form of F.V., which occurs solitary, and in chains of from two to six, rectangular in the middle, acuminate and truncate at the ends. On the margin, which is often slightly incurved, are seen denticulations arising from the cells of the disc or S.V. Length from 0·0005″ to 0·0015″; breadth of F.V. from 0·0003″ to 0·00045″. Diameter of disc,

0·0005″ to 0·0015″. Disc cellulate; cells largest in the centre, becoming regularly smaller towards the margin, arranged in quincunx, but, from the diminishing size of the cells, in curve lines. It often happens that the cells in the two inner thirds of the disc are conspicuously seen, inclosed within an equilateral triangle, formed by three slightly curved lines of cells, beyond which the cells and lines of cells are so small and hyaline, as to become obscure. Such discs are convex, and seen on the convex side. Others, especially the largest, are much flatter, and, when properly focussed, exhibit the whole cellular structure plainly.

This species is very frequent, even abundant, in Professor ALLMAN'S dredging from Lamlash Bay; and it is found more or less frequently in all the others. Possibly the disc is identical with the *Coscinodiscus minor* of EHRENBERG or KÜTZING; that of Professor SMITH being a fresh-water form. But the present species is unquestionably an *Orthosira*, notwithstanding the resemblance of the disc to *Coscinodiscus*.

## GROUP IV.

### DISCS, INCLUDING CAMPYLODISCI.

The new forms belonging to this group are not, in the materials examined, very numerous, but they are very interesting, and most of them are very fine species.

| | |
|---|---|
| 44. Melosira or Coscinodiscus (?) n. sp. | 50. Eupodiscus subtilis, Ralfs, n. sp. |
| 45. Coscinodiscus nitidus, n. sp., G. | 51. Campylodiscus centralis, n. sp. |
| 46. ... punctulatus, n. sp., G. | 52. ... Ralfsii (?) Sm. |
| 47. ... concavus, Ehr., G. | 53. ... angularis, n. sp. |
| 48. ... umbonatus, n. sp. | 54. ... eximius, n. sp. |
| 49. ... centralis, Ehr., G. | 55. ... limbatus, Bréb. |

44. *Melosira* or *Coscinodiscus* (?) (?), n. sp. Pl. X., fig. 44. Diameter of disc 0·003″. It is marked by fine, sharp, radiate lines, which are very numerous. These lines are strongest near the margin, which has the form of a broad, thick, raised rim, within which the valve seems to sink and then to rise, the middle part being apparently somewhat convex.

This disc occurs not very unfrequently in one of the Loch Fine dredgings, and more sparingly in one from Lamlash Bay. As no view of it is to be seen except the disc, or S.V., I have not been able to determine its genus, although the disc looks more like that of a filamentous form than a Coscinodiscus.

45. *Coscinodiscus nitidus*, n. sp. Pl. X., fig. 45. Diameter of disc 0·0012″ to 0·0025″. Margin entire, transversely striated. Striæ of margin about 16 in 0·001″, traceable to some small distance beyond the marginal band towards the centre. Surface of disc marked with distant and irregularly radiate lines of rather large, round, distant cells or granules. The rays are distinctly marked to-

wards the margin, but somewhat confused towards the centre. Puncta or granules larger towards the centre than at the margin. Aspect of valve glassy, puncta nitescent, very much as in *Cocconeis nitida* (fig. 26).

This pretty disc was figured without a name, from an imperfect specimen, in my last paper on the Glenshira Sand (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 50). Having found it tolerably frequent in Lamlash Bay, I now figure a perfect example, which, provisionally, I refer to Coscinodiscus.

46. *Coscinodiscus punctulatus*, n. sp. Pl. X., fig. 46. Diameter of disc 0·003″ to 0·0036″. It is marked by very fine and obscure lines, which, near the margin, are traceable as rays, but which soon become fainter, and apparently wavy at the same time, as they proceed towards the centre. Over the whole surface are scattered, sparsely, small puncta, which, in a certain focus, appear as points of light.

This disc was also figured, but not named, in my last paper on the Glenshira Sand (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 48); but the specimen here figured is a better one, and shows more of the very obscure structure. It is impossible, in the present state of our knowledge of it, to refer it with certainty to any genus; but it may be a Coscinodiscus. It occurs in Lamlash Bay and in Loch Fine, but is not very frequent. It may possibly prove to be the end view, or the dissepiment, of a Melosira or an Orthosira.

47. *Coscinodiscus concavus*, Ehr. Pl. X., fig. 47. Diameter of disc 0·0025″ to 0·0043″. Margin entire, transversely striated. Striæ about 10 in 0·001″. Surface concave, covered with large, equal, hexagonal cells, arranged as in a honeycomb.

This beautiful disc occurs rather sparingly both in Lamlash Bay and in Loch Fine. It agrees exactly with one of Ehrenberg's figures of *C. concavus*. But in my last paper on the Glenshira Sand, I have figured another disc (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 52), which differs from the present form in having a punctum in the centre of each cell, and in the margin being formed of very large cells, divided by strong bars, which appear to project from the plane of the valve, and which also extend beyond the outer margin. This disc I at first suspected to belong to some of the Polycystineæ; but I afterwards found it figured by Ehrenberg as *C. concavus*. I confess that I cannot believe these two discs to be of one species; but that represented in fig. 47 seems to be a true Coscinodiscus; and as it is identical with one of Ehrenberg's examples, we may consider it as the true *C. concavus*, leaving the other for farther investigation. (I have recently found, in some of the Clyde dredgings the other disc, just alluded to as having been figured in my paper on the Glenshira Sand, and as being also named by Ehrenberg, *C. concavus*.)

48. *Coscinodiscus umbonatus*, n. sp. Pl. X., fig. 48. Diameter of disc about 0·0045″. Surface densely cellulate, having a broad, nearly flat, marginal zone,

the central portion being almost or quite hemispherical. It is so convex, that, when the marginal zone is in focus, the middle part appears as if full of air. Cells in lines, generally radiate, rather small, irregular in outline, and unequal in size to some extent. As the rays diverge from each other towards the margin, the space is often filled up by a bifurcation of the rays, which gives an aspect of irregularity to the markings.

This fine disc occurs in one of the Lamlash Bay dredgings, in which, however, it is very scarce indeed. We must hope that it will be found in greater abundance. The broad marginal zone or brim, and the very convex middle part, give to it a great resemblance in shape to a "wide awake" hat; but I have named it from its resemblance to a shield with a large boss in the centre.

49. *Coscinodiscus centralis*, Ehr. Pl. XI., fig. 49. Diameter of disc 0·004″ to 0·009″, or even 0·01″. Surface regularly cellulate. Cells rather small, hexagonal, equal, except at the centre, where there are three large oblong cells, meeting in a point; and between these, a little farther from the centre, three more cells, a little smaller. The valve is remarkably transparent, and from this, and the small size of the cells, it is apt to be overlooked, although, when accurately focussed, the cells are very distinct. It has frequently a yellow or straw colour, in balsam, under the ½-inch objective.

This very beautiful disc is by no means rare in the Glenshira Sand; but when I described that deposit, I was not well acquainted with the discs figured by EHRENBERG, and supposed it to be a form of *C. radiatus*, or else *C. concinnus*. It agrees exactly with EHRENBERG's figure of *C. centralis*. It occurs not unfrequently, both in Lamlash Bay and in Loch Fine, and it reaches occasionally a diameter exceeding that above mentioned.

50. *Eupodiscus subtilis*, n. sp. (Ralfs.) Pl. XI., fig. 50. Diameter of disc 0·0033″. Surface apparently convex, very hyaline, and very densely marked with fine lines, and indications of minute cells, of which the lines are probably composed. In the centre is a rather large circular spot, and the usual pseudo-nodule of the genus is placed close to the margin.

This disc was first noticed by Mr RALFS in the early part of this winter (1856–57). He distributed specimens as probably *Coscinodiscus concinnus;* but when Dr GREVILLE and myself came to examine it with object-glasses of high power and of superior quality, it was soon recognised as an Eupodiscus. This, so far as I know, was first done by Dr GREVILLE. I had frequently observed a disc like it, with very delicate and obscure markings, in some of my dredgings; but it was Dr GREVILLE also who first ascertained that these discs, which I had taken for granted were *Coscinodiscus concinnus*, were really identical with the disc of Mr RALFS. As the form occurs in these dredgings, therefore, though less abundantly than in Mr RALFS' gathering, I figure it here as an Eupodiscus.

51. *Campylodiscus centralis*, n. sp. Pl. XI., fig. 51. Form orbicular, or nearly so.

Surface marked with strong canaliculi, which are broad near the margin, narrower towards the centre, near which they terminate in a portion of the middle of the median line; in length about two-fifths of the diameter. At about one-third of the radius, or a little more, from the margin, is a strong shade, probably a ridge or elevation, of a nearly square outline, placed diagonally to the median line, which passes through two of its angles. Half-way from this ridge to the centre is a second ridge of a circular form, inclosing the elongated centre, from which the canaliculi arise. The radiating canaliculi are fainter within the square ridge, stronger outside of it; they diverge as they approach the margin, near which their ends are joined by semicircular loops, forming a scalloped inner margin, beyond which is an entire outer margin. Diameter about 0·0021″. Canaliculi about 40 in a disc of that size.

This species occurs very sparingly in one of the Loch Fine dredgings, and none of those I have seen are larger than the one figured. It is probable, however, that it may occur of greater dimensions, as the next species does.

52. *Campylodiscus Ralfsii*, Sm. (?). Pl. XI., fig. 52. Diameter 0·003″ to 0·0045″. Form orbicular in many instances, but with all the modifications of the genus. From two points in the median line, near its extremities, arise two lines, diverging in the middle, so as to leave a long, narrow, vacant space, the width of which varies from 0·0003″ to 0·0005″, according to the size of the valve. To these lines the canaliculi reach. These canaliculi are narrow, very short near the ends of the median line, longest in the middle, where they reach a length of from 0·0013″ to 0·0017″. Near the margin each expands into a small round head, and beyond the line of these heads, the margin is entire. Canaliculi in a disc such as is here figured, 64, curved as they approach the ends, having their concavity turned from the centre. The valve is much undulated.

This fine and conspicuous form first occurred to me in the Glenshira Sand, where it is scarce. I afterwards found it, still scarce, in Professor ALLMAN's Lamlash Bay dredging; and still more recently, I found it frequent in three of the Loch Fine dredgings. I have referred it to *C. Ralfsii*, Sm., although it is so much larger than the form figured by him, and although there are other differences. Thus in *C. Ralfsii*, Sm., the canaliculi reach the median line, and the row of heads or expansions lie some distance from the margin. But these differences cannot be regarded as specific. The smaller variety figured by SMITH occurs also in some of the dredgings and in the Glenshira Sand. But the larger form seems to be the typical one, and for that reason, chiefly, I have here figured it.

53. *Campylodiscus angularis*, n. sp. Pl. XI., fig. 53. Valve orbicular, with the usual modifications. Diameter of disc from 0·0025″ to 0·0039″, the smaller sizes being the most frequent. The canaliculi from 160 to 180 in large examples, such as the one here figured. They are very short at the ends of the median line, and inclose a broad blank space, which, in the median line, occupies the whole

diameter, but in the middle, in a line at right angles to this, extends to less than half the diameter. Hence the canaliculi form a broad marginal band, except near the ends of the median line, where it is narrow; and the vacant space is elliptical, suddenly contracted at the ends to narrow processes, which traverse the band of canaliculi at its narrowest part to the very margin. At the roots of these processes, the canaliculi suddenly recede from the median line. They are much inclined at the extremities; and the longest are from 0·0008″ to 0·015″ in length. The true median line is visible, and is very delicate; but there are no other markings visible on the vacant middle space.

The surface of the valve, both above and below, that is, near both ends of the median line, is suddenly bent back, so as to form an angle with the rest of the valve. On the surface thus bent, short lines appear, apparently between the canaliculi, which lines terminate abruptly. The greater part of the valve, and especially that part on which are the canaliculi, is convex, which causes the canaliculi to appear curved.

This species is frequent in one of the Loch Fine dredgings, and occurs more sparingly in two others. I have named it from the angular bending back of the valve.

54. *Campylodiscus eximius*, n. sp. Pl. XI., figs. 54 and 54 *b*. Form nearly orbicular, and sometimes nearly square, with the usual modifications. I have seen one specimen spiral, like *C. spiralis*. Diameter of disc from 0·004″ to 0·007″ or even 0·008″. Canaliculi strong, very numerous, about 150 in average specimens; confined to a marginal band, the width of which is generally about 0·0007″ or 0·0008″. The middle space is pale and hyaline, and at first appears blank; but on close inspection is seen to be covered with pretty large, very transparent, round granules. which are not arranged in any order, but indiscriminately scattered, like the points in shagreen. The median line is marked by a raphe, the ends of which, as of the middle space, do not, as in the preceding species, traverse the band of canaliculi. There is, at each end, a small point of the middle space which indents the marginal band, that being a very little narrower there than it is elsewhere.

This very fine and conspicuous form is very frequent in one Loch Fine dredging, and less so in two others. At first I thought it might be identical with *C. Hodgsonii*, Sm., of which one example is figured as large as this form. But in his description, Professor SMITH describes the middle space as marked by strong radiate lines, formed of large granules; and although I have not seen any of *C. Hodgsonii* of this large size, yet I find the small ones to agree exactly with this description. Now in *C. eximius*, not only is the valve in the middle not marked with strong radiate lines, but under a low power it appears at first sight blank; and the very transparent granules are in general quite irregularly disposed, and only in some instances show faint traces of a linear arrangement close to the marginal band. As far as I have been able to judge, I am satisfied that the small

*C. Hodgsonii* is not of the same species as *C. eximius*. And with regard to the large *C. Hodgsonii* of the Synopsis, not having seen it, I am unable to decide. But if it be, as the figure indicates, as strongly marked as the small one, it cannot be *C. eximius*. If, on the other hand, it should prove to be like *C. eximius*, then it is probable that it will be found to differ specifically from *C. Hogdsonii*.

55. *Campylodiscus limbatus*, Bréb. Pl. XI., fig. 55. Form orbicular. Diameter from 0·005″ to 0·008″. Canaliculi marginal, short, broad, transversely sulcate, so as to appear, on close inspection, almost moniliform. Within this marginal band is another fainter band (the outer one being strong and black), which looks almost like the reflection in a mirror of the first, except that the bars in it are more decidedly moniliform, or formed of more distinct granules, which, near the median line, extend from both ends towards the centre, in a broad band, which becomes gradually narrower and fainter as it proceeds, and is lost on both sides before reaching the centre. In De Brebisson's figure these projecting bands reach the centre, but I have not been able to trace them so far, and have only done so with difficulty, so far as I have traced them.

This very fine and striking form is not unfrequent in two of the Loch Fine dredgings. About four years ago, I observed a fragment of a large Campylodiscus in a marine gathering from Oban, which was not known to Professor Smith; but I did not describe it, having waited for an entire specimen, which I did not meet with till this last October, when it proved to have been a fragment of *C. limbatus*, which De Brebisson had shortly before figured as occurring at Cherbourg.

## GROUP V.

### Amphiprorae.

Of the genus *Amphiprora*, the new species in these dredgings, though not very numerous, are very interesting. One form is very doubtfully referred to this genus.

| | | |
|---|---|---|
| 56. | Amphiprora | pusilla, n. sp. |
| 57. | ... | plicata, n. sp. |
| 58. | ... | elegans, Sm. |
| 59. | ... | lepidoptera, n. sp., G. |
| 60. | Amphiprora | obtusa, n. sp. |
| 61. | ... | maxima, n. sp., G. |
| 62. | ... | (?) complexa, n. sp., G. |

56. *Amphiprora pusilla*, n. sp. Pl. XII., figs. 56 and 56 *b*. Form of the F.V. nearly rectangular, a little incurved in the middle, and expanded at the extremities. Above each valve lies a plate, in shape like a narrow arc of a very large circle, the convex edge outwards, the middle of it slightly overlapping the central constriction, while the ends coincide with the inner terminal angles of the valve. Length about 0·0027″; breadth, including plates, at the middle, 0·0008″. S.V. lanceolate,

narrow, and, but for the absence of apiculi, a miniature of that of *Apr. lepidoptera*. (See farther on, fig. 59 *b*.) Both views, and also the plates, are marked with very fine transverse and parallel striæ, which, however, on the F.V. do not extend to the rectangular space between the valves. Striæ about 60 in 0·001″.

This pretty little species has occurred only in one of the Loch Fine dredgings, where it is rather scarce. It is remarkable for the occurrence of the lateral plates, which we shall find in several other species, and which perhaps ought to constitute a new genus. But the resemblance to Amphiprora is so great in other points, that I do not think it advisable to separate these species from that genus.

57. *Amphiprora plicata*, n. sp. Pl. XII., fig. 57. F.V. deeply constricted, with rounded extremities, which are very broad. Inner margin of valves straight. On each valve lies a plate extending from the inner margin of the valve to the nodule; its outer or convex margin being a little incurved in the middle, and bent forward at the ends, to join the inner angles of the valve. The rectangular space joining the two valves is marked with faint vertical lines or folds; and the valves and lateral plates, as well as the rectangular space, are all marked by fine transverse striæ, about 50 in 0·001″. Length 0·0037″; breadth from nodule to nodule 0·00075″, at broadest part 0·0016″. Outer margin double. S.V. not yet certainly known.

This species occurs with the preceding, and is equally scarce. It approaches nearest to *Apr. alata*, but differs from it in the folds of the middle space, and in the presence of the lateral plates.

58. *Amphiprora elegans*, Sm. Pl. XII., figs. 58 and 58 *b*. Form of S.V. linear-lanceolate, ends obtuse. Valve traversed by two longitudinal lines, and marked by fine but distinct transverse striæ. Length from 0·0085″ to 0·012″; breadth of S.V. about 0·0011″. F.V. rectangular, with two longitudinal lines. Striæ 44 in 0·001″, parallel.

I have figured the S.V. of this fine form, first noticed by Mr Bleakley, because, although described in vol. ii. of the Synopsis, no figure of it has yet appeared. It occurs both in the Glenshira Sand, and in more than one of the dredgings; but more frequently in one of those from Loch Fine than in any of the others. The valves are, in these deposits, generally detached, so that I have not met with the entire F.V. I am indebted to Mr Roper for an entire specimen, fig. 58 *b*.

59. *Amphiprora lepidoptera*, n. sp. Pl. XII., figs. 59, 59 *b*, and 59 *c*. Form of FV. linear, constricted in the middle, expanded at the apices, which are flatly rounded. Length from 0·0055″ to 0·008″; breadth of F.V. in the middle 0·0009″, near the ends 0·0015″. SV. lanceolate, with acute ends, terminating in small apiculi. Central nodule strongly marked; median lines somewhat curved, meeting in the nodule. Striation fine, parallel, about 48 in 0·001″; on the F.V. the striæ are absent from the middle space. The form of the F.V. with its long slender alæ, is very elegant.

(Since this paper was read, I have ascertained that the peculiar plates above mentioned in *Apr. pusilla*, and in *Apr. plicata*, occur also in this species. I had overlooked them, from their being very narrow. They rise from the inner margin of the valves, as seen on the F.V., from two points a short distance from the extremities. Their convex margin extends in the middle just beyond the constriction. They are so narrow as to be readily overlooked, but are quite distinct, and appear to be thicker at the outer or convex margin than at the inner or plane one. There is even some appearance of a second plate in each valve, rising from the same line, as that just mentioned, but apparently extending in a plane at right angles to the surface of the valve, so that, in the F.V., it is seen foreshortened. and appears as a dark line. But I am not yet satisfied about this second plate.

The plates first named constitute a very peculiar feature, both in this species, and in the two previously described. We shall see the same structure still more developed in another species, a very remarkable one, namely, *Apr. maxima*.)

The form represented in fig. 59 *c*, I am satisfied belongs to this species; but I do not quite understand its relations. It is of an elegant rhombic-lanceolate form, with two curve-lines on each side, which at the middle bend inwards, to join a strong stauros, interrupting the median line, and at the ends coalesce with the margin. The whole valve is marked with fine parallel striæ, except the stauros. This form, as will be seen, differs considerably from the usual S.V., (fig. 59 *b*), which latter often occurs with the longitudinal lines in it much more curved than in the example figured. Can the lateral lines in fig. 59 *c* be the outer margins of the two plates, above described, or perhaps of those suspected to lie in a different plane?

This species first occurred to me in the Glenshira Sand, and I figured the S.V. as *Apr. vitrea*, β? in my first plate of that deposit (*Mic. Jour.*, vol. iii., pl. iv., fig. 14, the larger of the two figures, which shows the curved lines above alluded to, with a third line to the side); and the F.V. in my third plate (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 39). But the former had not the striæ, and the latter was from a very inferior specimen, and, besides, did not show the peculiar plates. I have therefore figured both views here from good specimens, which are frequent in Mr MILES's Corallina gathering from Corriegills, near Lamlash Bay. It is at once distinguished from *Apr. didyma* by its much more elegant form, and by being twice as long. The S.V., also, is quite different. I am informed by Dr GREVILLE that he finds this form abundantly, even predominating, in a dredging or gathering from nearly the same locality, which he made early last summer.

60. *Amphiprora obtusa*, n. sp. Pl. XII., fig. 60. Form of F.V. linear, broad, with rounded ends, slightly incurved in the middle. The termination of the middle space projects a little beyond the general curve of the apices. The inner margin of the striated part of the valve is gently and gracefully curved, forming two concave lines towards the middle, which meet in a point of the inner margin

of the valve, in a narrow, elongated nodule, which is in a line with the terminal nodules, also long and narrow. But close to the outer margin, at the constriction, is another large round nodule. The narrow portions, which lie between the curve-lines just mentioned and the inner margin of the valve, appear, like the middle space, to be unstriated; but perhaps this is because they lie in a different plane from the outer compartments. And even in the last, the striæ are too fine to be resolved under a power of 400.

This remarkable form occurs in one of the Loch Fine dredgings, where, however, it is very scarce. I have not yet recognised with certainty the S.V., but there are some forms which may prove to belong to it.

61. *Amphiprora maxima*, n. sp. Plate XII., figs. 61 and 61 *b*. Form of F.V. rectangular, very broad, but deeply constricted in the middle, and rounded on the ends. Length 0·0068″; greatest breadth 0·0028″ to 0·003″, breadth of frustule at constriction 0·0017″. Central nodules large, situated close to the constriction in the outer margin. Terminal nodules on the inner margin of valves, conspicuous. Over each valve lies a strong, broad plate, arcuate outwards, straight on its inner margin, which coincides with that of the valve. Outer margin of plate prominent in the middle, and on each side of this prominence slightly incurved; thick, overlapping the valve at the constriction. Breadth from the margin of one plate at this point to that of the other 0·0023″. Valves and plates transversely striated. Striæ distinct, parallel, about 36 in 0·001″, thicker and apparently coarser at the margin of the plate. S.V. lanceolate, ends acute, with a trace of a constriction just before the apices. Greatest breadth 0·00125″. Two strong lines, as in *Apr. lepidoptera*, proceed from the apices, on one side, and curve inwards to join a large nodule at one-third of the width. Round this nodule the striæ are curved, as if pushed outwards. Between the nodule and the nearest margin the striæ are very short, and they leave a large blank space, extending to the margin. Valve thick and very convex on the S.V. Query, Are the curve-lines on the S.V., the outer margin of the plates, seen on the F.V? From the convexity of the S.V. its striation is more conspicuous than that of the F.V.

This, which is one of the finest and most interesting forms described in this paper, first attracted my attention in the Glenshira Sand. But I could only find there halves of the F.V; and I postponed the description of it. In Professor ALLMAN's Lamlash Bay dredging I again saw these halves occasionally; but it was not till I examined, in October last, the Loch Fine dredgings, that I found, in one of them, besides a number of the same halves, some entire frustules; and finally, the S.V. which, from its comparative narrowness, is seldom presented; the frustule or detached valve lying on its broadest side. There is something very beautiful in the entire form; and in its structure it is peculiarly interesting, as presenting the peculiar lateral plates, already noticed in *Apr. pusilla*, *Apr. plicata*, and *Apr. lepidoptera*, and that in a high degree of development. The occurrence of

these plates in four species, all of which have the general aspect of *Amphiproræ*, naturally leads to the inquiry, whether this remarkable structure may not be found in all the species of the genus, or whether the forms in which it occurs ought not to form a new genus. I understand from Mr ROPER, that he has found this species in a marine gathering from the coast of Wales, or of the south of England.

62. *Amphiprora* (?) *complexa*, n. sp. Pl. XII., figs. 62; 62, *b*; 62, *c*; 62, *d*; and 62, *e*. Form elliptical, broad, with a constriction in the middle, and broadly rounded ends. The frustule is composed of two arcuate and constricted segments, which are broad, thick at the outer margin, thin at the inner margin, and placed opposite each other, with a narrow interval between them. Over the middle of these two lateral segments is placed a complex mass, formed of five or six segments, converging inwards and on the ends, like the segments of an orange or melon. The thick backs of these central segments, marked with transverse striæ, are alone seen in the entire frustule, and those of the outer segments approach near to the outer margins of the lateral or flat lying segments, leaving only part of the surface of the latter exposed. A convex line joins the convergent ends of the central segments

When the frustule, as often happens, falls asunder, a number of segments are found lying near each other. Some of these have no constriction, and no nodule; these I take to be the segments of the central mass. Those with nodules at the middle of their outer margin, at the constriction, seem to be the lateral segments.

Length of frustule 0·0035″ to 0·004″; breadth 0·0028″. Segments arcuate, broad; some with a nodule and constriction on the thick convex or outer margin, others without. Surface of segments finely striated; striæ about 45 in 0·01, delicate, radiating from the thin or inner margin, and curved near the ends of the segment. At the margin, there is a row of conspicuous puncta, about 1 for every 2 striæ. The backs of the central segments, when *in situ*, are striated, but exhibit neither nodule nor puncta.

It is with much doubt that I refer this very curious form to *Amphiprora*; which I should not do, were it not that the lateral plates in four species already described may be regarded as the rudiments of the complex central mass in this species. One of the three alluded to, *Ap. plicata*, has even longitudinal folds in the middle part.

It is, however probable, that the remarkable structure of this species may render necessary the establishment of a new genus, a step which I do not venture to take without farther inquiry.

This form, like the preceding one, first caught my eye in the Glenshira Sand, where I could find, however, only detached segments, and one half frustule. Not being able, from these, to understand the structure, and not, indeed, perceiving the

connection between the segments and the half frustule, which only showed the backs of the segments, I postponed it with other forms, all of which I have now been able to establish as species. I found the present species tolerably frequent in Mr MILES's Corallina gathering, in which detached segments are much commoner than the entire frustules.

We shall see, farther on, that a similar structure prevails in a numerous section of Amphoræ, a few of which were described in my last paper on the Glenshira Sand.

## GROUP VI.

### AMPHORÆ.

The new forms of this genus in these materials are very numerous, since, in addition to almost all those (ten in number) which I had described in the Glenshira Sand, they have yielded about 32 additional undescribed species. And as I have again to describe and figure four of the Glenshira species, which are now better known, there are in all, 36 species of Amphoræ to be described and figured. As the whole of the British species figured in the Synopsis of Professor SMITH amounted only to eight, it appears that the Glenshira Sand, and the Clyde and Loch Fine dredgings, or the latter alone, without the sand, have yielded a five-fold addition to the British forms of this genus. This may serve to show what stores of undescribed forms are yet to be found in our estuaries; for all these have been obtained from two localities, namely, Lamlash Bay and the upper part of Loch Fine, just below Inveraray.

The remarkable group of complex Amphoræ, to which I have lately directed attention, and of which the first known example was *A. costata*, Sm., though the peculiarity of its structure seems to have been overlooked, has now become so large (one-half of the species here described belonging to it) that it is necessary to subdivide the genus. I shall, therefore describe the Amphoræ in two sub-groups, viz., A. Simple, and B. Complex Amphoræ.

(Since this paper was read, I have ascertained that two of the Amphoræ in the following list of simple species, namely, *A. monilifera*, and *A. spectabilis*, belong to the complex division. The latter is indeed one of the most curious of the complex Amphoræ.)

A. *Simple Amphoræ.*

63. Amphora turgida, n. sp.
64. ... nana, n. sp.
65. ... macilenta, n. sp.
66. ... angusta, n. sp.
67. ... binodis, n. sp.
68. ... ventricosa, n. sp.
69. ... monilifera, n. sp., G
70. ... lineata, n. sp.
71. ... Ergadensis, n. sp.
72. ... lævissima, n. sp.
73. Amphora, pellucida, n. sp.
74. ... lævis, n. sp.
75. ... exigua, n. sp.
76. ... dubia, n. sp.
77. ... truncata, n. sp.
78. ... oblonga, n. sp.
79. ... robusta, n. sp.
80. ... spectabilis, n. sp.
81. ... Proteus, n. sp.

63. *Amphora turgida*, n. sp. Pl. XII., fig. 63. Form nearly orbicular, with short, square, produced apices. Detached valves, nearly semicircular, with the vertical margin straight and the extremities capitate. Nodule conspicuous. Length from 0·001″ to 000·2″; breadth 0·008″ to 0·0015″. Valve marked with somewhat coarse radiate striæ. Striæ 24 in 0·001″.

This little form occurs in Lamlash Bay, but it is more frequent in one of the Loch Fine dredgings, which, though stony and unpromising in aspect, is yet singularly rich in undescribed species, and especially in Amphoræ. The detached valves or halves of this species, which are like small and very convex *Cymbellæ*, are much more frequent than the entire frustule.

64. *Amphora nana*, n. sp. Pl. XII., fig. 64. Form a narrow, linear-elliptic. Inner lines lying very near the ventral margin of valves, and nearly straight. Nodules small; near the ventral margin. Rectangular middle space very narrow. Length 0·001″ to 0·0016″; breadth 0·0004″ to 0·0005″. Striæ about 50 in 0·001″.

This small species occurs with the last, and is tolerably frequent.

65. *Amphora macilenta*, n. sp. Pl. XII., fig. 65. Form elliptic, long and narrow, contracting towards the ends, which are again slightly expanded, and terminate so that it has very short, square, produced ends. Valves very slender, arcuate on dorsal margin, straight on ventral. Middle space narrow, with a strong median line. Length 0·0018″ to 0·0022″; breadth of entire frustule 0·0005″ to 0·00086″. Striæ parallel, rather coarse, about 30 in 0·001″.

This species occurs with the two preceding, and is not at all unfrequent.

66. *Amphora angusta*, n. sp. Pl. XII., fig. 66. Form nearly rectangular, or rather linear-elliptic, narrow, truncate at the ends, so as to form a slender and somewhat elegant barrel shape. The inner curve-lines arise in each half or valve from the outer angle, and meet in a nodule situated half-way across the valve. Both valves are transversely striated across, up to the narrow rectangular space in the middle. Length 0·0015″; breadth of entire frustule 0·0004″. Striæ fine, about 44 in 0·001″.

This species occurs with the preceding ones, chiefly in the Loch Fine dredging above specified, where it is not very rare.

67. *Amphora binodis*, n. sp. Pl. XII., fig. 67. Form linear, with rounded apices, and two expansions half-way between the middle and the ends, between which it is deeply constricted. Length from 0·00175″ to 0·002″; greatest breadth about 0·0005″; breadth of narrow parts 0·0003″. Rectangular middle space narrow. The detached halves or valves, having the ventral margin straight, and two large rounded expansions, while the ends are linear, much resemble the frustules of *Eunotia Camelus*, as described by Professor Smith in vol. ii. of his Synopsis, and figured by Dr Greville in the Annals of Natural History; but the expansions are smaller and more rounded than in *E. Camelus*. The inner lines

run nearly parallel to the outer margin of the valve, but are less curved, and generally obscure. Striæ transverse, obscure, about 30 in 0·001″.

This very curious little form occurs by no means unfrequently in Professor ALLMAN's dredging from Lamlash Bay, and more sparingly in the Loch Fine dredging alluded to under *A. turgida*. The resemblauce of the detached valves to *Eunotia Camelus* has been mentioned. The entire frustule also resembles amini ature of *A. angularis*, Greg., figured by me in my first paper on the Glenshira Sand. (See *Micr. Jour.*, vol. iii., pl. iv., fig. 6.) But besides the much smaller size of *A. binodis*, the expansions in *A. angularis* are more angular; indeed, the figure represents them less so than they are; and the striation of *A. angularis* is not only distinct, but very much coarser, so that the whole aspect of the two forms is different.

68. *Amphora ventricosa*, n. sp. Pl. XII., fig. 68 and 68 *b*. Form linear-lanceolate, with obtuse apices, more or less expanded in the middle. Valves long, very slender, arcuate, with acute ends, marked with somewhat coarse transverse striæ. Length from 0·0023″ to 0·0035″; greatest breadth at the expansion in the middle 0·005″ to 0·008″. Inner lines generally obscure, but in some cases well seen. They arise from the inner angle of the valve, pass rapidly across, and, as shown in fig. 68 *b*, sometimes extend beyond the outer margin; then suddenly bend inwards to meet the obscure nodule near the inner margin. Rectangular middle space narrow. Striæ strong, about 22 in 0·001″; conspicuous.

This pretty and interesting species occurs not unfrequently both in Professor ALLMAN's dredging from Lamlash Bay, and in the Loch Fine one so often alluded to above. The detached valves resemble a very long and slender Cymbella, as may be seen in fig. 68, in which the form of the valve is plainly seen on each side of the frustule. By focussing, a transverse bar, or elongated nodule, may be seen in the middle of the valve; but it is obscured by the striæ, when they are in focus.

69. *Amphora monilifera*, n. sp. Pl. XII., fig. 69. Form elliptic, slightly recurved at the apices, which form very short produced extremities. The recurved ends of the valves do not meet, and the space between them is filled up by a transverse curve-line. Valves arcuate, very convex on dorsal margin, with recurved ends. Nodules on the ventral margin. Between the valves the frustule is marked by three to five longitudinal rows of distant round granules, giving to it a dotted aspect. If there be transverse striæ, they are very obscure. Length from 0·0017″ to 0·0026″; breadth from 0·0008″ to 0·0011″.

This pretty and well-marked species is tolerably frequent in the two dredgings mentioned under the last form, but chiefly in that from Lamlash Bay. It occurs also very sparingly in the Glenshira Sand, as may be seen by referring to my last plate (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 53). I had figured, from that deposit, an imperfect specimen, which at first I took for an *Amphora*, but subse-

quently supposed to be possibly not Diatomaceous. The Lamlash Bay dredging soon cleared up this point, and established it as a distinct species of *Amphora*.

(Since the preceding description was written, I have ascertained that this species belongs to the complex group. Like the other complex species, it is formed of a number of Cymbelloid segments, arranged like the segments of a melon, as will be more particularly described farther on, in some of the other species. The convergent longitudinal lines of dots prove to be the dotted backs of the central segments. I have not been able to find a detached segment, the frustule being usually entire, so that I cannot give a figure of the segment. I can see, however, that it has the form of a Cymbella.)

70. *Amphora lineata*, n. sp. Pl. XII., fig. 70. Form elliptic or elliptic-lanceolate, with short, produced apices, which are truncate. The valves are slender, arcuate on the dorsal, straight on the ventral margin. The nodules lie in the middle point of two strong lines, slightly curved, within the ventral margin. Outside of the nodule is another curve-line, dividing the valve longitudinally into two parts, the outer one being the broader. This outer portion is marked by fine longitudinal lines, of which there are generally four in each valve. The rectangular middle space is narrow, and has a sharp line down the middle. This line, with the inner margins of the valves, the slightly curved line with the nodule, the line beyond it, and the four exterior lines, give to the whole a lineate aspect, which is very characteristic. In certain views, the frustule appears lineate uniformly from one side to the other. Length from 0·0022″ to 0·003″; breadth of frustule 0·0007″ to 0·008″. The whole frustule is marked with transverse parallel striæ, which are fine, about 42 in 0·001″, and obscure, except at the margins, where they may generally be distinctly seen. As the longitudinal lines, however, are much more conspicuous, it is they which characterize the species.

This species is not unfrequent in the dredgings so often mentioned, and occurs also in the Glenshira Sand. In my last paper on that deposit, I described and figured it (*Trans. Mic. Soc.*. vol. v., pl. i., fig. 33). But by some mistake of mine, the form there figured was either not a characteristic specimen, or, more probably, a form of *A. salina*. I have therefore described it anew, and given, in fig. 70, an accurate representation of a very frequent size and shape of this species.

71. *Amphora Ergadensis*,* n. sp. Pl. XII., fig. 71. Form elliptic-lanceolate, narrow, with truncate apices, which are very slightly expanded. Valves in apposition, or nearly so, long, slender; the inner lines, on which lie the conspicuous central nodules, forming one gentle curve from one end of each valve to the other. Length about 0·0035″; breadth in the middle 0·00075″. The valves are marked by strong and conspicuous transverse striæ, about 24 in 0·001″.

* From Ergadia, Argyll.

This form, like most of the preceding Amphoræ, occurs both in Lamlash Bay and in Loch Fine; but it is not so frequent as most of the others. It is conspicuous from its length.

72. *Amphora lævissima*, n. sp. Pl. XII., fig. 72. Form elliptic, rather narrow, ends subtruncate. Aspect remarkably hyaline. Length 0·0025″ to 0·003″; breadth 0·0009″. The inner lines in each valve rise from the inner angles, bend suddenly outwards for a short distance, then gently inward to the nodule, leaving a very narrow inner compartment at each end. The middle of each valve is traversed by a strong bar or elongated nodule. The valves are transversely striated, but the striæ are so fine, and the valves so hyaline, that the striation has not yet been perfectly resolved.

This delicate species is not very rare in the Loch Fine dredging noticed under *A. turgida* as rich in Amphoræ. But the detached valves are much more frequent than the entire form. These detached valves have a resemblance in form to the valve of *A. elegans*, figured in my last paper on the Glenshira Sand (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 30). But *A. elegans* is a considerably larger form, and not particularly hyaline; while its striæ are much less fine, and thus easily seen.

73. *Amphora pellucida*, n. sp. Pl. XII., figs. 73 and 73 *b*. Form a broad oval. Length from 0·002″ to 0·003″; breadth from 0·0012″ to 0·0018″. Valves arcuate, very convex on the dorsal, slightly concave on the ventral surface. There is something very peculiar in the aspect of the terminal parts of the inner margins of the valves, as these coincide with the terminal nodules, which, from the delicacy and transparency of every other part of the inner margin of the valve, appear to project like horns, as the nodules not only coincide with the ends of the inner margin, but are narrow and much elongated. The inner lines follow the margin for a little, then bend rapidly outwards, and then as rapidly inwards to the central nodule on the ventral margin. The ventral margin, except at the nodules, is so hyaline as to be seen only on close inspection, as are also the spaces lying between it and the inner curve-lines. The space outside of these latter curve-lines is striated, the striæ rather coarse; but the whole is so hyaline, that the striæ are only seen on very close inspection, when they come out plainly. They are somewhat inclined, thicker and more easily seen at the outer margin, very delicate and nearly invisible towards the inner one, and about 30 in 0·001″. There is a narrow, rectangular space between the two valves. Sometimes the form of the frustule becomes nearly rectangular, but all its other characters continue as before.

The form of the valves, as well as of the entire frustule, in this species, is somewhat similar to that of *A. ovalis*. But, not to dwell on the marine habitat of *A. pellucida*, its very hyaline aspect, and the singular delicacy of the striæ, which are entire, and, towards the nodule, become so fine as to be hardly visible, effectually

distinguish it from that species, in which the striæ are conspicuous and moniliform.

This species is not unfrequent in the Loch Fine dredging mentioned under *A. turgida*. It has some resemblance, in the form of the valves, to *A. incurva*, Greg. figured in my first paper on the Glenshira Sand (See *Mic. Jour.*, vol. iii., pl. iv. fig. 5). But in *A. incurva* the striation is conspicuous, not hyaline, and the form of the valve is more elongated, and less projecting at the extremities. I am not yet acquainted with the entire frustule of *A. incurva*.

74. *Amphora lævis*, n. sp. Pl. XII., figs. 74, 74 *b*, and 74 *c*. Form of frustule rectangular, slightly incurved in the middle, sometimes with the ends rounded, but more commonly with nearly square ends. Length from 0·0017″ to 0·003″; breadth from 0·0007″ to 0·0012″. Aspect very hyaline. In each valve the inner curve line, rising from the inner angle of the valve, and following the margin outwards, bends inwards again in a long graceful curve to the central nodule placed just within the ventral margin. The nodule extends as a strong bar across the middle of the valve. The outer compartments are transversely striated, but the striæ are very fine, about 60 in 0·001″, very hyaline, and hardly to be seen with a power of 400.

I have added, in fig. 74 *d*, the figure of a fine *Amphora*, which probably belongs to this species. It is remarkable for the fact, that the curve line coincides with the outer margin till very near the middle, where it bends inwards to the nodule. There are also four longitudinal lines at considerable and equal distances. In this specimen, I have not seen the striæ as in the others; but believing it to be of the same species, I give the figure under the head *A. lævis*.

This species occurs with the last, and is more frequent, though by no means abundant. It frequently happens, as in one of the figures, that the valves are in apposition; and as this occurs in long examples, these become proportionally narrow.

75. *Amphora exigua*, n. sp. Pl. XIII., fig. 75. Form linear-elliptic, narrow, with somewhat obtuse ends. Length from 0·0015″ to 0·0022″; greatest breadth about 0·00035″. The valves are transversely striated, the striæ being strongest at the margin. Striæ about 28 in 0·001″.

This little form occurs, scattered, both in Lamlash Bay and Loch Fine. In size and form it approaches nearest to *A. lineata* (Pl. XII. fig. 70); but its markings are totally different. It has no striking characters, but I cannot refer it to the same species with any of the small *Amphoræ* I have here described.

76. *Amphora dubia*, n. sp. Pl. XIII., fig. 76. Form of entire frustule oval, flattened, or even a little incurved, on the sides. The valves are concave in the middle of the ventral margin; so that as they are in apposition, there is a longish rhombic opening in the middle, between them. They also diverge a little at the apices. Within each valve is a faint line, nearly parallel to the outer margin,

but lying nearer to the inner margin. On these lines are faint nodules, while at the middle of the inner margin there are distinct nodules. Length 0·0025″; breadth 0·0011″. The whole is marked by transverse striæ, which also traverse the openings between the valves, in which openings a median line is also visible. Striæ fine, subdistant, about 24 in 0·001″. The divergent extremities are joined by a convex curve line.

This very peculiar species occurs, but very sparingly, in Lamlash Bay. It has some analogy with *A. marina*, a form lately figured by Professor SMITH, in the *Annals of Natural History* (January 1857, vol. xix., pl. i., fig. 2.) But besides the coarser striation and distinct nodules of *A. dubia*, Professor SMITH describes *A. marina* as so much resembling *A. affinis*, that it has been confounded with that species on our coasts. Now *A. dubia* has no resemblance whatever to *A. affinis*, nor indeed to any other known species; so that I have some doubts as to its being really an Amphora. The figures of *A. marina* given by Professor SMITH are not satisfactory, for they do not at all resemble *A. affinis*.

77. *Amphora truncata*, n. sp. Pl. XIII., fig. 77. Form of frustule slightly barrel-shaped, broad, with truncate ends. Length about 0·0028″; breadth about 0·0017″. Valves arcuate on dorsal, straight on ventral margin. Inner curve lines arise from the terminal margin, and bend gently inwards to a small nodule, rather nearer the inner than the outer margins. Inner margin of valve marked by a longitudinal line of short transverse striæ. In the rectangular space between the two valves, which is broad, are two similar longitudinal lines of short striæ, near to, and parallel to those just mentioned, and between the two last-named lines or bars are traces of others. Valves transversely striated, but the striæ are more conspicuous on a band at the outer margin, than elsewhere. So that the frustule appears, at least in a certain focus, marked with longitudinal striated bands, formed of short striæ.

This species is not very rare, in either of the two gatherings so often named in connection with Amphoræ. It frequently happens, that the line joining the ends of the valves appears to be interrupted in the middle. But by careful focussing it may be seen.

The appearance of longitudinal striated bands on the middle part of the frustule seems to indicate a tendency in this species to the complex structure. Indeed, among the complex species there is one, *A. quadrata*, which has considerable analogy with the present form.

78. *Amphora oblonga*, n. sp. Pl. XIII., figs. 78, and 78 *b*. Form linear elliptic, rather broad, the ends obtusely acuminate. Length from 0·0034″ to 0·004″: breadth from 0·001″ to 0·0014″. The inner curve lines are strong and much curved, but they keep unusually near the outer margin of the valve, only in the middle projecting rather more than half way across. The central nodules, which are conspicuous, are situated outside of the curve lines, and nearer to the outer

margin. Valves transversely striated. Striæ about 24 in 0·001″, conspicuous. The portion of the valve outside of the curve line seems to be in a plane different from that of the inner portions, and the striæ on the latter are radiate.

This well-marked and conspicuous species, remarkable for the position of the central nodules, is by no means rare in one of the Loch Fine dredgings, in which, except this form, *Campylodiscus angularis*, and *C. Ralfsii*, but few forms are found. It occurs more sparingly in some of the other dredgings.

79. *Amphora robusta*, n. sp. Pl. XIII., figs. 79, 79 *b*, and 79 *c*. Form of frustule a broad oval with subtruncate extremities. Length from 0·0033″ to 0·0048″, occasionally even as much as 0·006. Breadth from 0·0018″ to 0·0024″. Valves arcuate, with the ends more or less obtuse; ventral margin straight or slightly concave. Inner curve lines very sharp and strong, rise from the inner angles, pass outwards, without reaching the outer margin, and bend suddenly inwards to the central nodules, which are within the ventral margin, by one-fifth of the width of the valve. In some specimens, or perhaps in a certain focus, the points where the two curve lines meet in the middle is on the inner side of a straight line, apparently forming the inner margin of the valve, so that a small blank triangular space is included between that straight line and the ends of the two curve lines. This produces a very peculiar aspect. Frustule thick, marked with strong striæ, which on the compartments outside the curve lines are transverse, but on the inner terminal compartments are somewhat radiate. Striæ subdistant, moniliform, about 16 in 0·001″. Figs. 79 *b*, and 79 *c*, represent two valves, in one of which the curve lines are seen, while it is evident that the inner compartments are in a different plane from the outer one. The other shows the entire valve viewed in a different focus, in which the striæ appear all in one plane.

This fine form, conspicuous for its size and the stoutness of its aspect, is not rare in the Loch Fine dredging mentioned under *A. turgida*, and occurs also in Lamlash Bay.

80. *Amphora spectabilis*, n. sp. Pl. XIII., figs. 80, 80 *b*, and 80 *c*. Form nearly rectangular, broad, with rounded angles; occasionally sub-elliptic. Length from 0·003″ to 0·0047″; breadth from 0·002″ to 0·0025″. The inner curve lines bend inwards from the outer margin very nearly to the inner margin of the valve, dividing the valve into a middle outer compartment, and two terminal inner ones. The detached valve has prominent obtuse rostra or beaks, not seen in the entire form. Outer compartment transversely striated, the striation being very coarse; inner compartments marked with radiate striæ, which are much finer. Aspect of the whole form soft and indistinct, so that in general only the marginal ends of the striæ in the outer compartments are easily seen, these striæ being thicker at that end, and the frustule very convex. Even at the margin they are not sharp but softened. Striæ in the outer compartment 14 to 16 in 0·001″: in the inner ones, 26 in 0·001″, and very obscure. As in the last species, and indeed in many Am-

phoræ, the inner compartments seem to be in a different plane from the outer ones. This species exhibits two frequent varieties, both smaller than the typical form, which are figured (figs. 80 *b*, and 80 *c*). One is long and narrow, its length being 0·0032″, its breadth hardly 0·0008″. The other is short and broad; its length being 0·002″, and its breadth 0·001″. Both have the soft, hazy, indistinct aspect of the larger form, and both, when carefully examined, exhibit the same characters, except that the striation is perhaps somewhat finer.

(Since the above was written, I have found that this species not only belongs to the complex group, but is one of the most interesting forms in that group. As will be seen in the descriptions of various complex *Amphoræ*, the complex structure is only seen in one focus, while in another, the frustule exhibits the characters of a simple *Amphora*. This is peculiarly the case in *A. spectabilis*. The simple view, shown in fig. 80, is so thoroughly simple, that it never occurred to me that it could possibly conceal a second and more complex structure. But when I happened to examine, under a rather high power, and with oblique light, a frustule well placed for bringing out this structure, I detected the appearance shown in fig. 80 *d*. In this case, the coarse transverse striæ of the middle part, as seen in fig. 80, were still traceable, at the margin only, on one side, because the frustule did not lie quite flat. In other examples, no such traces are visible on the complex view, although in general the very strong and elongated central nodules shine through. In fig. 80 *d*, it will be seen that the whole frustule is composed of parallel, longitudinal, very slightly convergent bars, with a narrow sulcus between each two bars. These bars are transversely striated, and the striæ, though much finer than even those of the inner and terminal compartments of the valves, are yet quite distinctly seen, even more so than the others, being apparently free from the haziness above alluded to. Striæ of the bars, in this view, about 50 in 0·001″.

In this remarkable form we have the unusual occurrence of three distinct systems of striation; 1*st*, The coarse, soft, and hazy transverse striæ of the outer and middle compartments of each valve, on the simple view, fig. 80; 2*dly*, On the same view, the finer and radiate striæ of the two terminal inner compartments of each valve, which lie in a plane inclined to that of the middle compartments; and, 3*dly*, The still finer and parallel striæ crossing the longitudinal bands, in the complex view, fig. 80 *d*, which lie again in a different plane. The frustule is also remarkable for its convexity, which is probably the cause, or one chief cause, of the indistinctness of the markings on the simple view.)

This beautiful species is tolerably frequent in Professor ALLMAN's dredging from Lamlash Bay, and in one from Loch Fine, less so in the other dredgings. The short variety is even more frequent than the typical form. The species has some resemblance to the preceding one in general form and coarseness of striation. But it is at once recognised by its hazy indistinct aspect, by the finer

striation on the inner compartments, by the beaked form of the detached valves, and finally, by its varieties. (Its complex structure, and its three systems of striæ, are even more decisive characters.)

81. *Amphora Proteus*, n. sp., Pl. XIII., figs. 81, 81 *b*; 81 *c*; 81 *d*; and 81 *e*. Form very variable, obtuse lanceolate, elliptical, barrel-shaped, broad and truncate, long and narrow, &c. It has usually the rectangular space between the valves, but sometimes the valves are in apposition, and then resemble twin frustules of *Cymbella*. Some of these modifications are figured. The size also varies prodigiously. Length from 0·0015″ to 0·006″; breadth from 0·0013 to 0·0024″. The broadest examples are generally short. Valves acute, sometimes with arcuate, at other times with obtuse apices. Inner curve lines and nodules strongly marked, and inner compartments of the valve in a different plane from the outer ones. There are two peculiarities which are found in all specimens, from the smallest to the largest. When the outer compartment is in focus, and its striæ conspicuous, the striæ of the inner compartments appear in narrow lines or bars, separated by white longitudinal lines or *raphes*; and the transverse striæ, which are finely moniliform, are, especially, in a certain focus, traversed by longitudinal wavy lines or striæ, produced by the circumstance that the granules of contiguous transverse striæ are not placed exactly opposite each other. In figs. 81 and 81 *b*, the same specimen, a long one, is shown as it is seen in two different foci, one of which brings out the curve lines and nodules, the other the transverse striæ, which extend across the whole valve. These striæ are about 22 in 0·001″; but in regard to the number of striæ there are very great variations in this species, as I have shown, in former papers, to occur in other species. In some of the smaller specimens the striæ are at least twice as numerous as in some of the larger, and even in the specimens of equal size they differ in this respect. But in all, the striæ exhibit the characters I have mentioned as peculiar and characteristic.

(I must here state, however, that there are some forms which, for the present, I include under *A. Proteus*, respecting which I have great doubts whether they ought not to form a distinct species. These forms have many characters in common with *A. robusta*, but have uniformly a much finer striation, and consequently a very different aspect.)

This species is very frequent in Lamlash Bay, and also in some of the Loch Fine dredgings. At first I was quite at a loss with the multitude of forms agreeing in striation, but when I had observed the characters above mentioned, I was able to trace all these forms into one another by gradual transition. Those here figured are some of them very different; but intermediate forms occur. One of the figures represents two valves in apposition, which I suspect to belong to this species (fig 80, *e*); but I am not quite certain about that one.

I may here direct attention to the fact, that such a form as that shown in fig. 81 *e*, resembles closely a twin frustule of a Cymbella or Cocconema. Yet it is con-

sidered as only a single frustule, while the Cymbella or Cocconema would be called a double one.

I confess I feel disposed to consider both in the same light, as double forms. If that be correct, then what is usually called the valve of an Amphora will be an entire frustule, and what is usually called the entire frustule (as in figs. 63 to 72, for example) will be considered as two frustules in the act of self-division, but still united by the connecting membrane.

One reason why I incline to this view is, that what, on this principle, are to be called single frustules, which are now regarded as halves or single valves, are often much more frequent than the entire, or, as I should say, double frustules. We never see the halves of Naviculæ separated in this way. *Secondly*, Each single valve, or as I should say frustule, has three nodules; but the entire or double frustules have six. *Thirdly*, In Amphoræ, the two halves of the double or entire frustule are almost always separated by a rectangular space, apparently homologous with the connecting membrane, which is seen in other genera when in a state of self-division. When this is absent, as in fig. 81 *e*, or in fig. 76, the form has exactly the appearance of a twin frustule of Cymbella. *Lastly*, If we regard the so-called valves as entire frustules, they become perfectly analogous, in general form and structure, to Cymbellæ or Cocconemata; and we shall find, in the remarkable group of Complex Amphoræ, next to be described, that the segments in many species, indeed in most, have a still greater resemblance to Cymbellæ or Cocconemata.

Let us now consider that group.

B. *Complex Amphoræ.*

Of this sub-group, as I have already stated, there are, in these materials, as many as of the simple group just described. These species are as follows:—

| | | | |
|---|---|---|---|
| 82. | Amphora lyrata, n. sp. | 92. | Amphora sulcata, Breb. |
| 83. | ... Milesiana, n. sp. | 93. | ... acuta, n. sp. |
| 84. | ... elongata, n. sp. | 94. | ... crassa, Greg. |
| 85. | ... quadrata, n. sp. | 95. | ... pusilla, n. sp. |
| 86. | ... excisa, n. sp. | 96. | ... granulata, n. sp. |
| 87. | ... nobilis, n. sp., G. | 97. | ... cymbifera, n. sp. |
| 88. | ... Arcus, Greg. | 98. | ... proboscidea, n. sp. |
| 89. | ... Grevilliana, Greg. | 99. | ... costata, Sm. |
| 90. | ... complexa, n. sp. | 100. | ... bacillaris, n. sp. |
| 91. | ... fasciata, n. sp. | | |

The first species of this group which was figured was *A. costata*, Sm. But the remarkable peculiarity of its structure was not specially noticed. I subsequently noticed several species in the Glenshira Sand, in which the complex structure had attracted my attention, such as *A. Arcus*, *A. Grevilliana*, and *A. crassa*, which I described and figured. I also pointed out that the same structure occurs in *A. costata*, Sm. In the new materials, besides acquiring a more accurate knowledge of the three new species just named, all of which I here figure a second

time, I have found a considerable number of new species, exhibiting the same peculiarities of structure.

The general character of all these species is, that the frustule is very convex, and formed, in the first place, of two segments or valves, in shape like Cymbellæ or the halves of simple Amphoræ, placed opposite each other, as in the simple species. The middle space between these is, in the next place, covered, to a greater or less extent, by a complex mass, like that seen in *Amphiprora complexa*, (fig. 61), and exhibiting, when the two lateral segments lie flat, a series of convergent longitudinal bars, which are the backs of other segments, grouped like those of an orange or a melon. The lateral segments exhibit all the characters of the halves of a simple Amphora; they have the inner curve lines and the nodules, in some cases elongated so as to form a cross bar. The segments forming the central mass appear to have neither curve lines, nor nodules, nor bars. The lateral segments are transversely striated, and the bars of the central mass, or backs of the central segments, are frequently also marked by transverse striæ; but in other cases have much coarser markings, approaching more to the nature of puncta or granules. In some species again, while the margins of the lateral segments are very coarsely marked, the backs of the middle segments are much more finely striated. In general, when the frustule lies so, that the two lateral segments are flat or nearly so, we see in one focus the lateral segments with their curve lines and nodules, while in another focus these become invisible, and the convergent bars alone are seen, which in many cases fill up the whole space between the lateral segments, of which only a part, that nearest the outer margin, can then be seen. This is shown in several of the figures.

This structure is so peculiar, that it seems as if it would be desirable to establish a new genus for the reception of these forms. But the form of the segments is so exactly that of the valves (or frustules) of simple Amphoræ, and even the entire complex forms, in a certain focus, are so like the others, that I think it will be sufficient to make a sub-genus for the complex forms.

I have already mentioned that some of these forms occur in the Glenshira Sand; others, as well as these, are found in the dredgings from Lamlash Bay. But by far the larger part of the complex species here described are to be found in that one Loch Fine dredging spoken of under *A. turgida*, as being rich in *Amphoræ*. Of the above list, thirteen species at least, along with *A. costata*, besides fourteen or fifteen of the simple group, occur in this one dredging, and several of these are found in it alone. It will be unnecessary to repeat this in detail for each species; and I shall briefly refer to Loch Fine as the locality of the species here alluded to as occurring in that particular dredging.

82. *Amphora lyrata*, n. sp. Pl. XIII., fig. 82. Form doubly lyrate, with truncate ends, and a notch in the middle, on each side, where the two lyrate halves meet base to base. The lateral segments have each a strong bar or elon-

gated nodule in the middle. Bars between the lateral segments four or five. Length 0·0011″; breadth 0·00075″. The whole form is transversely striated; striæ distinct, about 36 in 0·001″.

This pretty little form is not very rare in the Loch Fine dredging above mentioned, as containing most of the complex *Amphoræ*. I have seen it also in Lamlash Bay.

83. *Amphora Milesiana*, n. sp. Pl. XIII., fig. 83. Form nearly rectangular, with a constriction in the middle, and subtruncate ends. The sides are very slightly convex. There are several longitudinal bars between the lateral segments. Length 0·0023″; breadth 0·001″. The whole is transversely striated; striæ about 28 in 0·001″, conspicuous.

This species occurs with the last, both in Loch Fine and in Lamlash Bay.

84. *Amphora elongata*, n. sp. Pl. XIII., fig. 84. Form elliptic lanceolate; long, narrow, with truncate extremities. Length 0·0044″; breadth 0·0011″; Lateral segments very narrow; curve lines very near the outer margin. There are six bars in the middle, convergent on the ends. Striæ conspicuous, transverse, about 26 in 0·001″.

This species occurs both in Lamlash Bay and in Loch Fine. It is not frequent, nor yet very scarce.

85. *Amphora quadrata*, n. sp. Pl. XIII., fig. 85. Form nearly rectangular; the sides slightly convex, broad, ends truncated, as in *A. truncata*. Length about 0·0027″; breadth about 0·0018″. Lateral segments arcuate on the dorsal, straight on the ventral margin. Between them are about six broad vertical striated bars, which are not, as in the preceding forms, in apposition, but which are somewhat distant, being separated by narrow *raphes*. The structure, therefore, differs in some respects from that of the other complex species. The lateral segments and the vertical bars are transversely striated; striæ about 34 in 0·001″.

This species occurs both in Lamlash Bay and in Loch Fine, and is not very rare.

86. *Amphora excisa*, n. sp. Pl. XIII., fig. 86. Form rectangular. The curve line in each lateral segment keeps near the outer margin, except in the middle, where it bends inward to a nodule; but outside of this nodule is another, larger and more conspicuous, situated on the outer margin, which appears deeply notched at this point. Length 0·0025″ to 0·004″; breadth 0·0015″ to 0·0018″. There are a number of longitudinal bars, convergent at the ends. The whole form is hyaline, and marked with very fine transverse striæ, which are best seen, though with difficulty, at the outer margin. Striæ about 52 in 0·001″.

This fine species occurs in Lamlash Bay, where it is not unfrequent. It is also found more sparingly in Loch Fine.

87. *Amphora nobilis*, n. sp. Pl. XIII., fig. 87. Form barrel-shaped, broad, ends truncate. Length from 0·003″ to 0·0045″; breadth, 0·0013 to 0·0028″. La-

teral segments arcuate, ends acute, inner curve-lines strongly curved, nodule elongated into a transverse bar. The converging bars in the middle are numerous, in apposition, and the whole frustule very hyaline. It is marked by fine transverse striæ, easily seen on the lateral segments, and which may also be traced, by careful manipulation, across the whole frustule. Striæ about 40 in 0·001″.

I first observed this fine and conspicuous form in the Glenshira Sand, but postponed the description of it that I might examine it farther. I have found it more frequently in Lamlash Bay and in Loch Fine.

88. *Amphora Arcus*, Greg. Pl. XIII., fig. 88. Form of entire frustule barrel-shaped, ends truncate. Segments arcuate, sub-apiculate, marked by coarse moniliform striæ, which are also seen over the entire form. Length from 0·0035″ to 0·0045″; breadth about 0·002.″ Striæ 16 or 18 in 0·001″. The longitudinal bars in the middle between the lateral segments are about 16 or 17, closely set, and we can see that they are segments seen edgewise.

This fine form was rather frequent in the Glenshira Sand, but was only then known in the shape of detached segments, two of which I figured in my first paper on that deposit (*Mic. Jour.*, vol. iii., pl. iv., fig. 4), without at that time understanding its real structure. Subsequently I recognised the entire frustule in the Glenshira Sand, and also in the dredgings from Lamlash Bay. In my third paper I figured an imperfect specimen (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 37), and I now give a more perfect one, which shows that it really belongs to the complex group. See fig. 88.

89. *Amphora Grevilliana*, Greg. Pl. XIII., fig. 89. Form of entire frustule nearly oval, broad, slightly truncate at the extremities, which are broad, the line joining the apices of the lateral segments being concave, so that a short process stands out at each side. Length about 0·006″; breadth about 0·002″. In the specimen figured (fig. 89) the form of the lateral segments is perfectly seen, probably because the frustule is viewed from the flat side, as we view an orange cut in half from the cut side. These segments are precisely like that figured in my last plate from the Glenshira Sand (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 36*). They are broad, arcuate, with somewhat acute rostra, and three curve-lines on their surface, which are peculiar to this species. They are marked with strong transverse moniliform striæ. Striæ about 28 in 0·001″. Between the lateral segments are five or six convergent bars, which are the backs of other segments, and which are in apposition, except at the ends, where they diverge a little, from their apices becoming suddenly narrower.

This fine species occurs both in the Glenshira Sand and Lamlash Bay. But the entire frustule given under this name in the plate just referred to (at fig. 36) does not belong to this species, as may be seen at once by comparing it with fig. 89, and by the form of its segments, as seen in that figure. The segment in that plate, fig. 36*, is correct, as above stated.

The form figured in my last plate, at fig. 36, as the entire *A. Grevilliana*, is a new species, to which I now proceed.

90. *Amphora fasciata*, n. sp. Pl. XIII., fig. 91 ~~90.~~ Form of entire frustule barrel-shaped, that is, gently convex on the sides, broadly truncate on the ends. The form of the lateral segments, as well as that of the inner curve-lines, is well seen at the sides. Length about 0·005″; greatest breadth about 0·0023″. Between the lateral segments are seven or eight converging bars, separated by very narrow raphes; these are the backs of other segments. The detached segments are arcuate on the dorsal margin, nearly straight on the ventral margin, with acute ends. They have only one curve-line, bending forward in the middle to the nodule, which is near the ventral margin. Striæ about 34 in 0·001″, conspicuous.

This fine and conspicuous form is not unfrequent in the Glenshira Sand, and occurs also sparingly in Lamlash Bay and in Loch Fine. Unlike the preceding one, it is almost always entire.

91. *Amphora complexa*, n. sp. Pl. XIII., fig. 90 ~~91.~~ Form of entire frustule rectangular, with rounded extremities. Length about 0·005″; breadth about 0·0021.″ Lateral segments narrow, with the dorsal margin straight, except near the apices, where it bends inwards. Inner line strongly curved to the nodule, which lies close to the ventral margin. Between the lateral segments are seven longitudinal converging bars, the backs of other segments, which meet on the convex ends, entirely filling them up. These bars are separated by raphes, rather broader than in the preceding species. The whole form is marked by transverse striæ, which are strong and conspicuous; about 30 in 0·001″.

This very fine and conspicuous form, which is evidently nearly related to the two which precede it, *A. Grevilliana* and *A. fasciata*, while yet all these are distinct, as is seen by the figures, occurs, like the others, both in the Glenshira Sand and in Lamlash Bay, and is not very rare. For some time I confounded all these together in the Glenshira Sand, and it was only a careful examination, especially of the detached segments, which showed me that they were really different species.

92. *Amphora sulcata*, Bréb., n. sp. Pl. XIII., figs. 92 and 92 *b*. Form barrel-shaped, rather broad, ends truncate. Length of frustule 0·0041″; breadth 0·002″. Lateral segments broad, arcuate; inner lines strongly curved, nodule near ventral margin. Between the lateral segments—when viewed in a focus which brings out these segments clearly, as in one of the figures—are seen four or five converging bars. These bars, and all the lines of the lateral segments, are marked by short, transverse striæ, the rest of the surface being hyaline. In another focus, the lateral segments and their curve-lines disappear, and the whole width is taken up by seven or eight converging bars, which are now separated by very narrow lines of transverse striæ. The whole form is very hyaline and very convex. It

is transversely striated; but the striæ cannot be traced throughout without altering the focus. Striæ about 38 in 0·001″.

This remarkable form was first described by De Brebisson as occurring near Cherbourg. No form is better adapted to show the structure of a complex Amphora, on account of its transparency, and the breadth of the convergent bars. I have found it in Lamlash Bay, where it is not very frequent; and also sparingly in Loch Fine. No detached segments have yet occurred.

93. *Amphora acuta*, n. sp, Pl. XIV., figs. 93 and 93 *b*. Form of detached segment arcuate, dorsal margin convex; in some examples with a slight inflection just before the extremities; ventral margin straight, or slightly concave, ends acute. The inner lines are very near the ventral margin, and almost exactly parallel to it, but sometimes a little incurved, except in the middle, where the nodule meets them. Nodule elongated into a strong transverse bar. Length of segment 0·0035″ to 0·0055″; breadth of it 0·00075″. The segments are marked by transverse striæ, about 36 in 0·001″, which are distinctly moniliform.

I have not yet seen the entire frustule; but it is no doubt complex, for I found a good many half-frustules, as it were, formed of segments lying one over the other, to the extent of five or six. Sometimes no cross bar is seen, probably because the cross bar is only found on the lateral segments, which may have become detached, and left the middle ones by themselves. One figure shows a group or pack of segments.

This species occurs in Lamlash Bay, but is more frequent in Loch Fine. It is probable that the entire frustule will somewhat resemble *A. nobilis* in form, but not in its hyaline aspect. On comparing the segments of *A. acuta* with those of *A. nobilis*, as seen in the entire frustule, the curve lines in the latter are seen to be very deeply curved, and to be much farther from the inner margin of the segment, whereas the inner lines in the segments of *A. acuta* are straight, or very nearly so, and close to the ventral margin. In *A. acuta*, the striæ, though not coarse, are strongly moniliform, while the striæ of *A. nobilis* are much finer. Yet it is probable that these two species are related.

(I have very recently observed two specimens, apparently of *A. acuta*, in which two segments are placed opposite, and close to each other. I suppose this view to represent the flat side of the frustule, or the half frustule, like a cut orange, as in the figure of *A. Grevilliana*, fig. 89. But in these specimens of *A. acuta*, the two lateral segments are in apposition.)

94. *Amphora crassa*, Greg. Pl. XIV., figs. 94, 94 *b*, 94 *c*, 94 *d*. Form of frustule rectangular, broad, with rounded ends. Length from 0·0025″ to 0·004″; breadth from 0·0008″ to 0·0013″. Lateral segments linear, from 0·0005″ to 0·00075″ in breadth, straight, or very slightly incurved on the dorsal margin, which, at the apices, bends inwards, forming short, rounded beaks. Sometimes, as in one

of the figures, the dorsal margin is convex. Ventral margin of segments undulated. The inner curve lines arise from the point of the beaks, run a little outwards, then inwards to the nodule, placed very near the ventral margin. Markings entire, coarse, subdistant. Striæ about 12 in 0·001″. Between the lateral segments are from five to eight convergent bars, marked with the same subdistant, entire striæ. In one focus, not here figured, nothing is seen but bars from one side to the other, which are thus eight or nine in number.

This very pretty and interesting species occurs in the Glenshira Sand, but the figure given of it in my last plate (*Trans. Mic. Soc.*, vol. v., pl. i., fig. 35), is not, at all events, the usual form. In that figure the markings are too close, and it may possibly represent a different species. Indeed, I have some reason to think that there are two species which resemble each other in several points. But I have not yet ascertained this to be the case. I have found the form now figured, which is the true *A. crassa*, more frequent in Lamlash Bay, where, also, I have observed the detached segments, previously unknown to me, and which, as may be seen, are very peculiar. It occurs also in Loch Fine, though less frequently.

95. *Amphora pusilla*, n. sp. Pl. XIV., figs. 95 and 95 *b*. Form linear, with rounded ends. Length from 0·0014″ to 0·0021″; breadth 0·0004″ to 0·0006″. Lateral segments very narrow, dorsal margin straight, except at the ends, nodule not far from dorsal margin, inner curve line also near it; striation conspicuous. Between the lateral segments are five or six narrow bars, separated by very fine sharp lines, and marked by subdistant granules or very short striæ. Striæ about 24 in 0·001″, very strong at the margins of the frustule.

This little form is not very rare in the Loch Fine dredging, so often referred to; and occurs also, though more sparingly, in Lamlash Bay. In general aspect, it resembles a delicate miniature copy of the preceding species; but the form of the segments, the curve lines, and the striation, are all totally distinct.

96. *Amphora granulata*, n. sp. Pl. XIV., figs. 96; 96 *b*; 96 *c*; 96 *d*; 96 *e*; and 96 *f*. Form of frustule linear, broad, with slightly convex sides, and truncate extremities. Length from 0·0017″ to 0·003″; breadth 0·0008″ to 0·0013″. Lateral segments slightly arcuate on the dorsal margin, which is suddenly narrowed at the ends; ventral margin quite straight, or slightly concave, apices sub-acute. Segments marked with fine transverse striæ, which are parallel, and from 24 to 36 in 0·001″. Fig. 96 *f*, represents a detached segment. When the frustule is so focussed that the lateral segments are distinctly seen, and their striæ plainly resolved, the space between appears nearly blank. But in another focus, the whole frustule is seen to be made up of about twelve longitudinal bars convergent on the ends, the backs of which are marked by subdistant granules, from 14 to 18 in 0·001″. Hence the name. I have figured these two views of each of two frustules, one short and broad; the other longer and narrower. One figure, fig. 95 *e*, represents a somewhat larger form, which also exhibits granulated bars,

and lateral segments approaching in form to those of *A. granulata*. But on careful comparison, it seems very doubtful whether this form do not belong to a different species. As I have not yet had time to ascertain this precisely, I give it here *cum nota*.

*A. granulata* is tolerably frequent in the Loch Fine dredging I have named as being so rich in species of *Amphora*, especially of complex *Amphoræ*. I would observe that in this form the striation of the lateral segments is finer than that on the middle bars, whereas in the next species the reverse is the case. *A. granulata* sometimes attains a considerably larger size than in the two undoubted specimens here figured.

97. *Amphora cymbifera*, n. sp. Pl. XIV., figs. 97, 97 *b*, 97 *c*. Form of frustule elliptic, rather broad, with very short, produced, and truncate apices. Length 0·0025″ to 0·0045″; breadth in the middle, 0·0012 to 0·0016″. Lateral segments highly arcuate on the dorsal, almost straight on the ventral margin, the former being suddenly contracted at the ends, so as to produce round heads, with very short necks; thus the segments are capitate. Their form is elongated, and the curve regular and graceful. They are marked by somewhat coarse striæ, slightly inclined. Striæ about 22 in 0·001″. The nodules are on the inner margin, or just within it, and the inner lines are parallel to that margin, and close to it. The segments, when detached, as is seen in one of the figures, exactly resemble the frustules of an elegant Cymbella. Between the lateral segments lie from five to seven convergent bars, and, in one focus, the whole frustule is seen to be made up of these bars (fig. 97 *b*), which are marked by fine transverse striæ, considerably finer than those on the lateral segments, which became stronger and coarser near the margin, as may be seen by the figures. The bars, as in *A. sulcata* (fig. 91), appear to be separated by furrows, and in a certain focus these furrows may be seen marked by lines of short transverse striæ. Fig. 97 is the same frustule as that in 97 *b*, focussed so as to show the lateral segments. Fig. 97 *c*, is a detached segment.

This fine form is not unfrequent in the Loch Fine dredging above mentioned. The views of it are so different, according to its position, and the detached segments are so like *Cymbellæ*, that it was some time before I could see my way among these forms, especially mixed as they were with frustules and segments of the preceding, as well as of the next species, which have a similar structure.

98. *Amphora proboscidea*, n. sp. Pl. XIV., figs. 98; 98 *b*; 98 *c*; 98 *d*. Form of frustule nearly rectangular in the middle, contracted near the ends to truncate extremities. Length from 0·003″ to 0·005″; breadth from 0·001″ to 0·0015″. The longer specimens are narrower than those of middling length. Lateral segments arcuate on the dorsal, often slightly convex, or undulated, on the ventral surface, contracted at the ends so as to be capitate, the heads having longer necks than in the preceding species, which are bent forward at a very obtuse angle,

giving to the detached segment a very peculiar character. The segments are marked by strong, coarse striæ, about 20 in 0·001″. In one focus (fig. 98) the segments are well seen in the frustule; in another, fig. 98 *b*, the frustule is seen to consist of 9 or 10 convergent bars, which are coarsely granulate. In fig. 98 the lateral segments are seen to be nearly in apposition, with a narrow space between them, of varying width, from the undulations of the ventral margin. The inner line of each lateral segment is very slightly curved; the nodule lies nearest to the ventral margin. The detached segments, figs. 98 *c*, and 98 *d*, are precisely like Cymbellæ, and for a long time I considered them as such, with those of *A. granulata* and *A. cymbifera*. But the view in fig. 98 showed the real nature of these forms. Want of space alone prevents me from giving figures of the entire frustule corresponding to the segment in fig. 98 *d*. The reader is requested to compare fig. 98 with the corresponding view of *A. Grevilliana*, fig. 89.

This very striking species occurs only in the stony Loch Fine dredging, so often alluded to, where it is rather frequent, both entire and in detached segments.

99. *Amphora costata*, Sm. Pl. XIV., fig. 99. This species is described by Professor Smith in his *Synopsis*, vol. i., where the entire frustule, and also a half frustule, are figured. But little is said of its peculiar structure, and the detached segment is not figured. For this reason, and to show the analogy between this species and the three preceding ones, I have figured a detached segment (fig. 99). Form of segment highly arcuate, very broad in the middle. Ventral margin straight, or slightly concave, but often, as in the example figured, which is a rather small one, with a rounded prominence in the middle close to the nodule. The ends are capitate. Striæ coarse, conspicuous, about 14 or 16 in 0·001″, thicker and stronger near the dorsal margin. Length from 0·002″ to 0·0033″: breadth in the middle 0·0012″ to 0·0016″.

It will be seen that in this species also the segment resembles a Cymbella, although it is a very broad and highly arcuate one. When the segments are united, as in the entire frustule, it is not easy to see their real characters. The backs of these segments, or longitudinal bars, are, as in Professor Smith's accurate figures, marked by very coarse distant granules, which give no indication of the peculiar striation of the detached segments. Hence it was very long before I was able to detect the component parts of the frustule when detached, or to refer the form shown in fig. 99 to *A. costata*. But some specimens, as in the preceding species, when carefully focussed, clearly show their true nature.

100. *Amphora bacillaris*, n. sp. Pl. XIV., figs. 100 and 100 *b*. Form of frustule linear, narrow, with somewhat rounded ends, which are subacute. Length about 0·0017″; breadth about 0·0003″. In one focus it exhibits two lateral portions separated by a middle space, the sides of which are perfectly straight, the ends beautifully rounded. In another, the whole frustule is seen to be composed of very narrow bars, separated by very sharp lines, converging on the ends, and

marked with small granules. Striation transverse, fine; number of striæ not counted, but they are much finer than in *A. pusilla*. The detached segments are not yet known; but, as seen in fig. 98, the segments appear to be very narrow, and linear in form, the dorsal margin being hardly convex. The inner curve-lines and nodules are obscure. These characters, as well as the finer striations, the finer granulation of the bars in fig. 100 *b*, and the peculiar form of the middle space in fig. 100, sufficiently distinguish it from *A. pusilla*, the only form it resembles.

This species occurs in the same Loch Fine dredging with those which immediately precede.

---

The numerous examples here given of complex Amphoræ, to which, as we have seen, two have been added from the simple group, prove that this group of forms is by no means a small one, since so many have been obtained in one locality. It is worthy of remark, that the same dredging, which has yielded at least 12 or 13 of the forms just described, also contains *A. costata*, Sm., already alluded to as the first Complex Amphora ever figured in this country, though the peculiarities of its structure had not been fully appreciated. In fact, as we have seen *A. Grevilliana*, *A. complexa* and *A. fascata* to form a smaller group of closely allied species, so *A. granulata*, *A. proboscidea*, and *A. cymbifera* also form another such group, to which *A. costata* also belongs. It would almost seem as if the locality were favourable to these complex forms; for on the waters of the Clyde the whole of them occur. We have also in these waters four *Amphiproræ*, with the remarkable additions of plates lying on the valves, namely, *Apr. pusilla*, *Apr. lepidopteræ*, *Apr. plicata*, and *Apr. maxima*; and lastly we have the doubtfully named *Apr. complexa*, which exhibits the same complex structure in its middle portion as we find in so many species of Amphora, that, namely, of segments packed together, and converging on the ends, like those of an orange or melon. But we must also remember that the same locality is equally rich in new forms of simple Amphoræ.

## GROUP VII.

### Miscellaneous.

In this last group I shall describe a few forms of genera not yet named in this communication, and among them one or two whose real nature is doubtful. These are:—

101. Navicula (?) Libellus, n. sp.
102. Nitzschia (?) panduriformis, n. sp.
103. Nitzschia distans, n. sp., G.
104. ... hyalina, n. sp.
105. Pleurosigma (?) reversum, n. sp.
106. Sceptroneis Caduceus, Ehr.
107. Synedra undulata, Greg.
  Toxarium undulatum, Bail.
108. Synedra Hennedyana, n. sp. (?)

and, as an Appendix,

109. Creswellia Turris, n. sp., (Arnott).

101. *Navicula* (?) *Libellus*, n. sp. Pl. XIV., figs. 101, and 101 *b*. Form of F.V. rectangular, broad, with the angles rounded. The middle part is marked by longitudinal lines or folds, like the leaves of a book; and when the two halves of the F.V. separate, each retains a broad band of this lineate part. The breadth of the detached halves on the F.V. is so great, that, when united, they must, it would seem, mutually overlap, otherwise the resulting frustule would be much broader than it is. S.V. rhombic or elliptic-lanceolate, broad, with obtuse ends. Length from 0·003″ to 0·0035″; breadth of F.V. 0·0017″ to 0·002″. The S.V. is marked by very fine transverse striæ; striæ about 60 in 0·001″; median line distinct; nodule definite. When the edge of the S.V. is seen, as in fig. 101 *b*, the valve seems to be a compound one, formed of five or six, closely packed one over the other. I cannot ascertain if this be so or not.

This species is frequent in Lamlash Bay, and it much resembles the form figured by me in my second plate from the Glenshira Sand, under the name of *N. rhombica*, of which also I had figured two of the S.V. in my first plate (*Trans. Mic. Soc.*, vol. iv., pl. v., fig. 1, and *Mic. Jour.*, vol. iii., pl. iv., fig. 16). But I observe several uniform points of difference. *N. Libellus* is more obtuse and broader, and its striation is not only much finer, but the striæ are everywhere of uniform size and at a uniform distance; whereas in *N. rhombica*, they are near the middle of the valve, not only stronger, but so much more distant than in the rest of the valve as to be almost conspicuous. *N. Libellus* is also, on the whole, a larger form than *N. rhombica*.

But it is very doubtful whether either of them be really a Navicula. They have some resemblance, especially on the F.V., to *Schizonema Grevillii*, Sm., which, however, is a much smaller form. Still they may possibly belong to Schizonema, but this cannot be ascertained except in living, or at least quite recent and uninjured specimens. The F.V., with its foliated or complex structure, appears to me, however, to differ from that of a Schizonema.

I may here add, that there occurs in Lamlash Bay a much smaller form of the same shape, but not foliated, at least not distinctly so. This is perhaps the true *S. Grevillii*.

102. *Nitzschia* (?) *panduriformis*, n. sp. Pl. XIV., fig. 102. Form linear, broad, incurved in the middle, acuminate at the ends, which are usually obtuse and rounded, but sometimes acute and sub-apiculate. Length about 0·003″; breadth in the middle 0·001″; at the shoulders 0·0012″. The specimen here figured is longer than usual, and the only one I have seen of this length. Margin punctate. There is a faint indication of a double keel in the middle of the valve. Striation fine, both transverse and oblique; striæ about 48 in 0·001″.

This species occurs in several of the Loch Fine dredgings, and is not rare. The striation is similar to that of *Tryblionella constricta*, Grig. (*Mic. Jour.*, vol. iii., pl. iv., fig. 13); but the present form is much larger, and is distinguished

by the marginal puncta. Still it resembles a Tryblionella about as much as it does a Nitzschia, and I therefore give it with a query as to the genus.

103. *Nitzschia distans*, n. sp. Pl. XIV., figs. 103 and 103 *b*. Form of F.V. nearly rectangular; margin punctate, the puncta being very distant, some in pairs, others single, 5 or 6 in 0·001″. The punctate margin bends slightly inwards to each end, so that the ends would be narrower than the middle but for two small hyaline expansions at each end, which renders the extremities a very little broader than the middle. Length about 0·0058″; breadth about 0·001″. S.V. linear-lanceolate or rhombic-linear, narrow, broadest in the middle, where the breadth is 0·0005″; ends acute, keel central. The whole form is somewhat hyaline.

This species is not rare in the Glenshira Sand, and I was only prevented from figuring it in my last paper on that deposit by want of room for the figure. Since then I have found it frequent both in Lamlash Bay and in the remarkable stony dredging from Loch Fine, so often mentioned, from which the present figure is taken. It is probably striated, but I have not been able to resolve the striation.

I may here state that the form, of which a drawing was made, but not inserted in the plate just alluded to, seems to differ in some points from this, which is the more frequent. In that form, the puncta, though distant, were regular, and, as stated in the description, which was printed without the figure, appeared to be constricted, and to have fine lines proceeding from one constricted punctum to the constriction in the next. As the F.V. of this form had not the terminal expansions, a circumstance which at the time I attributed to accident, I am inclined to believe that that figure really represented a different species. This I have not had time to ascertain. But the present figure represents accurately the form which from the first I had named *Ntz. distans.*

104. *Nitzschia hyalina*, n. sp. Pl. XIV., figs. 104 and 104 *b*. Form of F.V. rectangular, with expansions at the extremities. On each margin is a row of small puncta. Length 0·0034″; breadth 0·0004″. S.V. linear, narow, and towards both ends contracted to long and still narrower terminations. Keel apparently double; but perhaps one is seen through the very hyaline valve. The whole form is so hyaline as to be easily overlooked.

This delicate species is tolerably frequent in the Loch Fine dredging so often mentioned. It is possible that it may be a *Homœocladia*, but I have no means at present of deciding this point. It is certainly not one of the species of *H.* figured in the Synopsis.

105. *Pleurosigma* (?) *reversum*, n. sp. Pl. XIV., figs. 105 and 105 *b*. Form linear-lanceolate, narrow, somewhat contracted on each side of the middle portion, and again expanding towards the ends, which are elongated, and have the expansion all on one side, but on opposite sides at the two apices. On the non-expanded side of the elongated ends, the margin is nearly straight, or slightly in-

curved. The whole form has a strange appearance, as if we were to take two long, narrow stockings, cut them across at the widest part, and join them at the cut ends, with the feet pointing opposite ways. From this last character I have named it. Length 0·005″ to 0·006″; greatest breadth 0·0006.″ Median line sigmoid, straight in the middle, and suddenly bent near the ends in opposite directions. Striation so fine that I have not yet succeeded in resolving it, and therefore not easily visible under a power of 400 diameters.

This singular form occurs in the stony Loch Fine gathering so often referred to. I have as yet only seen the two specimens here figured, and two more; but I have not searched for it, these being so remarkable, and so like each other, as to indicate sufficiently, in a general way, the existence of the species. I do not feel quite certain as to its genus; but I think it right to direct the attention of observers to it. It will probably be found more abundantly in some dredging or gathering from a different locality in the Clyde.

106. *Sceptroneis Caduceus*, Ehr. Pl. XIV., fig. 106. I cannot enter into a detailed description of this species, as the fragment here figured is the only specimen of it I have yet seen in these dredgings, or in any British gathering. And I figure it chiefly as evidence that this genus, which is frequent in several American fossil deposits, yet lives in our waters, although we have yet to find it in such abundance as will probably occur near its true habitat. EHRENBERG thus describes the genus (*Bericht der Berlener Akademie*, 1844, p. 264), "Animal e Bacillariis Echmelleis, affixum? Lorica simplex æqualiter bivalvis silicea stiliformis compressa, nonconcatenale, cuneata (viva facile pedicellata). Sutura laterum utrusque valvæ longitudinalis media, umbilicus nullus. Habitus *Meridii* non concatenati aut *Gomphonematis*, umbilico laterali carentis."

The species, *S. Caduceus*, is distinguished by its long slender form, having a central expansion, and another at one end, while the other end is long and narrow, and by its very coarse moniliform striæ. In this fragment we have the large end, which is unusually large, for it is commonly of a narrower and somewhat elliptical shape.

This form, which adds one to the list of British genera, occurs in the same Loch Fine dredging as the preceding one, and so many more.

107. *Synedra undulata*, Greg. *Toxarium undulatum*, Bail. Pl. XIV., figs. 107 and 107 *b*. Form of frustule very long, and very slender. F.V. rectangular, very narrow; S.V. with an elongated central expansion, and two small semi-elliptic terminal ones. Margin undulated. Striæ conspicuous, moniliform, in the expansions passing, towards the middle, into an indiscriminate punctation. Length 0·023″; greatest breadth of S.V. 0·00035″; breadth of the longer and narrower portions hardly 0·0001.″ So that the length of the frustule is about 70 times the width of the broadest part of the S.V., and more than 200 times that of the greater part of the valve.

Professor SMITH describes the S.V. as arcuate, as in fig. 107 *b*; but I find it very often quite straight, as in fig. 107. The arcuation seems to be accidental, due only to the great slenderness of the frustule, and therefore common; but it is most probably naturally straight in the S.V. as well as on the F.V. Professor BAILEY represents it as straight, although he figures a specimen of the enormous length of 0·0265.″ Those which are not straight are bent quite unequally, some very little, others considerably, others only at one end, and others more at one end than the other. I feel pretty sure, therefore, especially as straight examples are frequent, that it is not essentially an arcuate form.

This very remarkable species, the longest known Diatom, except a *Chætoceros*, figured along with it by BAILEY, which is as long, was first observed in this country, by me, in the Glenshira Sand, in which, however, I could not find, among some hundred specimens, one entire frustule. I figured three fragments, two of them nearly complete, in my first paper on the Sand (*Mic. Jour.*, vol. iii., pl. iv., fig. 23), and was able to calculate, that if entire, its length would be about one-fiftieth or one fortieth of an inch, or 0·02″ to 0·025″. The length of the specimen here figured lies between these measurements, that of Professor BAILEY's figure is a little above the highest of them. After my paper with the incomplete figures was published, I became acquainted with the earlier observations of Professor BAILEY, who had found it living on *Sargassum* on the American coast. I found one specimen of it also recent, but still fractured, before my paper was printed, in a gathering made by Professor SMITH on the south coast. Subsequently, Professor SMITH found it frequent in Cork harbour, though smaller than in America. Last year (1856) I found it frequent in Professor ALLMAN's Lamlash Bay dredging, and sparingly in the other dredgings. As no entire figure of it has yet appeared in this country, I have here given two figures, one arcuate, the other straight.

108. *Synedra Hennedyana*, n. sp (?) Pl. XIV., fig. 108. This form is in all respects similar to the preceding, except that the margin is not undulated. Fig. 108 represents it of the same length as *S. undulata*.

I first noticed this form along with *S. undulata*, in July 1856, in Professor ALLMAN's Lamlash Bay dredging, but I considered it as simply a variety of that species. I was led to do so by observing that in *S. undulata* it often happens that a considerable portion of the margin is devoid of undulations. But several other observers who had seen it, adopted the opinion that it was distinct from *S. undulata*. Mr ROPER was, I believe, one of these; and I rather think Professor WALKER-ARNOTT, and Mr HENNEDY have come to the same conclusion. Professor ARNOTT informs me that it occurs in a gathering from the Clyde, I believe near Cumbrae, without a single frustule of *S. undulata*. As this gathering was made by Mr HENNEDY, if I am not mistaken, and as he has at all events studied the form in question, I have figured it under his name, with a mark of doubt, as I am not yet quite satisfied that it is really a distinct species. In my material it

is mixed with *S. undulata;* and I know of no distinction beyond that of the absence of undulations on the margin, unless it be that the striæ in *S. Hennedyana* are perhaps a little finer than in *S. undulata.* Even of this I am not sure. But the figures, which are very accurate, will enable the reader to form his own conclusions.

Such are the results obtained, up to this time, by the examination of these 11 gatherings from the Firth of Clyde and Loch Fine, 10 of which are true dredgings, while the 11th is derived from *Corallina officinalis,* to which a good many Diatoms have adhered.

From the remarkable analogy between the Glenshira Sand and these gatherings, we may regard it simply as another dredging, the marine forms in which have been derived from Loch Fine. I have shown that the period at which it was deposited has not caused any material difference of composition, and that we may say, in general, that it does not differ more from the recent dredgings than they do from each other.

Considering, then, all as supplying us with existing forms, we are struck with the unexpectedly large number of undescribed species which this exploration of the waters of the Clyde, though very limited in the area whence the materials were derived, has yielded in a short space of time.

It is worthy of notice, that the great majority of these new forms are not only new as British species, but have not been observed elsewhere, although EHRENBERG and BAILEY have both described many rich marine gatherings from different parts of the world.

This proves that the existing stores of marine Diatoms have not yet been by any means fully explored. It is therefore highly desirable that dredgings or soundings from all seas and estuaries, and from every part of them, should be procured and carefully searched. From what has been already recorded, as well as from the results here given, it appears that estuaries and harbours, or other localities near the coast, are likely to be the richest in Diatoms, perhaps from the comparative shallowness of the water. But the conditions of the distribution of these organisms in the sea, and of the accumulation of their indestructible siliceous shells, are not yet known with certainty. Thus, while every one of these Clyde dredgings proved more or less rich in Diatoms, I have found several from the Long Narrows, in the Firth of Forth, kindly given me by Dr HECTOR, to be very poor in comparison, and indeed not worth the trouble of mounting. And while BAILEY has found many interesting forms of this class in soundings from a depth of 1700 fathoms, and even of 2700 fathoms, in the Kamtschatka Sea, a number of Atlantic soundings, from depths varying from 85 to 2000 fathoms, which, by the kindness of Professors W. THOMSON and ALLEN THOMSON, I was allowed to see, contain indeed Foraminifera and Polycystineæ, but are almost entirely destitute of Diatomaceæ. Yet BAILEY has found Diatoms in Atlantic soundings from

other localities. We have nothing for it, therefore, but to examine every specimen of sea-bottom that we can procure. And the example of the Firth of Clyde is sufficient to prove that much remains to be done.

It should also be stated here, that these Clyde dredgings are not exhausted. Indeed, it is a work both of much time and much labour fully to exhaust any such mixtures as these are.

While these sheets are passing through the press, I am in a position to state, that I have already collected, from the same materials, so considerable a number of additional undescribed forms, that it will be necessary to describe and figure them in a supplementary memoir. Of these forms, a large proportion are discs, many of which are small, or only of a medium size; but there are also Naviculoid forms, Amphoræ, and forms of a few other genera.

I would further direct attention to the fact, that these dredgings differ materially from each other, each being characterized by the prevalence of certain forms, although some forms are common to all. Thus, off Inveraray and Strachur, in Loch Fine, the proportion of large Campylodisci was very much greater in two gatherings than in all the rest, whether there or off Arran; while in Lamlash Bay, the material was remarkable for the great number and variety of Amphoræ, a character found in one only out of the seven dredgings from Loch Fine. This shows that the deposits may vary much in regard to species, and even genera, in localities at no great distance from each other, and points out the advisability of searching every corner.

Lastly, it appears probable that some genera, whether such as have been adopted by Ehrenberg, Kützing, Bailey, and others, or entirely new, will have to be added to Professor Smith's list of British genera. This is especially the case with the numerous new filamentous forms, hardly any of which agree with the genera in the Synopsis. I have not for the present ventured to introduce any entirely new genus, but I have added *Pyxidicula* and *Sceptroneis* of Ehrenberg, and, more doubtfully, *Diadesmis*, also admitted, in a recent paper, by Professor Smith. I refrain from doing more; because I believe that genera established in the present imperfect state of our knowledge of species as well as of genera, are not likely to be permanent. In one case, I have pointed out the possibility of uniting in one genus and in one species three forms, *Campylodiscus simulans*, *Surirella fastuosa*, and *Surirella lata*, at present referred to two genera and three species.

In distinguishing and describing the very numerous new forms figured in this communication, I have been careful to avoid unnecessary multiplication of species. In numerous cases I have united forms apparently distinct which a closer examination showed not to be so. And in every case where I have admitted a new species, it has been because I could not reconcile it with any figures or descriptions which were accessible to me. I have also had the great advantage of frequent consultation with Dr Greville, whose opinion has deservedly

very great weight with all students of the Diatomaceæ. I have further to thank Mr Roper for many useful hints, and for the use of some very accurate drawings of forms observed by him, in many cases identical with those I had myself described.

It is impossible to do full justice to the scrupulous accuracy and to the artistic beauty of the figures which Dr Greville has made of the forms which I have described, and to the signal success with which Mr Tuffen West has engraved them. I can only say that I have seen no figures of this kind equal to them in these respects, and that the chief value of communications like the present is derived from the presence of good figures. Without figures, descriptions are apt to be misunderstood, and inferior figures tend, more than any other cause, to lead observers to multiply species unnecessarily. Those who are in the habit of studying the Diatomaceæ will agree with me, that a large proportion of the figures in some works on the subject are worse than useless, and lead to hopeless confusion.

There is another point on which good figures throw much needful light. In many species, though by no means in all, the shape, as well as the size of the forms, and even the striation, all vary to a great extent. In such cases, it is most important that every author should figure a sufficient number of selected forms, to show the real extent of the species. These variable species ought to be thus treated individually, by which means many existing species would be got rid of and reduced to a smaller number. I have attempted something of the kind in *Navicula varians* (*Trans. Mic. Soc.*, vol. iii., p. 10), and in this paper I have done so partially in *Navicula Lyra*, *Nav. Smithii*, *Amphora Proteus*, and *Amphora lævis*. Such forms as *N. elliptica*, *N. didyma* (which I have in part illustrated in my last paper on the Glenshira Sand), *N. Crabro* and *P. Pandura*, for example, and even *N. Smithii*, besides others in different genera, require much fuller illustration than they have yet received.

Finally, I wish it to be understood, that in describing so many new species, I make no pretensions to deciding authoritatively on disputed or doubtful points. My sole object is to bring under the notice of observers, the *forms* which I meet with. To do this, I must needs give them names, and in this respect I endeavour to be as accurate as I can. I observe that Professor Smith, in his last paper in the *Annals*, objects to the establishment of new species, unless the specimens are frequent. But although I have given, as distinct species, some forms which are rare, I have not done so till after I had examined and compared many specimens of each, except in one or two cases, such as *Coscinodiscus umbonatus*, where the form is so striking and well-marked that even one specimen suffices.

If we confine our attention to one or two slides, then, indeed, rare forms can not be sufficiently studied. But in the researches made in connection with this paper, I have explored at least 1000 slides, most of them twice, many three times,

and even oftener. Thus it happens, that I have compared as many specimens of by far the greater number of the forms here mentioned as rare, as if they had been very frequent, and I had only seen one or two slides.

Compared with many forms, all the Complex Amphoræ would be considered rare, but of these, I have in every instance examined numerous specimens, and have satisfied myself of the constancy of their characters, which is the most important point.

I trust, therefore, that Naturalists will accept this paper as a simple contribution to our knowledge of Diatomaceous Forms. As such I present it, leaving to those who are better qualified for the task than I am, to decide on the conflicting claims of genera and species.

---

## APPENDIX.

For the following description and figure of a very beautiful new form, belonging to a new genus, I am indebted to my friend Dr GREVILLE. The form in question has not occurred to me as yet, but as Professor WALKER-ARNOTT has found it in the Clyde, it has a claim to be inserted in this account of new Clyde forms.

*Notice of a New Genus of Diatomaceæ.* By R. K. GREVILLE, LL.D., F.R.S.E., &c.

My friend Professor GREGORY having permitted me to introduce in this place the description of a new and most interesting diatomaceous form, I gladly avail myself of the privilege. Having been recently discovered in the Clyde, it may, indeed, be considered as possessing some claim to appear in company with the multitude of fine species described by Professor GREGORY in the preceding pages. This remarkable Diatom was communicated to me a few weeks ago, by my friend Professor WALKER-ARNOTT, for publication and illustration. It is to be regretted that he did not undertake this office himself; he has, however, very kindly supplied me with notes of his views regarding it, so that he has rendered my labour comparatively light. Professor WALKER-ARNOTT's attention was first directed to the form in question by the Rev. R. CRESWELL, who obtained it at Teignmouth, from the stomach of *Cynthia rustica* (*Phallusia rustica*, Flem.), along with *Biddulphia Baileyi* and other good things. It is, however, so scarce, that at present it must be reckoned among the rarissima of its tribe. Very fortunately the specimens which have been obtained are in a state to admit of satisfactory description. Like *Biddulphia* and *Isthmia*, it forms chains, the links or frustules of which are oblong, somewhat depressed at the ends, highly cellulate, separating transversely

into two equal valves. The frustules are united by means of a circle of numerous short, terminal processes of equal length, which ultimately separate in the middle; and the detached frustules then appear furnished at each end with a beautiful coronet, or circle of miniature turrets. This mode of connection is peculiar, although perhaps analogous to that in *Biddulphia*. In the subject of these remarks, however, the connecting processes do not appear to be so distinctly a continuation of the substance and structure of the body of the frustule, as are the horns of that genus, and must rather be regarded in the light of appendages.

With regard to the affinities of this beautiful little object, we may certainly assume, that if a solitary frustule had alone been formed it would have been referred by Kützing at once to the genus *Pyxidicula*. But little or nothing is known of the real nature of the variety of forms brought together under that name. Ehrenberg's own character for the original genus is as follows:—

"Animal e familia Bacillariorum, liberum, lorica simplici, bivalvi (silicea); solitarium, globosum (=Gallionella divisione spontanea perfecta aut nulla). *Die Infusionsthierchen*, p. 165.

But Ehrenberg subsequently constituted other genera or subgenera to receive the accumulating species; and as the work in which they appear is not generally accessible, I do not hesitate to give the characters verbatim in this place.

"*Dictyopyxis*, nov. gen. *Pyxidiculæ* generis ea bivalves subglobosæ aut turgidæ formæ, quæ valvularum testæ strictura simpliciter cellulosa insignis sunt ab iis, quæ continua et simplici membrana silicea includuntur, aut appendicibus variis instructæ sunt, gravius differunt et facillime distinguuntur. Cellulosas igitur in Dictyopyxidis subgenere colligendas senserim. *P. cruciata*, *Cylindrus*, *hellenica et Lens* huic subgeneri nunc inscribendæ sunt."—*Ehrenb. Bericht. der Berl. Akad.*, 1844. P. 262.

"*Stephanopyxis*, nov. subgenus. *Pyxidiculæ* generis bivalves turgidæ aut subglobosæ formæ, quæ valvularum testæ structura cellulosa insignes sunt et denticulorum, aculeorum aut membranæ coronam in media quavis valvula gerunt in hoc *Pyxidiculæ* subgenere colliguntur."—*Ehrenb.*, *l. c.*, 1844. P. 264.

"*Xanthopyxis*, nov. subgen. *Pyxidiculæ* subgenus bivalve turgidum subglobosum. Valvularum testæ silicæ continuæ integerrimæ nec cellulosæ, superficie hispida, setosa aut alata."—*Ehrenb.*, *l. c.*, 1844. P. 264.

Kützing, in his *Species Algarum* (1849), reunites the whole, giving twenty-two species, all of which, except two, are fossil. The frustules, according to him, are "non concatenata." Mr Creswell's Diatom is, therefore, by a most important character, excluded. Taking a simple frustule, and leaving the processes out of view, it much resembles *Dictyopyxis hellenica* (*Ehrenb. Microgeologie*, tab. xx., fig. 32), and also *Stephanopyxis appendiculata* of the same work, tab. xviii., fig. 4; but that species has only a single tooth at each end, and is provided with a sort of narrow zone or annulus. It resembles still more closely *Stephanopyxis apicu-*

*lata* (*Ehrenb.*, *l. c.*, tab. xix., fig. 13) which is represented with three terminal teeth; but these teeth can scarcely be the remains of a *corona*, as KÜTZING, in defining the frustule, says, "utroque fine medio apiculis elongatis hispido." Upon the whole, the safe course seems to be to regard the subject of this notice as not only specifically but generally new; and I gladly adopt the suggestion of Professor WALKER-ARNOTT, that it receive the name of *Creswellia* in honour of its discoverer, the Rev. R. CRESWELL, a gentleman well known to Algæologists, and to whom Professor HARVEY has already dedicated a new British *Schizothrise.* The following character will distinguish it at once from all its allies.

*Creswellia.* Frustules cylindrical, two-valved, cohering by short, filiform processes into a continuous filament. Valves cup-like, cellulate, destitute of any siliceous connecting band. Pl. XIV., fig. 109.

This singularly interesting Diatom, which may be called *Creswellia Turris*, has only been found in the locality already mentioned by Mr CRESWELL, and off the Island of Cumbræ, where it was dredged along with the nests of *Lima hians* by Mr HENNEDY, and a single frustule detected by Dr WALKER-ARNOTT.

The figure represents four frustules, the largest number which has been hitherto observed in connection. It will be perceived that in two of the frustules one of the valves is dark, and more or less opaque. This appearance we are quite unable to account for. It sometimes happens that the whole frustule is dark. Generally they are all beautifully clear. The structure is highly cellulate, the cells hexagonal. The length of the frustule is about ·0028″; the breadth about ·0016″.

## EXPLANATION OF PLATE IX.

Fig. 1. Navicula minor, n. sp.
— 2. ... Cluthensis, n. sp.
— 3. ... inconspicua, n. sp.
— 4. ... brevis, n. sp.
— 5. ... Claviculus, n. sp., S.V. × 800. 5 *b*, do., F.V. × 800. 5 *c*, do. S.V. × 400.
— 6. ... Musca, n. sp.
— 7. ... rectangulata, n. sp.
— 8. ... nebulosa, n. sp.
— 9. ... Barclayana, n. sp.
— 10. ... spectabilis, n. sp.
— 11. ... praetexta, Ehr.
— 12. ... Bombus, Ehr.
— 13 & 13 *b*, Lyra, Ehr.
— 14 & 14 *b*, Lyra, var. *β*.
— 15. ... Smithii, var. *β*, fusca.
Fig. 16. Navicula Smithii, var. *γ*, nitescens.
— 17. ... Smithii, var. *δ*, suborbicularis.
— 18 & 18 *b*, maxima, Greg., S.V. and F.V.
— 19. Pinnularia subtilis, n. sp.
— 20. ... rostellata, n. sp.
— 21. ... Allmaniana, n. sp.
— 22. ... Pandura, Bréb., var. *β*. elongata.
— 23. Cocconeis distans, Greg.
— 24. ... ornata, n. sp.
— 25. ... dirupta, n. sp.
— 26. ... nitida, n. sp.
— 27. ... pseudomarginata, n. sp.
— 28. ... major, n. sp.
— 29. ... splendida, n. sp.

All the above, except figs. 5 and 5 *b*, are magnified 400 diameters.

## EXPLANATION OF PLATE X.

Fig. 30. Denticula (?) interrupta, n. sp.
— 31. ... (?) capitata, n. sp.
— 32. ... (?) ornata, n. sp.
— 33, 33 *b*, & 33 *c*, ...(?) lævis.
Fig. 34. Denticula nana, n. sp., F.V. 34 *b*; do. S.V.
— 35, 35 *b*, & 35 *c*, minor, n. sp., F.V. 35 *d*; do. S.V.
— 36. ... distans, n. sp., F.V. 36 *b*; do. S.V.
— 37 & 37 *b*,... staurophora, n. sp., F.V. 37 *c*; do. S.V

Fig. 38. Denticula fulva, n. sp., F.V. 38 *b*; do. S.V.
— 39. ... marina, n. sp., 39 *b*; do. S.V.
— 40. Diadesmis (?) Williamsoni, F.V., 40 *b*; do. S.V.
— 41. Meridion (?) marinum, n. sp., F.V., 41 *b*; do. S.V.
— 42. Pyxidicula cruciata, Ehr.
— 43. Orthosira angulata, n. sp., F.V., 43 *b*; do. F.V.

Fig. 44. Melosira (?) or Coscinodiscus (?) qu. (?) sp. (?) n. sp.
— 45. Coscinodiscus nitidus, n. sp.
— 46. ... punctulatus. n. sp.
— 47. ... concavus, Ehr.
— 48. ... umbonatus, n. sp.

All the above are magnified 400 diameters.

## EXPLANATION OF PLATE XI.

Fig. 49. Coscinodiscus centralis, Ehr.
— 50. Eupodiscus subtilis, n. sp., Ralfs.
— 51. Campylodiscus centralis, n. sp.
— 52. ... Ralfsii (?) Sm.

Fig. 53. Campylodiscus angularis, n. sp.
— 54 & 54 *b*, ... eximius, n. sp.
— 55. ... limbatus, Bréb.

All the above are magnified 400 diameters.

## EXPLANATION OF PLATE XII.

Fig. 56. Amphiprora pusilla, n. sp., F.V., 36 *b*, do. S.V.
— 57. ... plicata, n. sp., F.V.
— 58. ... elegans, Sm. S.V., 58 *b*; do. F.V.
— 59. ... lepidoptera, n. sp., F.V., 59 *b*; do., S.V., 59 *c*; do. peculiar view.
— 60. ... obtusa, n. sp., F.V.
— 61. ... maxima, Greg., F.V., 61 *b*; do. S.V.
— 62 & 62 *b* (?) complexa, n. sp., F.V. entire; 62 *c*; do. half frustule, 62 *d* and 62 *e*; do. detached segments.
— 63. Amphora turgida, n. sp.

Fig. 64. Amphora nana, n. sp.
— 65. ... macilenta, n. sp.
— 66. ... angusta, n. sp.
— 67. ... binodis, n. sp.
— 68 & 68 *b*, ventricosa, n. sp.
— 69. ... monilifera, n. sp.
— 70. ... lineata, n. sp.
— 71. ... Ergadensis, n. sp.
— 72. ... lævissima, n. sp.
— 73. ... pellucida, n. sp., 73 *b*; half frustule of do.
— 74, 74 *b*, & 74 *c*, { lævis, n. sp., 74 *d*; Amphora, qu.? a form of A. lævis?

All the above are magnified 400 diameters.

## EXPLANATION OF PLATE XIII.

Fig. 75. Amphora exigua, n. sp.
— 76. ... dubia, n. sp.
— 77. ... truncata, n. sp.
— 78 & 78 *b*, oblonga, n. sp.
— 79. ... robusta, n. sp., 79 *b* and 79 *c*; half frustules.
— 80. ... spectabilis, n. sp.; 80 *b*; do. var. $\beta$, 80 *c*; do. var. $\gamma$; 80 *d*; do. view showing the complex structure of the species; 80 *e*; do. detached segment.
— 81, 81 *b*, 81 *c*, 81 *d*, & 81 *e*, } ...Proteus, n. sp.

Fig. 82. Amphora lyrata, n. sp.
— 83. ... Milesiana, n. sp.
— 84. ... elongata, n. sp.
— 85. ... quadrata, n. sp.
— 86. ... excisa, n. sp.
— 87. ... nobilis.
— 88. ... Arcus, Greg.
— 89. ... Grevilliana, Greg.
— ~~90.~~ 91 ... fasciata, n. sp.
— ~~91.~~ 90 ... complexa, n. sp.

All the above are magnified 400 diameters.

## EXPLANATION OF PLATE XIV.

Fig. 93. Amphora acuta, n. sp., detached segment; 93 *b*; do. pack of similar segments.
— 94. ... crassa, n. sp, 94 *b*, 94 *c*, and 94 *d*; do. detached segments.
— 95 & 95 *b*, { pusilla, n. sp., simple and complex views.

Fig. 96, 96 *b*, 96 *c*, & 96 *d*, { Amphora granulata, n. sp., simple and complex views of two frustules; 96 *f*, detached segment; 96 *e*; form qu? allied to A. granulata?

Fig. 97 & 97 *b*, Amphora cymbifera, n. sp., simple and complex views, 97 *c*; detached segment of do.
— 98 & 98*b*, ... proboscidea, n. sp., simple and complex views, 98 *c*, and 98 *d*; detached segmentsof do.
— 99. Amphora costata, Sm., detached segment.
— 100 & 100 *b*, ... bacillaris, n. sp., simple and complex views.
— 101. Navicula (?) Libellus, n. sp., 101 *b*; do, edge view.
— 102. Nitzschia (?) panduriformis, n. sp.
— 103. ... distans, n. sp., 103 *b*, do. S.V.
— 104. ... hyalina, n. sp., 104 *b*, do. S.V.
Fig. 105 & 105 *b*, Pleurosigma ? reversum, n. sp.
— 106. Sceptroneis Caduceus, Ehr.
— 107. Synedra undulata, Greg. (Toxarium undulatum, Bailey). Two specimens of the S.V., the one straight, the other arcuate.
— 108. ... Hennedyana, n. sp., S.V.

APPENDIX.

— 109. Creswellia (nov. gen.) Turris, n. sp., Arnott.
All the above are magnified 400 diameters.

---

## *Postscript.*

While the preceding pages were passing through the press, I have been able to examine with care numerous specimens of most of the forms there described, and I wish here to modify to a small extent some of the views I have expressed. In every instance, I speak after the comparison of a very large number of fine examples.

1. *Navicula nebulosa*, fig. 8. I wish to observe, that after a very careful comparison of this form with *N. Hennedyi*, I have no longer any doubts as to its being a distinct species. I find it remarkably uniform in its characters, and particularly in its oval form, with the ends on the whole broadly rounded, while it has a slight angularity in the middle, and a slight trace of acumination at the apices. It is equally uniform in the narrowness of the marginal band of striæ, in the fineness of the striation, and in its very peculiar colour and nebulous aspect. In all these points, *N. Hennedyi* differs from it, as I have stated. But while these points of difference appear trifling, and are difficult to express in words, I must observe, that there is no real resemblance between the forms, and that when, as often happens, both being frequent, they occur close together, and of equal size (though *N. Hennedyi* is usually a larger form), it is quite impossible, even under a low power, to confound them together, the whole aspect of the two forms being remarkably different.

2. *Navicula spectabilis*, fig. 10. Having found, in certain densities, many very fine specimens of this form, I have to state, that it occurs of nearly twice the size of the individual figured, and that it is perfectly uniform in its characters.

3. *Navicula Bombus*, Ehr., fig. 12. This form also has occurred abundantly in certain densities, and I am now quite satisfied that it is a distinct and well-marked species. In the description I have omitted to mention an important character, namely, that it is never, literally,—not in one out of thousands of examples—sym-

metrical. One-half is always more or less larger than the other, and the amount of surface on each side of the median line is unequal. This does not occur in any other panduriform *Navicula*, except only occasionally in *N. didyma*, which cannot be confounded with the present species. *N. splendida*, *N. incurvata*, *N. Musca*, and *N.* (or *P.*) *Pandura*, are all remarkable for symmetry. In addition to this want of symmetry, which is invariable, it may be stated that, although several Naviculæ, and even some of the panduriform group, vary a good deal in shape, there is no species which is more uniform in this respect than *N. Bombus*.

4. *Navicula Smithii*, var. *β, fusca*, fig. 15. A careful study of very numerous specimens, both of this form, and of that which I take to be the typical *N. Smithii*, has now entirely satisfied me that *N. fusca* is truly a variety of *N. Smithii*. But it must be added, that this, like the fresh-water *N. elliptica*, is one of the most variable species, not only in form, but also in the striation, which varies from what may be called fine to exceedingly coarse; in colour, which varies from colourless to dark brown; and in general aspect,—*N. Smithii* being usually destitute of the remarkable longitudinal ridge or shade on each side of the median line, so conspicuous in *N. fusca*. In all these points, a perfect gradation may be traced without difficulty.

5. *Navicula Smithii*, var. *γ, nitescens*, fig. 16. Having found this form abundantly in one density, I have now come to the conclusion that it is no variety, but a distinct species. I find it perfectly uniform in all its characters, and the remarkable peculiarity of the median line, which is invariably a broadish white line with perfectly parallel sides; while that of *N. Smithii*, including *N. fusca*, is always doubly conical, being much broader in the middle, and forming a very acute point at each apex, seems effectually to separate it from that species. The shining aspect of the striæ is also peculiar.

6. *Navicula Smithii*, var. *δ, suborbicularis*, fig. 17. This form has also occurred abundantly, and I am now able to state that it is so uniform in its characters, and so peculiar in its aspect, that it must be admitted as a distinct and well-marked species. The only variation, except one to be presently mentioned, is in size, as it now and then occurs of twice the length of the figure, or even more, in which case it is more oval in shape, though always very broad. But the peculiar structure about the median line, giving the appearance of two white, elliptical bands meeting in the nodule, or of one long elliptical band, suddenly constricted in the middle, seems to be quite invariable, and sufficient to distinguish it. The fact, also, that the striæ are hardly visible, except on a broad marginal band, where they are very conspicuous, having the shining aspect of those in *N. nitescens*, though coarser than in that species, as well as the permanence of its very peculiar form, seem to indicate that it ought to be separated. Neither in this

form nor in the preceding, have I seen the slightest trace of any tendency to pass into *N. Smithii*, or into its variety *N. fusca*.

I have, indeed, lately noticed one variety of the present form, namely, a panduriform variety, agreeing with the type in size, in general aspect, and in the peculiar median line. This I shall describe and figure on a future occasion.

I may here add, that I shall also have to describe and figure another new form of Navicula, occurring abundantly with the preceding ones, which at first I was disposed to refer, like them, to *N. Smithii.* But I find it so uniformly peculiar, that I must separate it also.

7. *Denticula* (?) *lævis*, fig. 33. I have some reason to think that I have detected the S.V. of this species. The F.V. is frequent in some densities, but it would appear that the entire frustule is so much broader on the F.V., that it never lies on the S.V., and that the valves are never, or hardly ever, separated. Even when separate, the S.V. must be so very narrow, and perhaps so convex, that this side is not usuallyseen. In one case, however, where one of a group has been partly turned, I think I can see that the S.V. resembles in shape that of *D. fulva*, only smaller and narrower.

---

I have also to add to the list of British species two forms, both remarkable, which occur in Lamlash Bay.

These are,—1. *Cocconeis Morrisiana*, Sm., a very curious species lately found by Professor SMITH, I believe, in a gathering from the Levant or from the Black Sea. 2. *Pleurosigma compactum*, Grev.; described and figured by Dr GREVILLE, from Trinidad. I propose, in a future paper, to figure these two species as British forms.

I have just now been able to add to the figures, one of the very remarkable detached segment of *Amphora spectabilis*, as described in the text. It will be found in Pl. XIII., fig. ~~70 e~~ 80 e.

*28th May* 1857.

## CORRIGENDA.

I have to request the reader's attention to the following corrections, which I wish to make in regard to some points in the preceding paper.

1. The form represented in fig. 52, Plate XI., as a modification of *Campylodiscus Ralfsii*, Sm., is, as I am now convinced, entirely distinct from that species, which in fact, occurs in some of the dredgings along with it; and, in addition to its being uniformly small, exhibits a very different aspect. Fig. 52 agrees with the description given by DE BREBISSON of his *C. decorus*, and I have no doubt belongs to that species, which must therefore be added to the list of British Diatoms.

2. Additional observations have satisfied me, that the form represented in fig. 95 *e*, Plate XIV., is not a form of *Amphora granulata*, but an entirely distinct species, to be more fully described at a future time.

3. I wish to mention, that although I cannot see any reason to separate the two forms, represented in figs. 80 *b*, and 80 *c*, Plate XIII., from *Amphora spectabilis*, fig. 80, so far as the simple view of the latter is concerned, I have not yet been able to trace the complex structure of *A. spectabilis*, shown in fig. 80 *d*, in these smaller forms. Whether this is because I have not yet employed the highest powers of the microscope for this purpose, since these small forms have much finer markings than the larger one, or whether the complex structure occurs in the larger alone, while the smaller remain always simple; or whether, finally, the two smaller forms belong to a different species, are questions which I cannot yet answer.

4. Having, by the kindness of Dr A. S. DONKIN, of Morpeth, been enabled to examine a most interesting marine gathering made by him, in which several of the forms described in this paper, as well as several of those yet to be described as occurring in the Clyde, are met with, I have now to state, that I find *Amphora Grevilliana* in that gathering, with almost exactly the form and aspect of *Amphora complexa*, fig. 90, Plate XIII., while detached segments also occur, evidently belonging to it, and having the straight dorsal margin, but yet in all other points agreeing with those of *A. Grevilliana*, as shown in fig. 36*, of my third plate of Glenshira forms, and as seen in the present paper in the entire *A. Grevilliana*, fig. 89, Plate XIII. I have therefore to withdraw *A. complexa* as a distinct species, and to request the reader to consider the figure (fig. 90), as representing one view of a straight-sided form of *A. Grevilliana*. In this variety, as seen in Dr DONKIN'S gathering, and as I have also observed in the Glenshira Sand and in the Clyde, the detached segments are much narrower than when the dorsum is convex. I have specimens of the convex segments, from the Clyde, nearly three times as broad as Dr DONKIN'S, with the straight dorsum. I would further observe, that in all probability, fig. 89 represents a frustule, or possibly a half-frustule, viewed from the flat or concave side, while the frustule in fig. 90 is seen from the convex side, so that the flat-lying lateral segments are not so distinctly seen. I must remind the

reader also, that the names and descriptions of figs. 90 and 91, *A. Complexa*, and *A. fasciata*, have by some mistake been transposed, as stated in the errata, which I did not discover till it was too late to correct it. Whether *A. fasciata*, fig. 91, shall also prove a form of *A. Grevilliana*, for which I gave it in my last paper on the Glenshira Sand, I cannot at present determine.

5. The form represented in fig. 74 *d*, Plate XII., is, as I am now quite satisfied, entirely distinct from *Amphora lævis*, and must be hereafter described as a new species.

6. I have been able to introduce, at fig. 68 *c*, Plate XII., a figure of the remarkable detached valve of *Amphora ventricosa*, which resembles a very slender Cymbella, and occurs even longer and narrower than in the figure. In one dredging, I find it tolerably frequent.

7. In the disc of *Coscinodiscus centralis*, Ehr., as represented in fig. 49, Plate XI., the central cells are shown considerably larger than is usually the case in the specimens which I have of this Diatom. Indeed the cells, in the figure, are more like what is seen in the centre of *C. Asteromphalus*, Ehr., and the question arises, whether the specimen figured may not belong to the last-named species, or whether these two species may not, in reality, be essentially one and the same. In a large number of specimens of *C. centralis* which I have lately examined, the central cells are invariably but a little larger than the rest, so that the form represented in fig. 49, if it be *C. centralis*, and I see no other difference, must have been, in this respect, abnormal. I may add, that in these beautiful discs, some of which are considerably larger than the one figured, the cells are distinctly arranged in spiral lines, as in engine-turning, and as is seen also in *C. radiatus*. This character is but slightly indicated in the figure.

W. G.

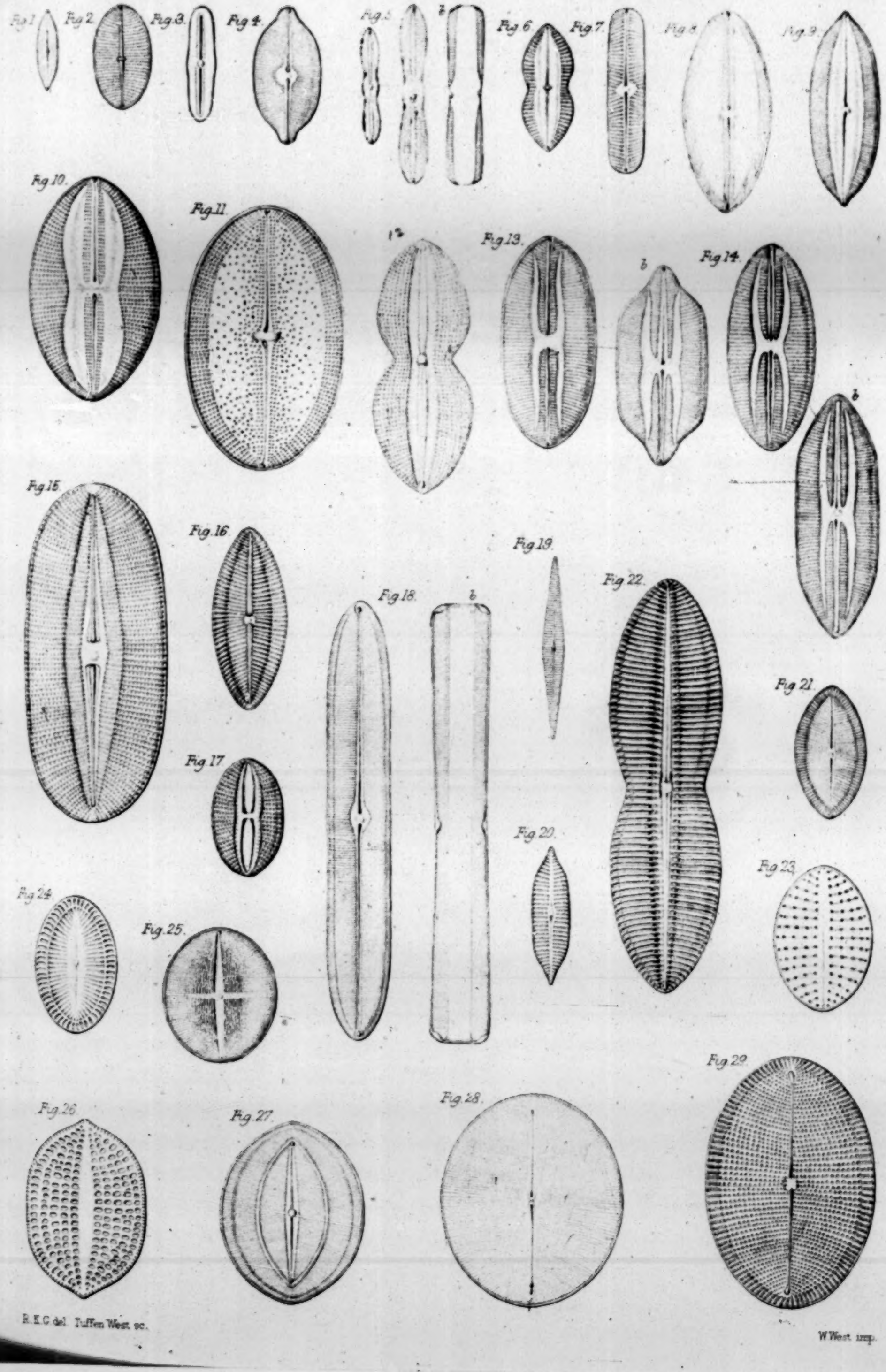
*1st August* 1857.

## EXPLANATION OF PLATE IX.

Fig. 1. Navicula minor, n. sp.
— 2. ... Cluthensis, n. sp.
— 3. ... inconspicua, n. sp.
— 4. ... brevis, n. sp.
— 5. ... Claviculus, n. sp., S.V. × 800. 5 *b*, do., F.V. × 800. 5 *c*, do. S.V. × 400.
— 6. ... Musca, n. sp.
— 7. ... rectangulata, n. sp.
— 8. ... nebulosa, n. sp.
— 9. ... Barclayana, n. sp.
— 10. ... spectabilis, n. sp.
— 11. ... praetexta, Ehr.
— 12. ... Bombus, Ehr.
— 13 & 13 *b*, Lyra, Ehr.
— 14 & 14 *b*, Lyra, var. $\beta$.
— 15. ... Smithii, var. $\beta$, fusca.

Fig. 16. Navicula Smithii, var. $\gamma$, nitescens.
— 17. ... Smithii, var. $\delta$, suborbicularis.
— 18 & 18 *b*, maxima, Greg., S.V. and F.V.
— 19. Pinnularia subtilis, n. sp.
— 20. ... rostellata, n. sp.
— 21. ... Allmaniana, n. sp.
— 22. ... Pandura, Bréb., var. $\beta$. elongata.
— 23. Cocconeis distans, Greg.
— 24. ... ornata, n. sp.
— 25. ... dirupta, n. sp.
— 26. ... nitida, n. sp.
— 27. ... pseudomarginata, n. sp.
— 28. ... major, n. sp.
— 29. ... splendida, n. sp.

All the above, except figs. 5 and 5 *b*, are magnified 400 diameters.




R. K. G. del. Tuffen West sc. W. West imp.

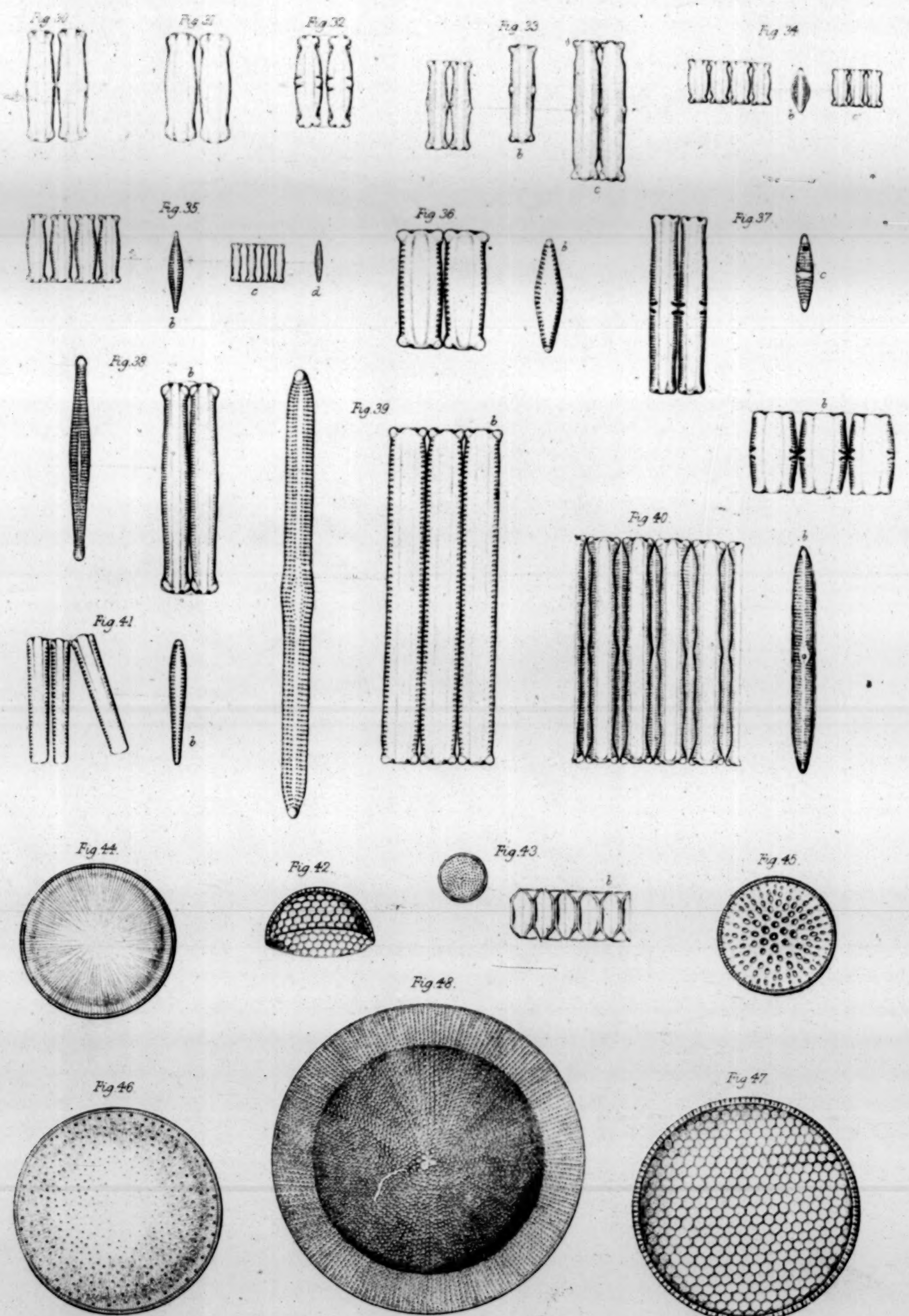
[illegible] 1 . 57

## EXPLANATION OF PLATE X.

Fig. 30. Denticula (?) interrupta, n. sp.
— 31. ... (?) capitata, n. sp.
— 32. ... (?) ornata, n. sp.
— 33, 33 *b*, & 33 *c*, ...(?) lævis.
— 34. ... nana, n. sp., F.V. 34 *b*; do. S.V.
— 35, 35 *b*, & 35 *c*, minor, n. sp., F.V. 35 *d*; do. S.V.
— 36. ... distans, n. sp., F.V. 36 *b*; do. S.V.
— 37 & 37 *b*,...staurophora, n. sp., F.V. 37 *c*; do. S.V.
— 38. ... fulva, n. sp., F.V. 38 *b*; do. S.V.
— 39. ... marina, n. sp., 39 *b*; do. S.V.

Fig. 40. Diadesmis (?) Williamsoni, F.V., 40 *b*; do. S.V.
— 41. Meridion (?) marinum, n. sp., F.V., 41 *b*; do. S.V.
— 42. Pyxidicula cruciata, Ehr.
— 43. Orthosira angulata, n. sp., F.V., 43 *b*; do. F.V.
— 44. Melosira (?) or Coscinodiscus (?) qu. (?) sp. (?) n. sp.
— 45. Coscinodiscus nitidus, n. sp.
— 46. ... punctulatus. n. sp.
— 47. ... concavus, Ehr.
— 48. ... umbonatus, n. sp.

All the above are magnified 400 diameters.




R. K. G. del. Tuffen West sc.

W West imp.

## EXPLANATION OF PLATE XI.

Fig. 49. **Coscinodiscus centralis, Ehr.**
— 50. **Eupodiscus subtilis, n. sp., Ralfs.**
— 51. **Campylodiscus centralis, n. sp.**
— 52. ... **Ralfsii (?) Sm.**
vide Corrigenda 541.

Fig. 53. **Campylodiscus angularis, n. sp.**
— 54 & 54 *b*, ... **eximius, n. sp.**
— 55. ... **limbatus, Bréb.**
**All the above are magnified 400 diameters.**

Fig 50

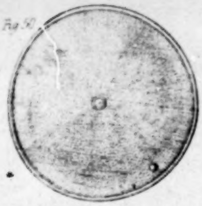


Fig 49

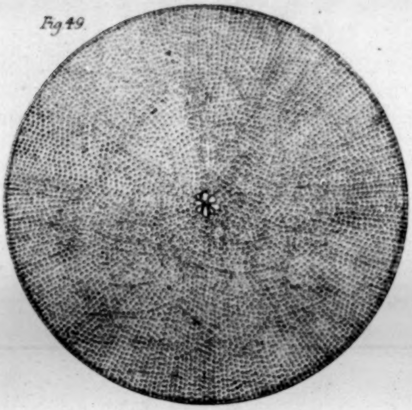


Fig 51

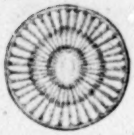


Fig 52

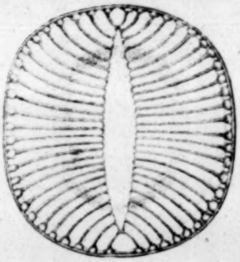


Fig 53

Fig 54 b

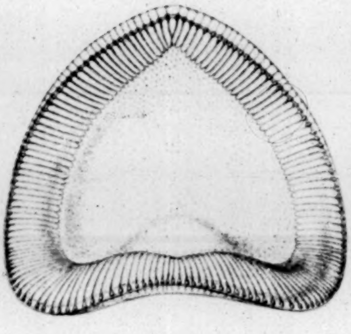


Fig 54

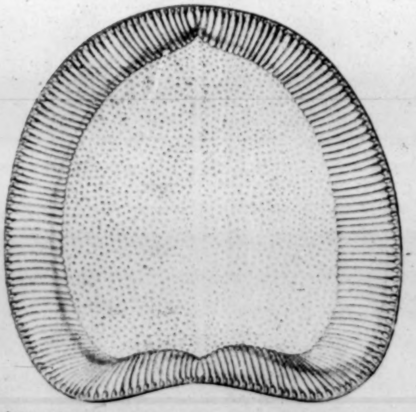
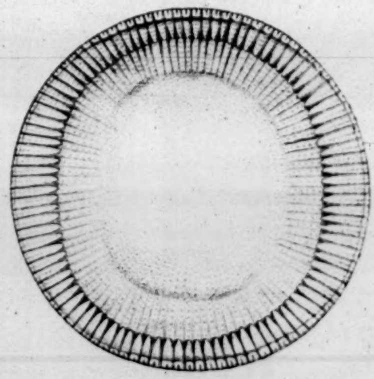


Fig 55



R. K. G del. Tuffen West sc. W West imp

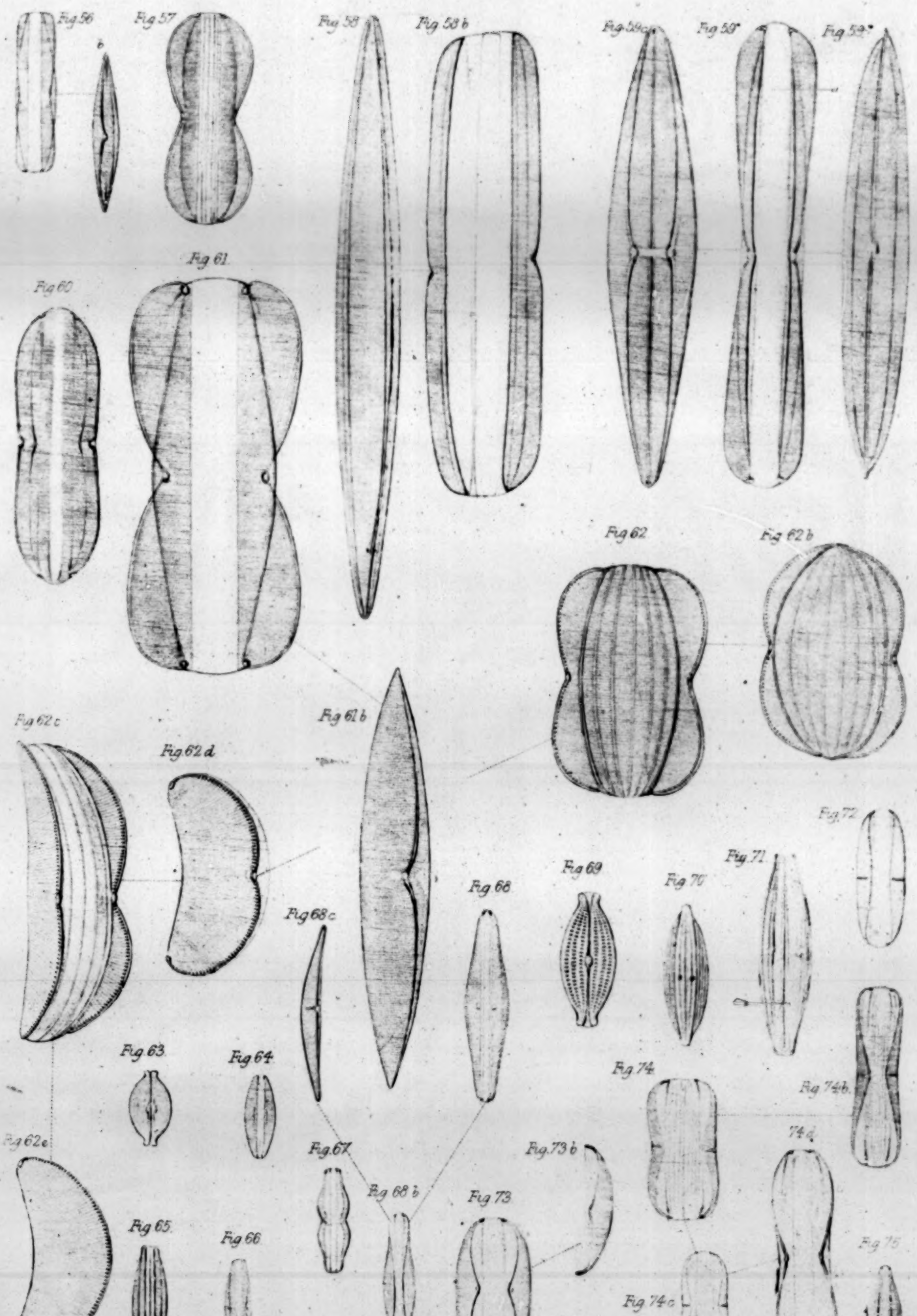
## EXPLANATION OF PLATE XII.

---

Fig. 56. Amphiprora pusilla, n. sp., F.V., 36 *b*; do. S.V.
— 57. ... plicata, n. sp., F.V.
— 58. ... elegans, Sm. S.V., 58 *b*; do. F.V.
— 59. ... lepidoptera, n. sp., F.V., 59 *b*; do., S.V., 59 *c*; do. peculiar view.
— 60. ... obtusa, n. sp., F.V.
— 61. ... maxima, Greg., F.V., 61 *b*; do. S.V.
— 62 & 62 *b* (?) complexa, n. sp., F.V. entire; 62 *c*; do. half frustule, 62 *d* and 62 *e*; do. detached segments.
— 63. Amphora turgida, n. sp.

Fig. 64. Amphora nana, n. sp.
— 65. ... macilenta, n. sp.
— 66. ... angusta, n. sp.
— 67. ... binodis, n. sp.
— 68 & 68 *b*, ventricosa, n. sp.
— 69. ... monilifera, n. sp.
— 70. ... lineata, n. sp.
— 71. ... Ergadensis, n. sp.
— 72. ... lævissima, n. sp.
— 73. ... pellucida, n. sp., 73 *b*; half frustule of do.
— 74, 74 *b*, & 74 *c*, { lævis, n. sp., 74 *d*; Amphora, qu. ? a form of A. lævis ?

All the above are magnified 400 diameters.




R. F. C. & F. C. S. Roper del. Tuffen West sc.

W West imp

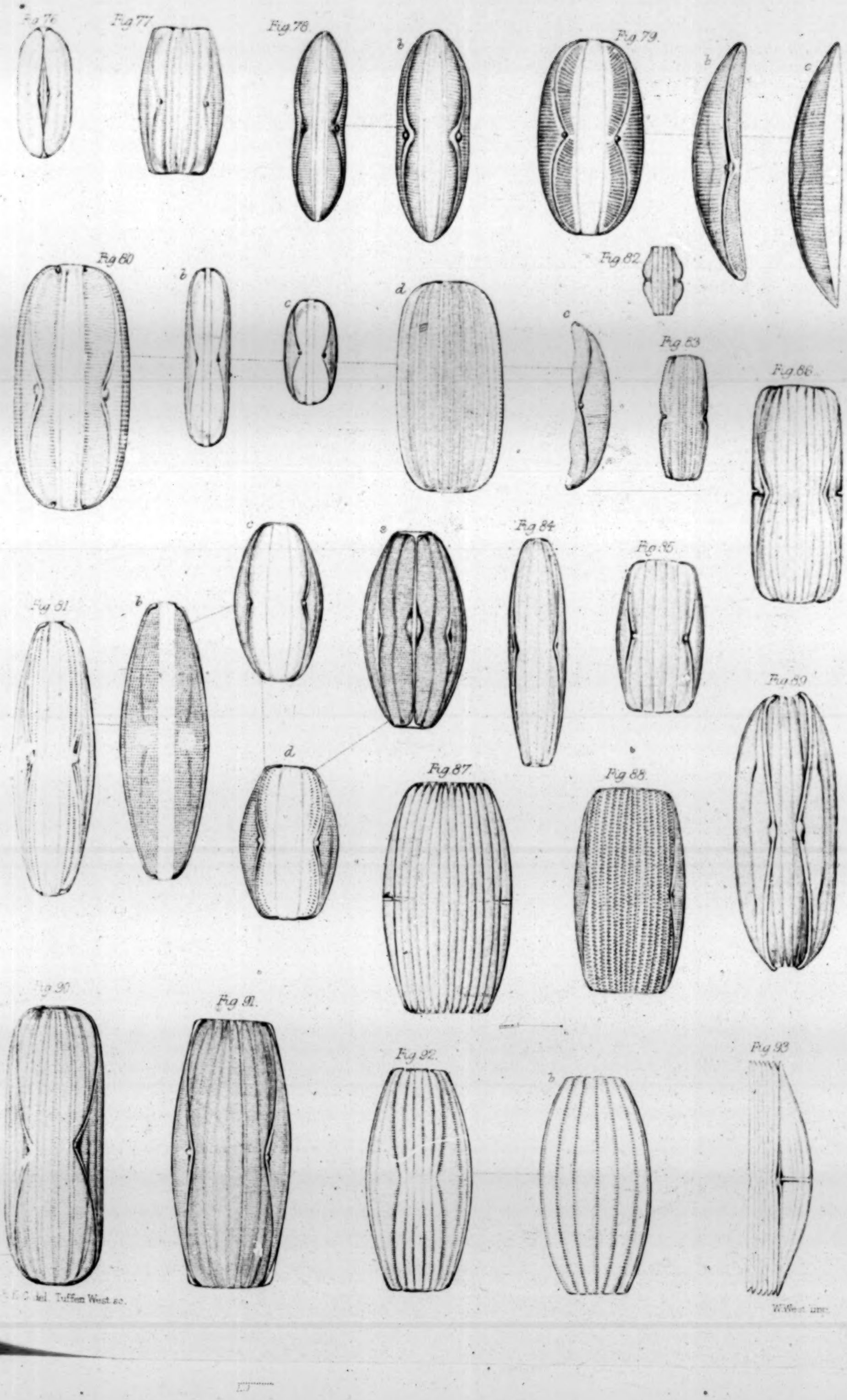
## EXPLANATION OF PLATE XIII.

---

Fig. 75. Amphora exigua, n. sp.
— 76. ... dubia, n. sp.
— 77. ... truncata, n. sp.
— 78 & 78 *b*, oblonga, n. sp.
— 79. ... robusta, n. sp., 79 *b* and 79 *c*; half frustules.
— 80. ... spectabilis, n. sp.; 80 *b*; do. var. $\beta$, 80 *c*; do. var. $\gamma$; 80 *d*; do. view showing the complex structure of the species; 80 *e*; do. detached segment.
— 81, 81 *b*, 81 *c*, 81 *d*, & 81 *e*, ...Proteus, n. sp.

Fig. 82. Amphora lyrata, n. sp.
— 83. ... Milesiana, n. sp.
— 84. ... elongata, n. sp.
— 85. ... quadrata, n. sp.
— 86. ... excisa, n. sp.
— 87. ... nobilis.
— 88. ... Arcus, Greg.
— 89. ... Grevilliana, Greg.
— 90. 91 ... fasciata, n. sp.
— 91. 90 ... complexa, n. sp.

All the above are magnified 400 diameters.




R. K. G. del. Tuffen West sc. W. West imp.

## EXPLANATION OF PLATE XIV.

Fig. 93. Amphora acuta, n. sp. detached segment; 93 *b*; do. pack of similar segments.

— 94. ... crassa, n. sp, 94 *b*, 94 *c*, and 94 *d*; do. detached segments.

— 95 & 95 *b*, ...pusilla, n. sp., simple and complex views.

— 96, 96 *b*, 96 *c*, & 96 *d*, ...granulata, n. sp., simple and complex views of two frustules; 96 *f*, detached segment; 96 *e*; form qu.? allied to A. granulata?

— 97 & 97 *b*, ...cymbifera, n. sp., simple and complex views, 97*c*; detached segment of do.

— 98 & 98*b*, ...proboscidea, n. sp., simple and complex views, 98 *c*, and 98 *d*; detached segments of do.

— 99. ... costata, Sm., detached segment.

Fig. 100 & 100 *b*, Amphora bacillaris, n. sp., simple and complex views.

— 101. Navicula (?) Libellus, n. sp., 101 *b*; do, edge view.

— 102. Nitzschia (?) panduriformis, n. sp.

— 103. ... distans, n. sp., 103 *b*, do. S.V.

— 104. ... hyalina, n. sp., 104 *b*, do. S.V.

— 105 & 105 *b*, Pleurosigma ? reversum, n. sp.

— 106. Sceptroneis Caduceus, Ehr.

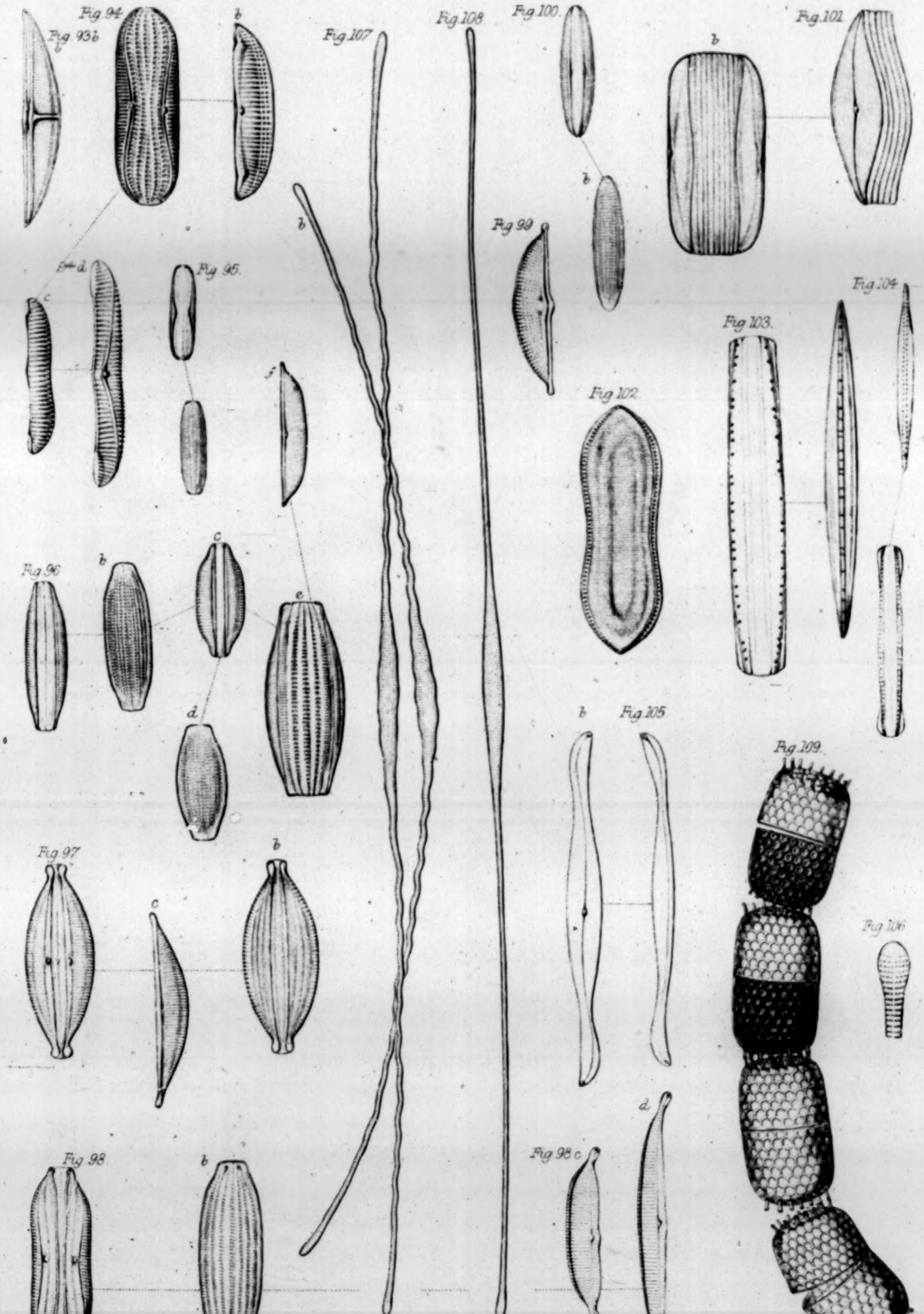
— 107. Synedra undulata, Greg. (Toxarium undulatum, Bailey). Two specimens of the S.V., the one straight, the other arcuate.

— 108. ... Hennedyana, n. sp., S.V.

### APPENDIX.

— 109. Creswellia (nov. gen.) Turris, n. sp., Arnott.

All the above are magnified 400 diameters.




R. K. G. del. Tuffen West sc.

W. West imp.

## XXXII.—*On the Urinary Secretion of Fishes, with some Remarks on this Secretion in other Classes of Animals.* By John Davy, M.D., F.R.SS. Lond. and Edin., &c.

(Read 2d February 1857.)

Notwithstanding the progress made of late years in animal chemistry in connection with comparative anatomy, I am not aware of any observations that have yet been published on the urinary secretion of fishes. The neglect of this inquiry probably has arisen from several circumstances,—the nature of the element inhabited, the peculiarities of the urinary organs, the difficulty of collecting the matter voided, and its having no well-marked distinctive qualities obvious to the senses.

For some years, as leisure and opportunities offered, I have given attention to the subject, and in the paper which now I have the honour to submit to the Society, I beg to communicate the observations I have made. Few and imperfect as these are, they are given mainly with the hope of attracting notice to the inquiry and of inducing others more favourably situated to engage in its prosecution.

The fishes I have examined in search of their urinary secretion have been the following,—the salmon, sea-trout, charr, common trout, pike, and perch; the skate, ling, conger, cod, pollack, haddock, turbot, bream, and mackerel.

Of these the salmonidæ, pike, perch, ling, and ray, have a small urinary bladder; and in all but the last communicating directly with the kidneys. In the last mentioned, the ray, the communication appears to be indirect, after the manner observable in some of the batrachians, in which the ureters terminate in the cloaca.

The other fishes named seem to be destitute of a urinary bladder, or, if possessed of one, it was so small as to have escaped observation. The ureter in these, when distinct, was found to terminate near the verge of the anal aperture; in several instances it was so large and dilated as to serve the place of a bladder.

In the small urinary bladder of the salmonidæ (so small as to be little more than rudimentary), I have never found any fluid collected. In the bladder of a trout (*Salmo fario*) taken in June, in Windermere, when in highest condition, there was seen a little whitish mucus-like matter. Tested by nitric acid and heat properly graduated, it became yellow, without the slightest purplish tinge, indicative of the presence of lithic acid.

The urinary bladder of the perch (*Perca fluviatilis*) is larger, and internally plicated and spongy, and has been found to contain a fluid. In that of one,—a fish,

weighing about a pound and a half, taken in the same lake, and in the same month as the trout,—there was a little mucus-like matter suspended in its fluid contents. The fluid was rendered turbid by admixture with alcohol. It cleared on rest, from the subsidence of the precipitated matter. The clear solution, decanted and evaporated gently, yielded crystals approaching in form those afforded by a weak solution of muriate of ammonia similarly treated. Redissolved on the addition of a minute portion of nitric acid, and again evaporated, crystalline plates were obtained very like those of the nitrate of urea. Subjected to the temperature required for detecting the presence of lithic acid, the result was negative,—the hue produced was yellow, without the slightest tinge of purple;—and the mucus-like matter similarly tested afforded a like result.

The urinary bladder of the pike (*Esox lucius*) is very small. I have always found it empty. In the ureter* of one of about two pounds, taken in Windermere in May, a few delicate yellowish flakes were detected. These, under the microscope, exhibited no characteristic appearance; acted on by dilute nitric acid, however, they were in great part dissolved; and when evaporated with a graduated heat to dryness on a support of thin glass, the purple stain distinctive of lithic acid was produced, and it was so strong, that it coloured a proportionally large quantity of water.

The ling (*Lota molva*) has a comparatively large urinary bladder. From the bladder of one,—a fish of about four feet long, taken in the Mount's Bay, in Cornwall, in the month of June,—a small quantity, about a drachm, of nearly colourless fluid was obtained, in which a few flakes resembling lymph were suspended. These flakes were tested for lithic acid, but with a negative result. The fluid was coagulated by heat, by nitric acid, and by alcohol, indicating the presence of a notable proportion of albumen. The alcoholic solution, after the separation of the precipitated albumen, evaporated to dryness at a low temperature, yielded, after the addition of a minute portion of nitric acid, crystals which, seen under the microscope—they were too small to be seen without this aid—resembled so closely those of nitrate of urea, that I had little hesitation in coming to the conclusion that they were this compound.

The common ray (*Raia batis*) is provided with two small bladders, each distinct, and neither of them communicating directly with the kidneys. In a male, examined in November, they were found distended with a nearly colourless limpid fluid, in which, placed under the microscope, were seen many small globules, and a few spermatozoa. This fluid, evaporated at a low temperature, yielded a colourless residue, in which were minute crystals of common salt; and, acted on

* Professor OWEN, in his *Lectures on the Comparative Anatomy of the Vertebrate Animals* (Part i., p. 223), describes the bladder of the pike as communicating with the kidneys by a single common ureter; in most instances I have found the communication such, but in one fish, one of six pounds, it was by two.

by alcohol and nitric acid, indications were afforded of the presence also of a little albumen and urea, but without any trace of lithic acid.

Of the fishes before named, destitute of a urinary bladder, the ureter, in the instance of the haddock (*Morrhua æglefinus*), of the cod (*Morrhua vulgaris*), of the pollack (*Merlangus pollachius*), of the turbot (*Rhombus maximus*), was found so capacious, that it might answer the purpose of a receptacle or bladder. In each its inner surface was wet; but only in one, that of the turbot, was there any fluid collected. The quantity obtained, by cutting out the duct, after a ligature had been passed above and below, was about ten drops. It was colourless, not quite clear, and had suspended in it a few white flakes. These were not dissolved by nitric acid, nor did they, when the acid was evaporated by heat, afford any the slightest indications of lithic acid. The residue was yellow; nor could urea be detected in the minute portion of fluid.

Of the bream (*Pagillus centrodontus*), the ureter is narrow, and of little capacity; as is also that of the conger (*Conger vulgaris*), and that of the mackerel (*Scomber scombrus*). Of all three the ureter was found merely moist—wet—as if a fluid had passed; in neither could any solid matter be detected. At the termination of the ureter of the bream a minute portion of whitish matter was seen adhering, suggesting lithate of soda or ammonia, but not confirmed when tested; for, when acted on by nitric acid and heat, the colour acquired was yellow, without the slightest tinge of purple.

I may mention, generally, that in most of the fishes, the names of which have been given, I did not omit examining the cloaca, but with results so unsatisfactory, that they might be said to have been negative. Often there was an appearance as if of the presence of an alkaline lithate; but, when tested, it was found to be different, and the matter chiefly intestinal excrement. In the instance of one only, and that a sea-trout (*Salmo trutta*), was a trace of urea indicated, judging from the form of the minute microscopic crystals obtained on evaporation, after treatment with alcohol and nitric acid.

I may also mention, generally, that in each fish I carefully inspected the structure of the kidneys; but without success as to the finding of any matter conspicuous to the eye, such as is commonly seen in the same organs in the instance of serpents and lizards, viz., the opaque lithate.

In one instance only, that of the haddock, have I examined these organs chemically. The result, too, was negative. The trial was made, first by digesting the kidneys in alcohol, decanting the clear spirit, evaporating it at a low temperature, and to the concentrated extract obtained adding nitric acid; secondly, by digesting the organs with aqua ammoniæ, filtering the solution, and testing the little extract obtained by nitric acid and heat.

If any conclusions are permissible from the preceding few and imperfect observations, I would venture to submit the following:—1*st*, that the urinary secre-

tion of fishes is very limited as to quantity; *2dly*, that it is commonly liquid; *3dly*, that the nitrogenous compound eliminated is variable,—either urea or a lithate (the latter probably very seldom), or some nearly allied compound of azote.

A brief glance at this secretion in other classes of animals may here not be out of place, as bearing on these conclusions. I need not dwell on the importance of the urinary secretion, denoted by its generality, and how, in all the great divisions of the animal kingdom in which it has hitherto been examined, viz., the mammalia, birds, reptiles, insects, spiders, the mollusca, it has been found to consist chiefly of compounds abounding in nitrogen, authorizing the commonly-received conclusion that the secerning organs are depurating in their function, and the main channel by which the excess of this element (nitrogen) is removed from the system.

The differences however compatible with this intent, — differences in the nature of the secretion,—are not a little remarkable. I allude merely to the quality—to the chemical ingredient; and they seem to be regulated more by the structure of the urinary apparatus, or secerning vessels, than by any other circumstance, not even excepting the kind of diet, whether animal or vegetable, or an admixture of the two.

In the mammalia, provided with an ample urinary bladder, the normal secretion is seen to be entirely liquid, and the principal ingredient, so far as it has yet been determined, always soluble urea: Such it has been found to be in man; such in the carnivorous animals; such in the herbivorous; with the addition, in that of some of them, of the hippuric acid.

In birds, on the contrary, and in those reptiles which, like them, are destitute of a urinary bladder, viz., snakes and lizards, invariably the secretion, judging from my own pretty extended experience, is chiefly solid,—a soft, plastic one, owing its consistence to admixture with water, and composed principally of lithate of ammonia and lithic acid. Yet in others of the latter class, which have a receptacle corresponding to the urinary bladder, and destined to hold the secretion,* the secretion is fluid, as in the instance of the toads and frogs; and the nitrogenous matter eliminated is again the soluble urea. The same remark applies to the tortoises, with this difference, that sometimes, though their food be vegetable solid matter, flakes of a lithate are occasionally found suspended in the fluid contents of their urinary bladder.

In insects, also in spiders and scorpions, all which, it is presumed, have no

* Whether this receptacle be considered,—as it is by Mr T. R. JONES, in his *General Outlines of the Animal Kingdom* (p. 585)—the unobliterated remains of the allantois, or a true urinary bladder, its primary use, I apprehend, can hardly now be questioned, since all the later examinations that have been made of the fluid contained in it prove that in composition it is urinous, as stated above: whether, in the instance of the frog, it may not subserve to aid, as some distinguished physiologists suppose, by transpiration in keeping the skin duly moist, is open to question.

receptacle for the secretion but the cloaca, we find it in consistence analogous to that of birds, snakes, and lizards, a soft solid; in insects, as far as my observations have extended, and they have been numerous,* it is composed chiefly of an alkaline lithate; but in the others, the spiders and scorpions, of guanine.†

Of the secretion in the mollusca, also without a urinary bladder, I can venture to say little. In two instances I have found it to be lithic acid; the individuals in the excrement of which I detected this compound were our common slug (*Lima agrestis*), and the large snail of Tobago (*Helix oblonga?*).

Of animals lower in the organic scale, the only ones I have examined with any positive result have been two of the Myriapoda,—the common centipede of the West Indies (*Scolopendra morsitans*), and our millipede (*Iulus terrestris*), the one voracious, feeding on insects, the other feeding on vegetable matter. In the mixed excrement of the scolopendra, lithate of ammonia in abundance was detected;‡ but in that of the millipede, merely a trace of lithic acid.

In this brief notice of the urinary secretion in the several classes of animals mentioned, I have, as I premised, taken notice only of its principal ingredient; I would further beg to remark, that in stating that the quality of the secretion is independent of the quality of the food, I would wish to be understood as not holding the opinion that it is not in some measure modified by the kind of food,—especially as regards the quantity of matter eliminated. As might be expected, the larger the proportion of nitrogen in the food consumed, the larger, *cæteris paribus*, seems to be the quantity of the nitrogenous compound excreted, and *vice versa*. Moreover, when the food is entirely vegetable, there seems to be in some instances a tendency towards the production of the hippuric acid rather than of the lithic. MM. Magnon and Lehmann have found this compound in the urine of the tortoise feeding on lettuce;§ and have found it mixed with lithic acid in the urine of caterpillars feeding exclusively on vegetables,—a result which accords with my own experience.

In the animal economy we see commonly, amongst the different classes of animals, a certain relation and accordance of functions conducive in action to the elaboration and wellbeing of each individual structure. Such a relation is manifest between the kidneys and the lungs; the former the depurator of nitro-

* *Trans. Ent. Society*, vol. iii., N. S.

† When I first examined the excrement of spiders and scorpions in 1847–1848, operating on minute quantities, I inferred that it consisted chiefly of xanthic oxide: Guanine was not then known. Since its discovery by Bodo Unger, I have re-examined portions of the excrement of each, which I brought from the West Indies, and have satisfied myself that the principal ingredient of both is this compound; I have also found it, in accordance with the researches of Will and Gorup-Besanez, to form the chief portion of the excrement of our spiders. The very low degree in which this excrement is soluble in *cold* muriatic acid may account for its having been first confounded with the xanthic oxide.

‡ *Edin. Phil. Jour.*, vol. xlv. p. 383.

§ Lehmann's *Physiological Chemistry*, vol. ii., p. 458.

gen, as much as the latter is of carbonic acid. How strongly is this exemplified in birds;—of high temperature, consuming much atmospheric air, evolving much carbonic acid,—their urinary secretion, also, is remarkably abundant, and abounding in nitrogen.* And in other classes of animals, such as insects in their several stages, such as serpents and lizards, and the hybernating ones of different classes, whether active or torpid, a like accordance, though perhaps not so strongly shown, is yet clearly observable.

Reasoning hence, guided by analogy, might it not be expected that in the instance of fishes, inasmuch as their temperature is low, and the quantity of carbonic acid evolved small, that their urinary secretion also would be small—proportionally small? And, granted that it is so, as the results of the experiments described would seem to indicate, does it not lead to another conclusion, viz., that subsisting, with few exceptions, exclusively on animal food, this their food, under the influence of a high digestive power, is almost entirely assimilated, and that no more is expended on the urinary secretion than is requisite to balance the small amount consumed in carrying on the aërating process? And if this be admitted, does it not help to explain some of their peculiarities,—their remarkable rapidity of growth when supplied with abundance of food,—their little waste of substance when sparingly supplied, and their long endurance without loss of life, under a total, or nearly total, privation of aliment?

The history of the salmon and its congeners, which of late years has been so carefully and successfully studied, might be adduced in illustration,—exemplifying, 1*st*, The great activity and power of the organs carrying on the digestive functions,—the stomach itself of the captured fish, with the parietes adjoining, being found more or less dissolved by the action of the gastric juice in the short space of a few hours, and in being always found empty in the migrating fish; 2*dly*, The extraordinary increase in weight during the short sojourn of the young salmon in the sea, when, without stint of food, it passes from the smolt stage of growth to that of the grilse; and, 3*dly*, The comparatively very slow growth of the young salmon in its parr stage, during the months of winter and early spring, when its food is scarce.

LESKETH HOW, AMBLESIDE, *Dec.* 1, 1856.

* I may mention as an instance the swallow, feeding like the trout, when the food of the latter is chiefly insects, and, as regards the secretion in question, showing a remarkable difference. From the nest of a pair I had an opportunity of observing, the young of which were only a few days old, the droppings on a flag-stone beneath, in one day, were as many as forty-five; those collected and dried thoroughly weighed 78·3 grains; the following day, the droppings were seventy. They consisted chiefly of lithate of ammonia with a little urea, and of the indigestible remains of insects,—the urinous portion by far the largest. The excrement, it may be inferred, was chiefly from the young birds, as the parent birds were almost constantly on the wing providing food. How large in quantity was this excrement in comparison with the bulk of the birds! I have found an old swallow to weigh only about 300 grains, and when thoroughly dried no more than 105 grains, so that the amount of excrement in two days exceeded considerably in weight one of the old birds!

# XXXIII.—*On the Minute Structure of Involuntary Muscular Fibre.* By Joseph Lister, Esq., F.R.C.S. Eng. and Edin., Assistant-Surgeon to the Royal Infirmary, Edinburgh. Communicated by Dr Christison.

(Read 1st December 1856.)

It has been long known that contractile tissue presents itself in the human body in two forms, one composed of fibres of considerable magnitude, and therefore readily visible under a low magnifying power, and marked very characteristically with transverse lines at short intervals, the other consisting of fibres much more minute, of exceedingly soft and delicate aspect, and destitute of transverse striæ. The former variety constitutes the muscles of the limbs, and of all parts whose movements are under the dominion of the will; while the latter forms the contractile element of organs, such as the intestines, which are placed beyond the control of volition. There are, however, some exceptions to this general rule, the principal of which is the heart, whose fibres are a variety of the striped kind.

Till within a recent period the fibres of unstriped or involuntary muscle were believed to be somewhat flattened bands of uniform width and indefinite length, marked here and there with roundish or elongated nuclei; but in the year 1847, Professor Kölliker of Würzburg announced that the tissue was resolvable into simpler elements, which he regarded as elongated cells, each of somewhat flattened form, with more or less tapering extremities, and presenting at its central part one of the nuclei above mentioned. These "contractile" or "muscular fibre-cells," as he termed them, were placed in parallel juxtaposition in the tissue, adhering to each other, as he supposed, by means of some viscid connecting substance. In the following year the same distinguished anatomist gave a fuller account of his discovery in the 1st volume of the *Zeitschrift für Wissenschaftliche Zoologie*, and described in a most elaborate manner the appearances which the tissue presented in all parts of the body where unstriped muscle had been previously known to occur, and also in situations, such as the iris and the skin, where its existence had before been only matter of conjecture, but where the characteristic form of the fibre-cells, and of their "rod-shaped" nuclei had enabled him to recognise it with precision. Confirmations of this view of the structure of involuntary muscular fibre were afterwards received from various quarters, one of the most important being the observation made in 1849 by Reichert, a German histologist, that dilute nitric or muriatic acid loosens the cohesion of the fibre-cells, and enables them to be isolated with much greater facility. In 1852 I wrote a paper "On the Contractile Tissue of the Iris," published in the *Micro-*

*scopical Journal*, in which I gave an account of the involuntary muscular fibre contained in that organ in man and some of the lower animals, stating that the appearances I had met with corresponded exactly with Kölliker's descriptions, and illustrating my remarks with careful sketches of several fibre-cells from the human iris, isolated by tearing a portion of the sphincter pupillæ with needles in a drop of water. In 1853, another paper by myself appeared in the same Journal, "On the Contractile Tissue of the Skin," confirming Kölliker's recent discovery of the "*arrectores pili*," and describing the distribution of those little bundles of unstriped muscle in the scalp. These and other investigations into the involuntary muscular tissue convinced me of the correctness of Kölliker's observations, and led me to regard his discovery as one of the most beautiful ever made in anatomy; and this is now, I believe, the general opinion of histologists.

Still, however, there are those who are not yet satisfied upon this subject. In Müller's Archives for 1854, is a paper by Dr J. F. Mazonn of Kiew, in which the author expresses his belief that the muscular fibre-cells of Kölliker are created by the tearing of the tissue in preparing it, and denies the existence of nuclei in unstriped muscle altogether; but he gives so very obscure an account of his own ideas respecting the tissue, that his objections seem to me to carry very little weight, more especially as the appearances which he describes require, according to his own account, several days' maceration of the muscle in acid for their development. In June of the present year (1856), Professor Ellis of University College, London, communicated to the Royal Society of London a paper entitled "Researches into the Nature of Involuntary Muscular Fibre." In the abstract given in the "Proceedings" of the Society, recently issued, we are informed that, "having been unable to confirm the statements of Professor Kölliker respecting the cell-structure of the involuntary muscular fibre, the author was induced to undertake a series of researches into the nature of that tissue, by which he has been led to entertain views as to its structure in vertebrate animals, but more especially in man, which are at variance with those now generally received." In the "summary of the conclusions which the author has arrived at," we find the following: "In both kinds of muscles, voluntary and involuntary, the fibres are long, slender, rounded cords of uniform width . . . ." "In neither voluntary nor involuntary muscle is the fibre of the nature of a cell, but in both is composed of minute threads or fibrils. Its surface-appearance, in both kinds of muscle, allows of the supposition that in both it is constructed in a similar way, viz., of small particles or "sarcous elements," and that a difference in the arrangement of these elements gives a *dotted* appearance to the involuntary, and a transverse striation to the voluntary fibres." "On the addition of acetic acid, fusiform or rod-shaped corpuscles make their appearance in all muscular tissue; these bodies, which appear to belong to the sheath of the fibre, approach nearest in their characters to the corpuscles belonging to the yellow or elastic fibres which pervade va-

rious other tissues; and from the apparent identity in nature of these corpuscles in the different textures in which they are found, and especially in voluntary, as compared with involuntary muscle, it is scarcely conceivable that in the latter case exclusively they should be the nuclei of oblong cells constituting the proper muscular tissue."

Mr Ellis, then, agrees with Mazonn in believing that the tapering fibre-cells of Kölliker owe their shape to tearing of the tissue; and he regards the nuclei as mere accidental accompaniments of the proper muscular structure, probably belonging to the sheath of the fibres, which, according to him, are of rounded form and uniform width.

The distinguished position of Mr Ellis as an anatomist makes it very desirable that his opinion on this important subject should be either confirmed or refuted, and the object of the present paper is to communicate some facts which have recently come under my observation, and which, I hope, may prove to others as unequivocally as they have done to myself, the truth of Kölliker's view of this question.

In September last, being engaged in an inquiry into the process of inflammation in the web of the frog's foot, I was desirous of ascertaining more precisely the structure of the minute vessels, with a view to settling a disputed point regarding their contractility.

Having divided the integument along the dorsal aspect of two contiguous toes, I found that the included flap could be readily raised, so as to separate the layers of skin of which the web consists, the principal vessels remaining attached to the plantar layer. Having raised with a needle as many of the vascular branches as possible, I found, on applying the microscope, that they included arteries of extreme minuteness, some of them, indeed, of smaller calibre than average capillaries. A high magnifying power showed that these smallest arteries consisted of an external layer of longitudinally arranged cellular fibres in variable quantity, an internal exceedingly delicate membrane, and an intermediate circular coat, which generally constituted the chief mass of the vessel, but which proved to consist of neither more nor less than a single layer of muscular fibre-cells, each wrapped in a spiral manner round the internal membrane, and of sufficient length to encircle it from about one-and-a-half to two-and-a-half times. Fig. 18. (Plate XV.) represents one of these vessels as seen under a rather low power, and shows the general spiral arrangement of the fibres of the middle coat. Fig. 19. is a camera lucida sketch of the same artery highly magnified, in which I have for the most part traced the outline of the fibres on the nearer side of the vessel only, but one fibre-cell is shown in its entire length wrapped round nearly two-and-a-half times in a loose spiral. In some other vessels the muscular elements were arranged in closer spirals, as in figs. 20 and 21. They are seen to have more cr less pointed extremities, and are provided with an oval nucleus at

their broadest part, discernible distinctly, though somewhat dimly, without the application of acetic acid. The tubular form of the vessels enables the observer, by proper adjustment of the focus, to see the fibre-cells in section; they are then observed to be substantial bodies, often as thick as they are broad, though the latter dimension generally exceeds the former. Here and there a nucleus is so placed in the artery as to appear in section with the fibre-cell, as shown in figs. 20, 22, and 23. The section of the nucleus is in such cases invariably found surrounded by that of the substance of the fibre-cell, though occasionally placed eccentrically in it. From the circular form of its section the nucleus appears to be cylindrical. These fibre-cells are from $\frac{1}{200}$ inch to $\frac{1}{100}$ inch in length, from $\frac{1}{2500}$ inch to $\frac{1}{2000}$ inch in breadth, and about $\frac{1}{2500}$ inch in thickness, measurements on the whole rather greater than those given by KÖLLIKER for the human intestine, the chief difference being that in the frog's arteries they are somewhat broader and thicker.

Now, the middle coat of the small arteries is universally admitted to be composed chiefly of involuntary muscular fibre; but in the vessels just described it consists of nothing whatever else than elongated, tapering bodies, corresponding in dimensions with KÖLLIKER's fibre-cells, and each provided with a single cylindrical nucleus embedded in its substance. Considering, then, that no tearing of the tissue had been practised in the preparation of the objects, but that the parts were seen undisturbed in their natural relations, it appeared to me that the simple observation above related settled the point at issue conclusively.

It was, however, suggested to me by an eminent physiologist, that the various forms in which contractile tissue occurs in the animal kingdom forbid our drawing any positive inference regarding the structure of human involuntary muscle from an observation made on the arteries of the frog. Being anxious to avoid all cavil, and understanding that Mr ELLIS's researches had been directed chiefly to the hollow viscera, I thought it best to examine the tissue in some such organ. For this purpose I obtained a portion of the small intestine of a freshly killed pig, selecting that animal on account of the close general resemblance between its tissues and those of man. The piece of gut happened to be tightly contracted, and on slitting it up longitudinally, the mucous membrane, which was thrown into loose folds, was very readily detached from the subjacent parts. I raised one of the thick, but pale and soft fasciculi of the circular coat, and teased it out with needles in a drop of water, reducing it without difficulty to extremely delicate fibrils. On examining the object with the microscope, I found that it was composed of involuntary muscular fibre, almost entirely unmixed with other tissue, reminding me precisely of what I had seen in the human sphincter pupillæ, except that the appearances were more distinct, especially as regards the nuclei, which were clearly apparent without the application of acetic acid. Several of the fibre-cells were isolated in the first specimen I ex-

amined, each one presenting tapering extremities about equidistant from a single elongated nucleus. The fibre-cells were of soft and delicate aspect, generally homogeneous or faintly granular, with sometimes a slight appearance of longitudinal striæ, such as is represented in fig. 4.

I had now seen enough to satisfy my own mind that the involuntary muscular fibre of the pig's intestine was similarly constituted with that of the human iris and the frog's artery: but before throwing up the investigation, I thought it right to examine carefully some short, substantial-looking bodies of high refractive power, which at first sight appeared, both from their form and the aspect of their constituent material, totally different in nature from the rest of the tissue. Several of these bodies are represented in figs. 10–15. Each is seen to be of somewhat oval shape, with more or less pointed extremities, and presents several strongly marked, thick, transverse ridges upon its surface; and each, without exception, possesses a roundish nucleus whose longer diameter lies across that of the containing mass. Yet between these bodies and the long and delicate homogeneous fibre-cells above described, every possible gradation could be traced. Figs. 8 and 9, are somewhat longer than those just indicated, and are also remarkable for their regularity. In figs. 5, 6, and 7, are represented fibre-cells of considerable length, marked here and there with highly refracting transverse bands, in the intervals of which they are of soft and delicate aspect. In several cells one half was short, with closely approximated rugæ, the other half long and homogeneous. Hence it was pretty clear that the appearances in question were due to contraction of the fibre-cells, and that the shortest of these bodies were examples of an extreme degree of that condition; their substantial aspect and considerable breadth being produced by the whole material of the long muscular elements being drawn together into so small a compass. The rounded appearance of the nuclei was accounted for by supposing either that they had themselves contracted, or that they had been pinched up by the contracting fibres, of which explanations the latter appears the more probable.

In order to place the matter if possible beyond doubt, I prepared two contiguous portions of the circular coat of a contracted piece of intestine in different ways; the one by simply cutting off a minute portion with sharp scissors, so as to avoid as much as possible any stretching of the tissue, the other by purposely drawing out a fasciculus to a very considerable length, and then teasing it with needles. In the former preparation, the fibre-cells appeared all of them more or less contracted, except in parts where the slight traction inseparable from any mode of preparation had stretched the pliant tissue, which in the fresh state appears to yield as readily to any extending force as does a relaxed muscle of a living limb. In the other object, where the tissue had been purposely stretched, most of the fibre-cells were extended, and possessed elongated nuclei. Here and there one

would be seen of excessive tenuity, scarcely broader at its thickest part than the nucleus, looking, under the highest magnifying power, like a delicate thread of spun glass. To how great a length the fibre-cells admit of being drawn out in this way without breaking I cannot tell. Fig. 1 represents a portion of such a fibre with the contained nucleus. Among these extended fibres, however, there lay, here and there, an extremely contracted one, the result, I have no doubt, of the irritation produced by the needles upon the yet living tissue. In order to guard against this source of fallacy, I kept a piece of contracted gut 48 hours, and then examined two contiguous parts of the circular coat in the way above described. The muscle was much less readily extended than in the fresh state, and I found that, where stretching of the tissue had been avoided as much as possible, it was composed entirely of fibre-cells marked with transverse ridges of varying thickness and proximity; a minute fibril having, under a rather low power, the general aspect represented in fig. 17. But I saw no distinct examples of the extreme degree of contraction so frequent in muscle from the same piece of intestine in the fresh state. This confirmed my suspicion that the latter had been induced by the irritation of the mode of preparation. On the other hand, a fully stretched fasciculus showed its fibres everywhere destitute of transverse rugæ, so that the point was now distinctly proved. Kölliker, in his original article in the *Zeitschrift für Wissensehaftliche Zoologie*, figured some long fibre-cells with transverse lines upon them,—" knotty swellings," as he termed them, which he supposed probably due to contraction, and he repeats this hypothesis in the part of his *Mikroskopische Anatomie*, published in 1852. The *proof* of the correctness of this idea is now, I believe, given for the first time.

The bearings of these observations on the main question respecting the structure of involuntary muscular fibre are obvious and important. In the first place, if the short, substantial bodies were mere contracted fragments of rounded fibres of uniform width, we should expect them to be as thick at their extremities as at the centre, instead of which they are always more or less tapering, and often present a very regular appearance of two cones applied to each other by their bases. Secondly, the uniform central position of the nuclei in the contracted fibres, proves clearly that the former are no accidental appendages of the latter, to which it seems difficult to refuse Kölliker's appellation of *cells*.

The effect of acetic acid on the involuntary muscular tissue is to render the fibres indistinct, but the nuclei more apparent; and if this reagent be applied to a piece of contracted muscle, many of the nuclei are seen to be of more or less rounded form. The deviation of the nuclei from the " rod-shape" has hitherto been a puzzling appearance, but is now satisfactorily accounted for.

In examining a fasciculus that had been fully stretched, 48 hours after death, I met with several good specimens of isolated fibre-cells, two of which are represented in figs. 2 and 3. I would draw particular attention to the delicate, spirally-twisted extremities of the fibre-cell 3, such as no tearing of a continuous fibre could possibly have produced. Though these fibres are very long, yet we have no reason to believe that anything near the extreme degree of extension has been attained in them, and we cannot but contemplate with amazement the extent of contractility possessed by this tissue.

In fig. 16 is represented a portion of a fibre-cell curled up, which has been introduced for the sake of the clear manner in which it shows the position of the nucleus embedded in it. Just as in the case of the fibres wrapped round the arteries of the frog's foot, this cell might be seen in section by proper adjustment, and that section is observed to be oval; proving that the fibre is not round, but somewhat flattened. It happens that the nucleus appears at this point; its section is circular, and is surrounded on all sides by the substance of the cell.

The pig's intestine seems to be a peculiarly favourable situation for the investigation of unstriped muscle. Judging from Kölliker's measurements, the fibres appear to be of much larger size there than in the same situation in the human body. The length of the fibre-cell 3 is $\frac{1}{37}$ inch. The fibre 2 is imperfect at one extremity; but, taking the double of the distance from its pointed end to the nucleus, its length is $\frac{1}{33}$ inch. These measurements are between three and four times greater than any which Professor Kölliker has given for the human intestine, and considerably exceed the length of the "colossal fibre-cells" which he describes as occurring in the gravid uterus. The individual fibre-cells, with their nuclei and transverse markings, if they have any, are quite distinctly to be seen with one of Smith and Beck's $\frac{4}{10}$ object-glasses. But in order to examine their structure minutely, a higher power is required: that which I use is a first-rate $\frac{1}{12}$, made several years ago by Mr Powell of London. All the figures in Plate XV., except 17 and 18, are from camera lucida sketches, reduced to the same scale. The principal measurements of the fibre-cells from the pig's intestine are as under:—

| | |
|---|---|
| Length of fibre-cell, 3, | $\frac{1}{37}$ inch. |
| Breadth of ditto, | $\frac{1}{3300}$ " |
| Length of nucleus of ditto, | $\frac{1}{1000}$ " |
| Breadth of ditto, | $\frac{1}{8000}$ " |
| Breadth of fibre-cell, 16, | $\frac{1}{3000}$ " |
| Thickness of ditto, | $\frac{1}{4000}$ " |
| Length of fibre-cell, 13, | $\frac{1}{750}$ " |
| Breadth of ditto, | $\frac{1}{1250}$ " |
| Longitudinal measurement of nucleus of ditto, | $\frac{1}{5500}$ " |
| Transverse, ditto, | $\frac{1}{3500}$ " |
| Length of fibre-cell, 15, | $\frac{1}{1000}$ " |

Hence it appears that the length of the most contracted fibre-cell is the same as that of the nucleus of an extended one. The fibres vary somewhat in breadth, independently of the results of contraction. Thus, one in the extended condition which I sketched, but which is not here shown, measured only $\frac{1}{4000}$ inch across. The nuclei of the uncontracted fibres are very constantly of the same length, and are good examples of the rod-shape to which KÖLLIKER has directed particular attention. They always possess one or two nucleoli, and have often a slightly granular character; occasionally, as in fig. 21, they present an appearance of transverse markings. One frequently sees near the nucleus of a fibre that has been artificially extended from the contracted state, an appearance of a gap in the substance of the cell, forming a sort of extension of the nucleus, as if the fibre generally had been stretched more completely than the nucleus: an example of this is presented by fig. 7. Mr ELLIS lays great stress on a dotted appearance which he considers characteristic of involuntary muscular fibre. I must say I agree with KÖLLIKER in finding the fibre-cells, for the most part, homogeneous when extended, or faintly marked with longitudinal striæ.* No doubt dots are present in abundance; but these, so far as I have observed them in the pig's intestine, are distinctly exterior to the fibres, though adherent to their surface; and I suspect them to be little globules of a tenacious connecting fluid. That the fibre-cells do stick very tightly together, may be seen by drying a minute portion of the tissue, after which they will be found shrunk, and slightly separated from one another, but connected more or less by minute threads.

To sum up the general results to which we are led by the facts above mentioned. It appears that in the arteries of the frog, and in the intestine of the pig, the involuntary muscular tissue is composed of slightly-flattened elongated elements, with tapering extremities, each provided at its central and thickest part with a single cylindrical nucleus embedded in its substance.

Professor KÖLLIKER's account of the tissue being thus completely confirmed in these two instances, and the description here given of its appearance in the arteries of the frog's foot being an independent confirmation of the general doctrine, there seems no reason any longer to doubt its truth.

* The longitudinal striæ above referred to, are probably due to a fine fibrous structure in the substance of the fibre-cells. When in London, last Christmas, I had, through the kindness of Dr SHARPEY, the opportunity of examining a specimen of muscle from the stomach of a rabbit, which he had prepared after REICHERT's method. The nitric acid had not only detached the fibre-cells from one another, but also brought out very distinctly in each muscular element the appearance of minute parallel longitudinal fibres, which seemed to make up the entire mass of the fibre-cell except the nucleus. In a plate accompanying the paper on the Iris, before referred to, I gave figures of some fibre-cells with distinct granules arranged in longitudinal and transverse rows. This appearance, which, however, so far as my experience goes, is exceptional, and is hardly sufficiently marked to deserve the appellation "dotted," is probably caused by unequal contractions in the constituent material.—2*d April* 1857.

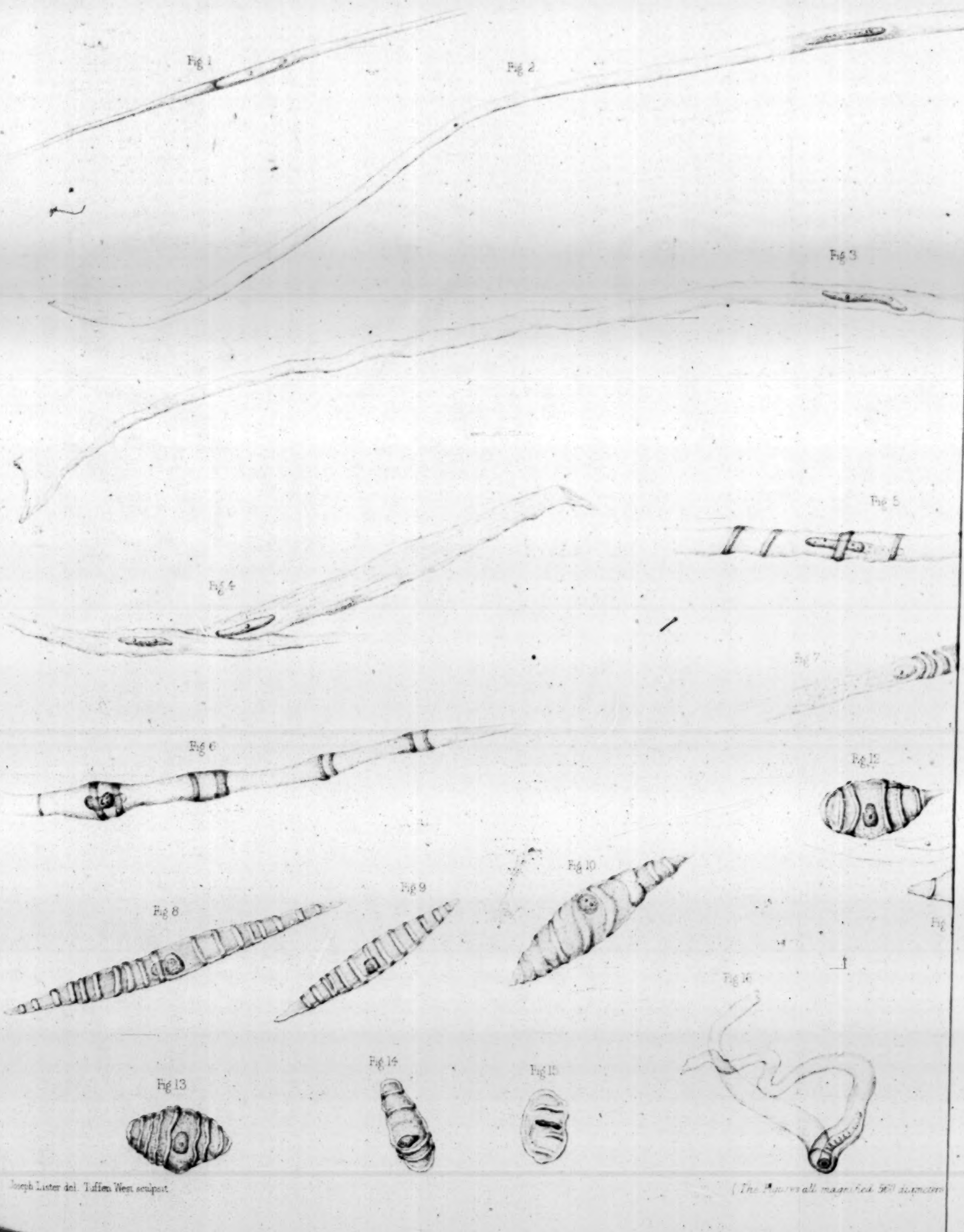
It further appears, that in the pig's intestine the muscular elements are, on the one hand, capable of an extraordinary degree of extension, and, on the other hand, are endowed with a marvellous faculty of contraction, by which they may be reduced from the condition of very long fibres to that of almost globular masses In the extended state they have a soft, delicate, and usually homogeneous aspect, which becomes altered during contraction by the supervention of highly refracting transverse ribs, which grow thicker and more approximated as the process advances. Meanwhile, the "rod-shaped" nucleus appears to be pinched up by the contracting fibre till it assumes a slightly oval form, with the longer diameter transversely placed.

I will only further remark, that these properties of the constituent elements of involuntary muscular fibre explain, in a very beautiful manner, the extraordinary range of contractility which characterizes the hollow viscera.

---

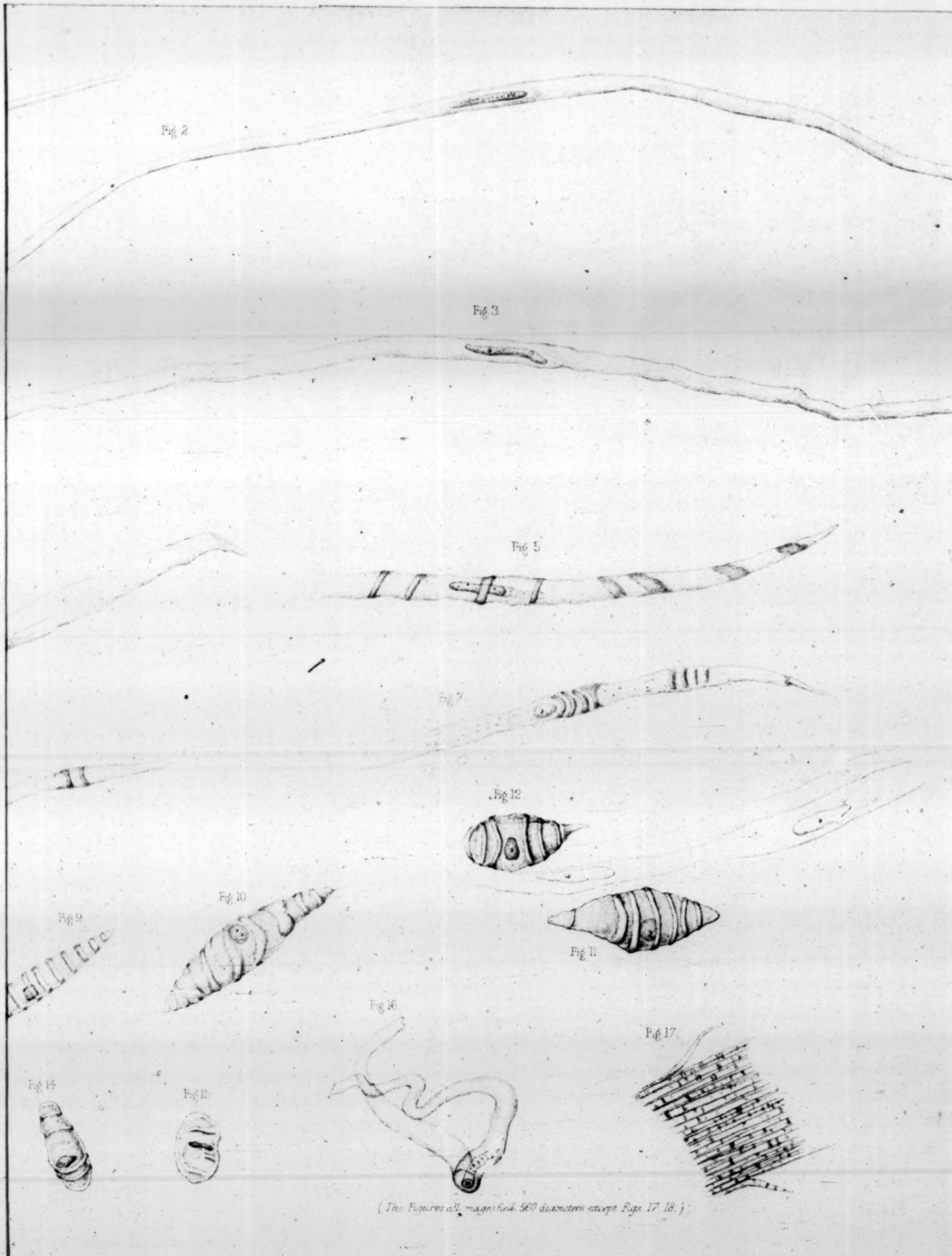
## EXPLANATION OF PLATE XV.

Fig. 1 represents part of a fibre-cell from the pig's intestine, drawn out into a very fine thread.
Figs. 2 and 3, fibre-cells from the same situation, considerably extended.
Fig. 4, fibre-cells exhibiting faint longitudinal striation.
Figs. 5, 6, and 7, fibre-cells imperfectly contracted.
Figs. 8 and 9, small fibre-cells considerably contracted.
Figs. 10, 11, 12, 13, 14 and 15, fibre-cells extremely contracted.
Fig. 16, a fibre-cell curled up, showing the position of the nucleus embedded in its substance.
Fig. 17, part of a moderately contracted fasciculus of unstriped muscle from the pig's intestine, as seen under a rather low magnifying power.
Fig. 18, a small artery from the frog's web, under a rather low magnifying power.
Fig 19, part of the same vessel highly magnified, showing the spiral arrangement of the muscular fibre-cells.
Figs. 20 and 21, muscular fibre-cells from another artery. In fig. 20, the spirals are much closer than in fig. 19; and in fig. 21, the spiral is quite close.
Figs. 22 and 23 represent some fibre-cells in arteries of extreme minuteness, and show the section of the nucleus surrounded by that of the fibre-cell.




Joseph Lister del. Tuffen West sculpsit.

(The Figures all magnified 500 diameters




( The Figures all magnified 560 diameters except Figs. 17. 18. )

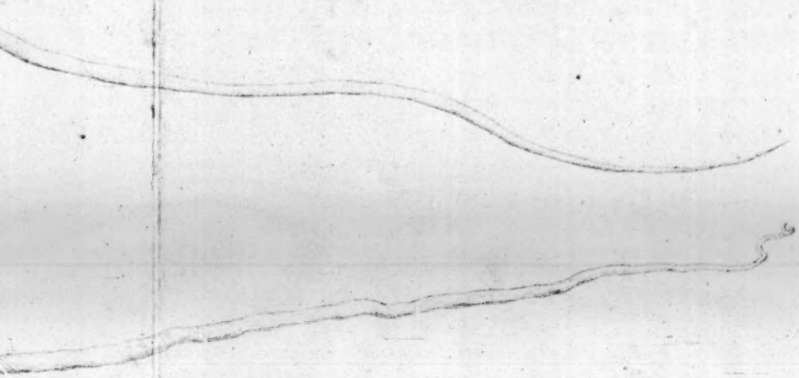


Fig 18.

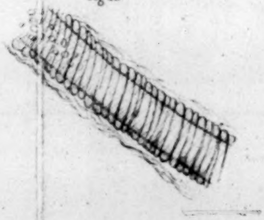


Fig 19.

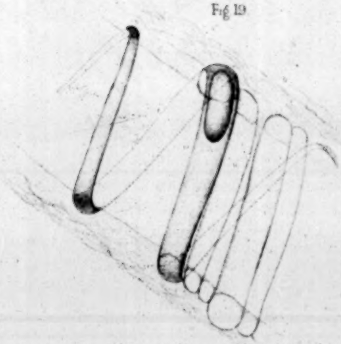


Fig 20.

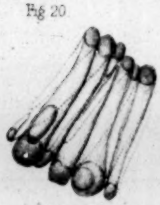
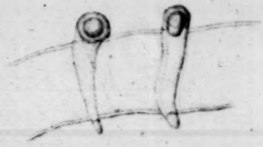


Fig 21.

Fig 22.

Fig 23.



W West imp. London

# XXXIV.—*On a Dynamical Top, for exhibiting the phenomena of the motion of a system of invariable form about a fixed point, with some suggestions as to the Earth's motion.* By J. C. Maxwell, B.A., Professor of Natural Philosophy in Marischal College, Aberdeen.

(Read 20th April 1857.)

To those who study the progress of exact science, the common spinning-top is a symbol of the labours and the perplexities of men who had successfully threaded the mazes of the planetary motions. The mathematicians of the last age, searching through nature for problems worthy of their analysis, found in this toy of their youth, ample occupation for their highest mathematical powers.

No illustration of astronomical precession can be devised more perfect than that presented by a properly balanced top, but yet the motion of rotation has intricacies far exceeding those of the theory of precession.

Accordingly, we find Euler and D'Alembert devoting their talent and their patience to the establishment of the laws of the rotation of solid bodies. Lagrange has incorporated his own analysis of the problem with his general treatment of mechanics, and since his time M. Poinsot has brought the subject under the power of a more searching analysis than that of the calculus, in which ideas take the place of symbols, and intelligible propositions supersede equations.

In the practical department of the subject, we must notice the rotatory machine of Bohnenberger, and the nautical top of Troughton. In the first of these instruments we have the model of the Gyroscope, by which Foucault has been able to render visible the effects of the earth's rotation. The beautiful experiments by which Mr J. Elliot has made the ideas of precession so familiar to us are performed with a top, similar in some respects to Troughton's, though not borrowed from his.

The top which I have the honour to spin before the Society, differs from that of Mr Elliot in having more adjustments, and in being designed to exhibit far more complicated phenomena.

The arrangement of these adjustments, so as to produce the desired effects, depends on the mathematical theory of rotation. The method of exhibiting the motion of the axis of rotation, by means of a coloured disc, is essential to the success of these adjustments. This optical contrivance for rendering visible the nature of the rapid motion of the top, and the practical methods of applying the theory of rotation to such an instrument as the one before us, are the grounds on which I bring my instrument and experiments before the Society as my own.

I propose, therefore, in the first place, to give a brief outline of such parts of

the theory of rotation as are necessary for the explanation of the phenomena of the top.

I shall then describe the instrument with its adjustments, and the effect of each, the mode of observing of the coloured disc when the top is in motion, and the use of the top in illustrating the mathematical theory, with the method of making the different experiments.

Lastly, I shall attempt to explain the nature of a possible variation in the earth's axis due to its figure. This variation, if it exists, must cause a periodic inequality in the latitude of every place on the earth's surface, going through its period in about eleven months. The amount of variation must be very small, but its character gives it importance, and the necessary observations are already made, and only require reduction.

*On the Theory of Rotation.*

The theory of the rotation of a rigid system is strictly deduced from the elementary laws of motion, but the complexity of the motion of the particles of a body freely rotating renders the subject so intricate, that it has never been thoroughly understood by any but the most expert mathematicians. Many who have mastered the lunar theory have come to erroneous conclusions on this subject; and even NEWTON has chosen to deduce the disturbance of the earth's axis from his theory of the motion of the nodes of a free orbit, rather than attack the problem of the rotation of a solid body.

The method by which M. POINSOT has rendered the theory more manageable, is by the liberal introduction of "appropriate ideas," chiefly of a geometrical character, most of which had been rendered familiar to mathematicians by the writings of MONGE, but which then first became illustrations of this branch of dynamics. If any further progress is to be made in simplifying and arranging the theory, it must be by the method which POINSOT has repeatedly pointed out as the only one which can lead to a true knowledge of the subject,—that of proceeding from one distinct idea to another, instead of trusting to symbols and equations.

An important contribution to our stock of appropriate ideas and methods has lately been made by Mr R. B. HAYWARD, in a paper, "On a Direct Method of estimating Velocities, Accelerations, and all similar quantities, with respect to axes, moveable in any manner in Space." (*Trans. Cambridge Phil. Soc.* vol. x. part i.)

* In this communication I intend to confine myself to that part of the subject which the top is intended to illustrate, namely, the alteration of the position of the axis in a body rotating freely about its centre of gravity. I shall, therefore, deduce the theory as briefly as possible, from two considerations only,—the per-

* 7th May 1857.—The paragraphs marked thus have been rewritten since the paper was read.

manence of the original *angular momentum* in direction and magnitude, and the permanence of the original *vis viva*.

* The mathematical difficulties of the theory of rotation arise chiefly from the want of geometrical illustrations and sensible images, by which we might fix the results of analysis in our minds.

It is easy to understand the motion of a body revolving about a fixed axle. Every point in the body describes a circle about the axis, and returns to its original position after each complete revolution. But if the axle itself be in motion, the paths of the different points of the body will no longer be circular or re-entrant. Even the velocity of rotation about the axis requires a careful definition, and the proposition that, in all motion about a fixed point, there is always one line of particles forming an instantaneous axis, is usually given in the form of a very repulsive mass of calculation. Most of these difficulties may be got rid of by devoting a little attention to the mechanics and geometry of the problem before entering on the discussion of the equations.

Mr HAYWARD, in his paper already referred to, has made great use of the mechanical conception of Angular Momentum.

DEFINITION.—*The Angular Momentum of a particle about an axis is measured by the product of the mass of the particle, its velocity resolved in the normal plane, and the perpendicular from the axis on the direction of motion.*

* The angular momentum of any system about an axis is the algebraical sum of the angular momenta of its parts.

As the *rate of change* of the *linear momentum* of a particle measures the *moving force* which acts on it, so the *rate of change* of *angular momentum* measures the *moment* of that force about an axis.

All actions between the parts of a system, being pairs of equal and opposite forces, produce equal and opposite changes in the angular momentum of those parts. Hence the whole angular momentum of the system is not affected by these actions and re-actions.

* When a system of invariable form revolves about an axis, the angular velocity of every part is the same, and the angular momentum about the axis is the product of the *angular velocity* and the *moment of inertia* about that axis.

* It is only in particular cases, however, that the *whole* angular momentum can be estimated in this way. In general, the axis of angular momentum differs from the axis of rotation, so that there will be a residual angular momentum about an axis perpendicular to that of rotation, unless that axis has one of three positions, called the principal axes of the body.

By referring everything to these three axes, the theory is greatly simplified. The moment of inertia about one of these axes is greater than that about any other axis through the same point, and that about one of the others is a mini-

mum. These two are at right angles, and the third axis is perpendicular to their plan, and is called the mean axis.

* Let A, B, C be the moments of inertia about the principal axis through the centre of gravity, taken in order of magnitude, and let $\omega_1\ \omega_2\ \omega_3$ be the angular velocities about them, then the angular momentum will be $A\omega_1$, $B\omega_2$ and $C\omega_3$.

Angular momentum may be compounded like forces or velocities, by the law of the "parallelogram," and since these three are at right angles to each other, their resultant is

$$\sqrt{A^2\omega_1{}^2+B^2\omega_2{}^2+C^2\omega_3{}^2}=H \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and this must be constant, both in magnitude and direction in space, since no external forces act on the body.

We shall call this axis of angular momentum the *invariable axis.* It is perpendicular to what has been called the invariable plane. Poinsot calls it the axis of the couple of impulsion. The *direction-cosines* of this axis in the body are,

$$l=\frac{A\omega_1}{H} \quad m=\frac{B\omega_2}{H} \quad n=\frac{C\omega_3}{H}$$

Since $l$, $m$, and $n$ vary during the motion, we need some additional condition to determine the relation between them. We find this in the property of the *vis-viva* of a system of invariable form in which there is no friction. The *vis-viva* of such a system must be constant. We express this in the equation

$$A\omega_1{}^2+B\omega_2{}^2+C\omega_3{}^2=V. \quad . \quad . \quad . \quad (2)$$

Substituting the values of $\omega_1\ \omega_2\ \omega_3$ in terms of $l$, $m$, $n$

$$\frac{l^2}{A}+\frac{m^2}{B}+\frac{n^2}{C}=\frac{V}{H^2}$$

Let $\dfrac{1}{A}=a^2$, $\dfrac{1}{B}=b^2$, $\dfrac{1}{C}=c^2$, $\dfrac{V}{H^2}=e^2$

and this equation becomes

$$a^2\ l^2+b^2\ m^2+c^2\ n^2=e^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the equation to the cone, described by the invariable axis within the body, is

$$(a^2-e^2)x^2+(b^2-e^2)\ y^2+(c^2-e^2)z^2=0 \quad . \quad . \quad . \quad . \quad (4)$$

The intersections of this cone with planes perpendicular to the principal axes are found by putting $x$, $y$, or $z$, constant in this equation. By giving $e$ various values, all the different paths of the pole of the invariable axis, corresponding to different initial circumstances, may be traced.

* In the figures, I have supposed $a^2=100$, $b^2=107$, and $c^2=110$. The first figure represents a section of the various cones by a plane perpendicular to the axis of $x$, which is that of greatest moment of inertia. These sections are ellipses having their major axis parallel to the axis of $b$. The value of $e^2$ corresponding to each of these curves is indicated by figures beside the curve. The ellipticity

increases with the size of the ellipse, so that the section corresponding to $e^2=107$ would be two parallel straight lines (beyond the bounds of the figure), after which the sections would be hyperbolas.

* The second figure represents the sections made by a plane, perpendicular to the *mean* axis. They are all hyperbolas, except when $e^2=107$, when the section is two intersecting straight lines.

The third figure shows the sections perpendicular to the axis of least moment of inertia. From $e^2=110$ to $e^2=107$ the sections are ellipses, $e^2=107$ gives two parallel straight lines, and beyond these the curves are hyperbolas.

* The fourth and fifth figures show the sections of the series of cones made by a cube and a sphere respectively. The use of these figures is to exhibit the connexion between the different curves described about the three principal axes by the invariable axis during the motion of the body.

* We have next to compare the velocity of the invariable axis with respect to the body, with that of the body itself round one of the principal axes. Since the invariable axis is fixed in space, its motion relative to the body must be equal and opposite to that of the portion of the body through which it passes. Now the angular velocity of a portion of the body whose direction-cosines are $l$, $m$, $n$, about the axis of $x$ is

$$\frac{\omega_1}{1-l^2}-\frac{l}{1-l^2}(l\,\omega_1+m\omega_2+n\omega_3)$$

Substituting the values of $\omega_1$, $\omega_2$, $\omega_3$, in terms of $l$, $m$, $n$, and taking account of equation (3), this expression becomes

$$\mathrm{H}\frac{(a^2-e^2)}{1\ \ l^2}\,l$$

Changing the sign and putting $l=\frac{\omega_1}{a^2\,\mathrm{H}}$ we have the angular velocity of the invariable axis about that of $x$.

$$=\frac{\omega_1}{1-l^2}\quad\frac{e^2-a^2}{a^2}$$

always positive about the axis of greatest moment, negative about that of least moment, and positive or negative about the mean axis according to the value of $e^2$. The direction of the motion in every case is represented by the arrows in the figures. The arrows on the outside of each figure indicate the direction of rotation of the body.

* If we attend to the curve described by the pole of the invariable axis on the sphere in fig. 5, we shall see that the areas described by that point, if projected on the plane of $y\,z$, are swept out at the rate

$$\omega_1\frac{e^2-a^2}{a^2}$$

Now the axes of the projection of the spherical ellipse described by the pole are,

$$\sqrt{\frac{e^2-a^2}{b^2-a^2}} \text{ and } \sqrt{\frac{e^2-a^2}{c^2-a^2}}$$

Dividing the area of this ellipse by the area described during one revolution of the body, we find the number of revolutions of the body during the description of the ellipse—

$$=\frac{a^2}{\sqrt{b^2-a^2}\,\sqrt{c^2-a^2}}$$

The projections of the spherical ellipses upon the plane of $yz$ are all similar ellipses, and described in the same number of revolutions; and in each ellipse so projected, the area described in any time is proportional to the number of revolutions of the body about the axis of $x$, so that if we measure time by revolutions of the body, the motion of the projection of the pole of the invariable axis is identical with that of a body acted on by an attractive central force varying directly as the distance. In the case of the hyperbolas in the plane of the greatest and least axis, this force must be supposed repulsive. The dots in the figures 1, 2, 3, are intended to indicate roughly the progress made by the invariable axis during each revolution of the body about the axis of $x$, $y$, and $z$ respectively. It must be remembered, that the rotation about these axes varies with their inclination to the invariable axis, so that the angular velocity diminishes as the inclination increases, and therefore the areas in the ellipses above mentioned are not described with uniform velocity in absolute time, but are less rapidly swept out at the extremities of the major axis than at those of the minor.

* When two of the axes have equal moments of inertia, or $b=c$, then the angular velocity $\omega_1$ is constant, and the path of the invariable axis is circular, the number of revolutions of the body during one circuit of the invariable axis, being

$$\frac{a^2}{b^2-a^2}$$

The motion is in the same direction as that of rotation, or in the opposite direction, according as the axis of $x$ is that of greatest or of least moment of inertia.

* Both in this case, and in that in which the three axes are unequal, the motion of the invariable axis in the body may be rendered very slow by diminishing the difference of the moments of inertia. The angular velocity of the axis of $x$ about the invariable axis in space is

$$\omega_1\frac{e^2-a^2l^2}{a^2(1-l^2)}$$

which is greater or less than $\omega_1$, as $e^2$ is greater or less than $a^2$, and, when these quantities are nearly equal, is very nearly the same as $\omega_1$ itself. This quantity indicates the rate of revolution of the axle of the top about its mean position, and is very easily observed.

FIG. 3.

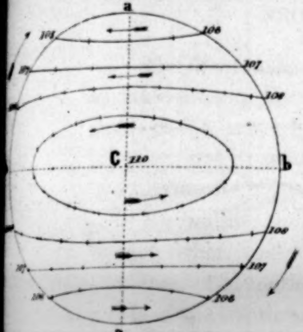



FIG. 2.

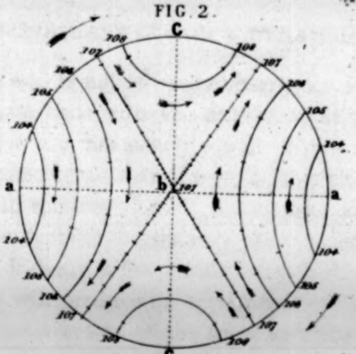
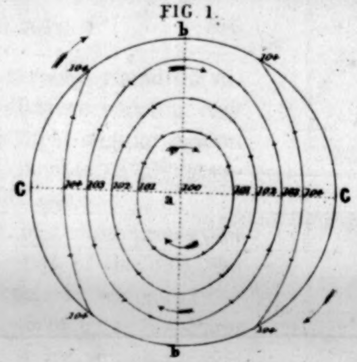



FIG. 1.



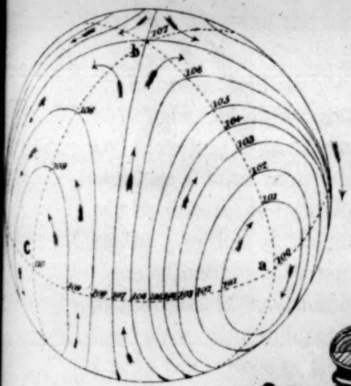




FIG. 5.

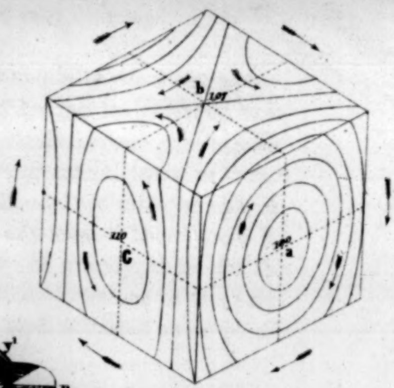



FIG. 4.

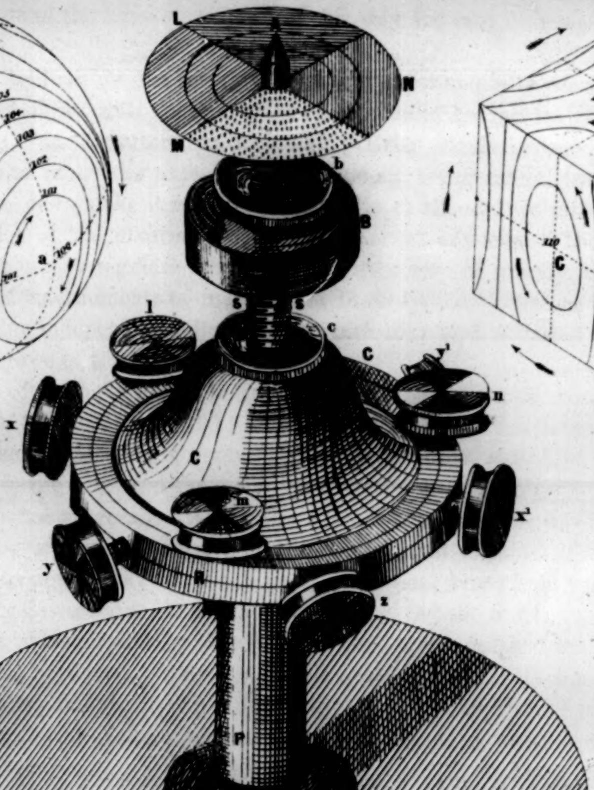



FIG. 6.

* The *instantaneous axis* is not so easily observed. It revolves round the invariable axis in the same time with the axis of $x$, at a distance which is very small in the case when $a$, $b$, $c$, are nearly equal. From its rapid angular motion in space, and its near coincidence with the invariable axis, there is no advantage in studying its motion in the top.

* By making the moments of inertia very unequal, and in definite proportion to each other, and by drawing a few strong lines as diameters of the disc, the combination of motions will produce an appearance of epicycloids, which are the result of the continued intersection of the successive positions of these lines, and the cusps of the epicycloids lie in the curve in which the instantaneous axis travels. Some of the figures produced in this way are very pleasing.

In order to illustrate the theory of rotation experimentally, we must have a body balanced on its centre of gravity, and capable of having its principal axes and moments of inertia altered in form and position within certain limits. We must be able to make the axle of the instrument the greatest, least, or mean principal axis, or to make it not a principal axis at all, and we must be able to *see* the position of the invariable axis of rotation at any time. There must be three adjustments to regulate the position of the centre of gravity, three for the magnitudes of the moments of inertia, and three for the directions of the principal axes, nine independent adjustments, which may be distributed as we please among the screws of the instrument.

The form of the body of the instrument which I have found most suitable is that of a bell, (Plate XVI. fig. 6.) C is a hollow cone of brass, R is a heavy ring cast in the same piece. Six screws, with heavy heads, $x$, $y$, $z$, $x'$, $y'$, $z'$, work horizontally in the ring, and three similar screws, $l$, $m$, $n$, work vertically through the ring at equal intervals. AS is the axle of the instrument, SS is a brass screw working in the upper part of the cone C, and capable of being firmly clamped by means of the nut $c$. B is a cylindrical brass bob, which may be screwed up or down the axis, and fixed in its place by the nut $b$.

The lower extremity of the axle is a fine steel point, finished without emery, and afterwards hardened. It runs in a little agate cup set in the top of the pillar P. If any emery had been embedded in the steel, the cup would soon be worn out. The upper end of the axle has also a steel point by which it may be kept steady while spinning.

When the instrument is in use, a coloured disc is attached to the upper end of the axle.

It will be seen that there are eleven adjustments, nine screws in the brass ring, the axle screwing in the cone, and the bob screwing on the axle. The advantage of the last two adjustments is, that by them large alterations can be made, which are not possible by means of the small screws.

The first thing to be done with the instrument is, to make the steel point at the end of the axle coincide with the centre of gravity of the whole. This is done roughly by screwing the axle to the right place nearly, and then balancing the instrument on its point, and screwing the bob and the horizontal screws till the instrument will remain balanced in any position in which it is placed.

When this adjustment is carefully made, the rotation of the top has no tendency to shake the steel point in the agate cup, however irregular the motion may appear to be.

The next thing to be done, is to make one of the principal axes of the central ellipsoid coincide with the axle of the top.

To effect this, we must begin by spinning the top gently about its axle, steadying the upper part with the finger at first. If the axle is already a principal axis the top will continue to revolve about its axle when the finger is removed. If it is not, we observe that the top begins to spin about some other axis, and the axle moves away from the centre of motion and then back to it again, and so on, alternately widening its circles and contracting them.

It is impossible to observe this motion successfully, without the aid of the coloured disc placed near the upper end of the axis. This disc is divided into sectors, and strongly coloured, so that each sector may be recognised by its colour when in rapid motion. If the axis about which the top is really revolving, falls within this disc, its position may be ascertained by the colour of the spot at the centre of motion. If the central spot appears red, we know that the invariable axis at that instant passes through the red part of the disc.

In this way we can trace the motion of the invariable axis in the revolving body, and we find that the path which it describes upon the disc may be a circle, an ellipse, an hyperbola, or a straight line, according to the arrangement of the instrument.

In the case in which the invariable axis coincides at first with the axle of the top, and returns to it after separating from it for a time, its true path is a circle or an ellipse having the axle in its *circumference*. The true principal axis is at the *centre* of the closed curve. It must be made to coincide with the axle by adjusting the vertical screws $l$, $m$, $n$.

Suppose that the colour of the centre of motion, when farthest from the axle, indicated that the axis of rotation passed through the sector L, then the principal axis must also lie in that sector at half the distance from the axle.

If this principal axis be that of *greatest* moment of inertia, we must *raise* the screw $l$ in order to bring it nearer the axle A. If it be the axis of least moment we must *lower* the screw $l$. In this way we may make the principal axis coincide with the axle. Let us suppose that the principal axis is that of greatest moment of inertia, and that we have made it coincide with the axle of the instrument. Let us also suppose that the moments of inertia about the other axes are equal,

and very little less than that about the axle. Let the top be spun about the axle and then receive a disturbance which causes it to spin about some other axis. The instantaneous axis will not remain at rest either in space or in the body. In space it will describe a right cone, completing a revolution in somewhat less than the time of revolution of the top. In the body it will describe another cone of larger angle in a period which is longer as the difference of axes of the body is smaller. The invariable axis will be fixed in space, and describe a cone in the body.

The relation of the different motions may be understood from the following illustration. Take a hoop and make it revolve about a stick which remains at rest and touches the inside of the hoop. The section of the stick represents the path of the instantaneous axis in space, the hoop that of the same axis in the body, and the axis of the stick the invariable axis. The point of contact represents the pole of the instantaneous axis itself, travelling many times round the stick before it gets once round the hoop. It is easy to see that the direction in which the instantaneous axis travels round the hoop, is in this case the same as that in which the hoop moves round the stick, so that if the top be spinning in the direction L, M, N, the colours will appear in the same order.

By screwing the bob B up the axle, the difference of the axes of inertia may be diminished, and the time of a complete revolution of the invariable axis in the body increased. By observing the number of revolutions of the top in a complete cycle of colours of the invariable axis, we may determine the ratio of the moments of inertia.

By screwing the bob up farther, we may make the axle the principal axis of *least* moment of inertia.

The motion of the instantaneous axis will then be that of the point of contact of the stick with the *outside* of the hoop rolling on it. The order of colours will be N, M, L, if the top be spinning in the direction L, M, N, and the more the bob is screwed up, the more rapidly will the colours change, till it ceases to be possible to make the observations correctly.

In calculating the dimensions of the parts of the instrument, it is necessary to provide for the exhibition of the instrument with its axle either the greatest or the least axis of inertia. The dimensions and weights of the parts of the top which I have found most suitable, are given in a note at the end of this paper.

Now let us make the axes of inertia in the plane of the ring unequal. We may do this by screwing the balance screws $x$ and $x^1$ farther from the axle without altering the centre of gravity.

Let us suppose the bob B screwed up so as to make the axle the axis of least inertia. Then the mean axis is parallel to $xx^1$, and the greatest is at right angles to $xx^1$ in the horizontal plane. The path of the invariable axis on the disc is no longer a circle but an ellipse, concentric with the disc, and having its major axis parallel to the mean axis $xx^1$.

The smaller the difference between the moment of inertia about the axle and about the mean axis, the more eccentric the ellipse will be; and if, by screwing the bob down, the axle be made the mean axis, the path of the invariable axis will be no longer a closed curve, but an hyperbola, so that it will depart altogether from the neighbourhood of the axle. When the top is in this condition it must be spun gently, for it is very difficult to manage it when its motion gets more and more eccentric.

When the bob is screwed still farther down, the axle becomes the axis of greatest inertia, and $xx^1$ the least. The major axis of the ellipse described by the invariable axis will now be perpendicular to $xx^1$, and the farther the bob is screwed down, the eccentricity of the ellipse will diminish, and the velocity with which it is described will increase.

I have now described all the phenomena presented by a body revolving freely on its centre of gravity. If we wish to trace the motion of the invariable axis by means of the coloured sectors, we must make its motion very slow compared with that of the top. It is necessary, therefore, to make the moments of inertia about the principal axes very nearly equal, and in this case a very small change in the position of any part of the top will greatly derange the *position* of the principal axis. So that when the top is well adjusted, a single turn of one of the screws of the ring is sufficient to make the axle no longer a principal axis, and to set the true axis at a considerable inclination to the axle of the top.

All the adjustments must therefore be most carefully arranged, or we may have the whole apparatus deranged by some eccentricity of spinning. The method of making the principal axis coincide with the axle must be studied and practised, or the first attempt at spinning rapidly may end in the destruction of the top, if not of the table on which it is spun.

### *On the Earth's Motion.*

We must remember that these motions of a body about its centre of gravity, are *not* illustrations of the theory of the precession of the Equinoxes. Precession can be illustrated by the apparatus, but we must arrange it so that the force of gravity acts the part of the attraction of the sun and moon in producing a force tending to alter the axis of rotation. This is easily done by bringing the centre of gravity of the whole a little below the point on which it spins. The theory of such motions is far more easily comprehended than that which we have been investigating.

But the earth is a body whose principal axes are unequal, and from the phenomena of precession we can determine the ratio of the polar and equatorial axes of the "central ellipsoid;" and supposing the earth to have been set in motion about any axis except the principal axis, or to have had its original axis disturbed

in any way, its subsequent motion would be that of the top when the bob is a little below the critical position.

The axis of angular momentum would have an invariable position in space, and would travel with respect to the earth round the axis of figure with a velocity $=\omega\frac{C-A}{A}$ where $\omega$ is the sidereal angular velocity of the earth. The apparent pole of the earth would travel (with respect to the earth) from west to east round the true pole, completing its circuit in $\frac{A}{C-A}$ sidereal days, which appears to be about 325·6 solar days.

The instantaneous axis would revolve about this axis in space in about a day, and would always be in a plane with the true axis of the earth and the axis of angular momentum. The effect of such a motion on the apparent position of a star would be, that its zenith distance would be increased and diminished during a period of 325·6 days. This alteration of zenith distance is the same above and below the pole, so that the polar distance of the star is unaltered. In fact the method of finding the pole of the heavens by observations of stars, gives the pole of the *invariable axis*, which is altered only by external forces, such as those of the sun and moon.

There is therefore no change in the apparent polar distance of stars due to this cause. It is the latitude which varies. The magnitude of this variation cannot be determined by theory. The periodic time of the variation may be found approximately from the known dynamical properties of the earth. The epoch of maximum latitude cannot be found except by observation, but it must be later in proportion to the east longitude of the observatory.

In order to determine the existence of such a variation of latitude, I have examined the observations of *Polaris* with the Greenwich Transit Circle in the years 1851-2-3-4. The observations of the upper transit during each month were collected, and the mean of each month found. The same was done for the lower transits. The difference of zenith distance of upper and lower transit is twice the polar distance of Polaris, and half the sum gives the co-latitude of Greenwich.

In this way I found the apparent co-latitude of Greenwich for each month of the four years specified.

There appeared a very slight indication of a maximum belonging to the set of months,

March, 51. Feb. 52. Dec. 52. Nov. 53. Sept. 54.

This result, however, is to be regarded as very doubtful, as there did not appear to be evidence for any variation exceeding half a second of space, and more observations would be required to establish the existence of so small a variation at all.

I therefore conclude that the earth has been for a long time revolving about

an axis very near to the axis of figure, if not coinciding with it. The cause of this near coincidence is either the original softness of the earth, or the present fluidity of its interior. The axes of the earth are so nearly equal, that a considerable elevation of a tract of country might produce a deviation of the principal axis within the limits of observation, and the only cause which would restore the uniform motion, would be the action of a fluid which would gradually diminish the oscillations of latitude. The permanence of latitude essentially depends on the inequality of the earth's axes, for if they had been all equal, any alteration of the crust of the earth would have produced new principal axes, and the axis of rotation would travel about those axes, altering the latitudes of all places, and yet not in the least altering the position of the axis of rotation among the stars.

Perhaps by a more extensive search and analysis of the observations of different observatories, the nature of the periodic variation of latitude, if it exist, may be determined. I am not aware of any calculations having been made to prove its non-existence, although, on dynamical grounds, we have every reason to look for some very small variation having the periodic time of 325·6 days nearly, a period which is clearly distinguished from any other astronomical cycle, and therefore easily recognised.

## NOTE.

*Dimensions and Weights of the parts of the Dynamical Top.*

| | |
|---|---|
| I. Body of the top— | |
| Mean diameter of ring, 4 inches. | |
| Section of ring, $\frac{1}{3}$ inch square. | |
| The conical portion rises from the upper and inner edge of the ring, a height of $1\frac{1}{2}$ inches from the base. | |
| The whole body of the top weighs . . . . . . . | 1 lb. 7 oz. |
| Each of the nine adjusting screws has its screw 1 inch long, and the screw and head together weigh 1 ounce. The whole weigh . . | 9 „ |
| II. Axle, &c.— | |
| Length of axle 5 inches, of which $\frac{1}{2}$ inch at the bottom is occupied by the steel point, $3\frac{1}{2}$ inches are brass with a good screw turned on it, and the remaining inch is of steel, with a sharp point at the top. The whole weighs . . . . . . . . . . . | $1\frac{1}{2}$ „ |
| The bob B has a diameter of 1·4 inches, and a thickness of ·4. It weighs . | $2\frac{3}{4}$ „ |
| The nuts $b$ and $c$, for clamping the bob and the body of the top on the axle, each weigh $\frac{1}{2}$ oz. . . . . . . . . . | 1 „ |
| Weight of whole top | 2 lb. $5\frac{1}{4}$ oz. |

The best arrangement, for general observations, is to have the disc of card divided into four quadrants, coloured with vermilion, chrome yellow, emerald green, and ultramarine. These are bright colours, and, if the vermilion is good, they combine into a grayish tint when the revolution is about the axle, and burst into brilliant colours when the axis is disturbed. It is useful to have some concentric circles, drawn with ink, over the colours, and about 12 radii drawn in strong pencil lines. It is easy to distinguish the ink from the pencil lines, as they cross the invariable axis, by their want of lustre. In this way, the path of the invariable axis may be identified with great accuracy, and compared with theory.

## XXXV.—*On the Products of the Destructive Distillation of Animal Matters.* Part IV.

By Thomas Anderson, Professor of Chemistry, University of Glasgow.

(Read 20th April 1857.)

Owing to the great length of time over which the investigation of the products of the destructive distillation of animal substances has stretched, and various circumstances which it is unnecessary to detail, the inquiry has been pursued in a somewhat fragmentary manner, and with less continuity than might have been desired. The difficulties attending many of the experiments, and the occasional exhaustion of materials prepared by laborious processes, extending in many instances over considerable periods, have occasioned long intervals in the regular course of the inquiry which it became necessary to occupy with the examination of such matters as could be taken up at the moment. In this way a number of facts required to complete the history of the bases already described have gradually been accumulated, some of the products of their decomposition examined, and the pyrrol so frequently adverted to in the previous parts of this paper has been subjected to a full investigation. The details of these experiments form the subject of the present communication.

It has been already shown that the whole series of the alcohol bases, from methylamine to butylamine, can be obtained from bone oil, and the probable existence of amylamine in the portion boiling, about 200°, has been pointed out. The quantity of base obtained at that temperature is by no means large; but enough was collected not only to prove the existence of amylamine, but to substantiate the fact that it was unquestionably that base, and not one of its isomeres. After sufficient rectification it gave, with bichloride of platinum, an extremely beautiful platinum salt, which, when the fluid was sufficiently concentrated, deposited itself after some time in fine golden yellow scales, very soluble in water. The mother liquor, on evaporation, yielded another crop, agreeing with the first in properties and composition. A platinum determination of each gave the subjoined results:—

I. 5·39 grains of the platinum salt gave 1·815 grains of platinum.
II. 2·99 grains gave 1·010 grs. platinum.

| | Experiment. I. | Experiment. II. | Calculation. | | |
|---|---|---|---|---|---|
| Carbon, | ... | ... | 20·46 | $C_{10}$ | 60 |
| Hydrogen, | ... | ... | 4·77 | $H_{14}$ | 14 |
| Nitrogen, | ... | ... | 4·77 | N | 14 |
| Chlorine, | ... | ... | 36·34 | $Cl_3$ | 106·5 |
| Platinum, | 33·67 | 33·77 | 33·66 | Pt | 98·7 |
| | | | 100·00 | | 293·2 |

When the base was treated with iodide of amyle in a sealed tube, it rapidly dissolved; and, on cooling, the fluid became filled with fine crystalline plates. These crystals, when treated with potash, evolved a smell quite distinct from that of amylamine, much more pleasant, and devoid of that putrid odour which distinguishes the whole of the alcohol amide bases. When a quantity of the iodide was introduced into a retort, with an excess of potash, and distilled into a moderately dilute solution of hydrochloric acid, a salt immediately deposited itself as a crystalline powder of sparing solubility, and possessing all the characters of the hydrochlorate of diamylamine. Analysis gave the following results:—

$$\left\{\begin{array}{l}\text{6·400 grains dried at 212° gave}\\ \text{14·505 grains of carbonic acid and}\\ \text{7·280 grains water.}\end{array}\right.$$

$$\left\{\begin{array}{l}\text{8·240 grains dried at 212° gave}\\ \text{6·165 grains of chloride of silver.}\end{array}\right.$$

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, . . . . | 61·80 | 62·10 | $C_{20}$ | 120· |
| Hydrogen, . . . . | 12·63 | 12·40 | $H_{24}$ | 24· |
| Nitrogen, . . . . | ... | 7·16 | N | 14· |
| Chlorine, . . . . | 18·50 | 18·34 | Cl | 35·5 |
| | | 100·00 | | 193·5 |

These experiments prove incontestably that the base is amylamine, and they afford an indirect refutation of the opinion expressed by some chemists, that the substance described in a previous part of this paper as propylamine, might possibly be trimethylamine. The occurrence of the whole series of the alcohol bases, with their proper boiling points, as well as many facts observed during the investigation, had fully convinced me of the accuracy of my original opinion; but the experiments now detailed, showing that one of them is really an amide base, may be taken as affording the strongest possible evidence that the others are similarly constituted.

Various attempts have been made to ascertain whether any of the higher members of the alcohol series of bases, and particularly caprylamine, exist in bone oil, but without success. Some difficulty attends the examination, because the boiling points of these substances do not differ very greatly from those of the different members of the pyridine series, and a small quantity existing along with the latter might easily escape detection, but a careful examination of the first portions passing over during the distillation of pyridine, and which ought to contain any caprylamine, has satisfied me that it is not present. Taking into account the great difference in the quantity of hydrogen in these two bases; the former containing 6·3, and the latter 14·9 per cent. of that element, we should anticipate, that in the analysis of the first portions of pyridine, the hydrogen would be in excess if it contained caprylamine, and farther, as the bases of the alcohol series are stronger than those of the pyridine series, it would follow

that, if a mixture of two such bases were partially saturated by an acid, the salt produced should consist chiefly of the stronger base, and consequently should give a large excess of hydrogen. Salts prepared in this way gave the exact results required for pyridine, as will be seen in a subsequent page.

*Pyridine and its Compounds.*

In the second part of this paper a very cursory account was given of pyridine and its platinum salt, at a time when I had obtained this beautiful base in comparatively small quantity. Subsequent experiments have afforded me a much larger supply, and rendered it possible to submit it and its compounds to a more minute investigation. It is a transparent and colourless oil, with a powerful pungent smell, soluble in water in all proportions, and obtained absolutely dry only with some difficulty. It boils at 242°, and its specific gravity at 32° F. is 0·9858. It precipitates the salts of zinc, iron, manganese, and alumina in the cold, nickel only on the application of heat, and the precipitate dissolves in excess. Copper gives a pale blue precipitate, soluble in excess of base with a deep blue colour, not distinguishable from that produced by ammonia. It has a remarkable tendency to form double salts, most of which are highly crystallizable, and retain the metallic oxide in a state in which it cannot be precipitated by excess of pyridine. An analysis gave—

$$\begin{cases} 3{\cdot}175 \text{ grains of carefully dried pyridine gave} \\ 8{\cdot}830 \quad \ldots \quad \text{carbonic acid, and} \\ 1{\cdot}950 \quad \ldots \quad \text{water.} \end{cases}$$

| | | Calculation. | | |
|---|---|---|---|---|
| Carbon, | 75·84 | 75·94 | $C_{10}$ | 60 |
| Hydrogen, | 6·82 | 6·33 | $H_5$ | 5 |
| Nitrogen, | ... | 17·73 | N | 14 |
| | | 100·00 | | 79 |

The density of the vapour of pyridine determined by DUMAS's method, gave—

| | I. | II. |
|---|---|---|
| Temperature of the air, | 14° cent. | 15° c. |
| „ vapour, | 164 „ | 143° |
| Excess of weight of the balloon, | 0·3088 grammes. | 0·4060 gr. |
| Capacity of the balloon, | 305 c. c. | 324 c. c. |
| Barometer, | 765 m. m. | 752 m. m. |
| Residual air, | 14 c. c. | „ |
| Density of vapour, | 2·912 | 2·920 |

The formula $C_{10}$ $H_5$ N requires

$$\begin{aligned} 10 \text{ vol. carbon vapour} &= 0{\cdot}8290 \times 10 = 8{\cdot}2900 \\ 10 \ \ldots \text{ hydrogen} \ldots &= 0{\cdot}0692 \times 10 = 0{\cdot}6920 \\ 2 \ \ldots \text{ nitrogen} \ldots &= 0{\cdot}9713 \times \ \, 2 = 1{\cdot}9426 \end{aligned}$$

$$\frac{10{\cdot}9246}{4} = 2{\cdot}734$$

These experimental results are somewhat in excess of the theoretical density; but this is in all probability due to the presence of a small quantity of picoline, which, from the nature of the experiment, must necessarily remain in the balloon, and tend to produce an appreciable error in the density, even when its quantity is far too minute to be distinguished by an ordinary analysis. The specimen of pyridine used in these experiments had been purified with great care, and its platinum salt gave results corresponding completely with theory.

### *Salts of Pyridine.*

*Hydrochlorate of Pyridine.*—When hydrochloric acid is saturated with pyridine, and the solution evaporated on the water bath, the salt remains in the form of a thick syrup, so long as it is warm, but on cooling, crystals slowly make their appearance, and gradually shoot through the fluid, which is eventually converted into a hard radiated mass. The salt deliquesces when exposed to moist air, and sublimes unchanged at a high temperature. It is very soluble in alcohol, but less so than in water. It is insoluble in ether.

*Hydriodate of Pyridine.*—This salt crystallizes in tabular crystals, readily soluble both in water and alcohol, but not deliquescent. An analysis of the salt in an impure state will be afterwards given.

*Hydrobromate of Pyridine.*—A deliquescent salt, obtained on evaporation as a mass of acicular crystals.

*Nitrate of Pyridine.*—This salt is easily obtained by mixing nitric acid and the base. If the acid be concentrated, and the base dry, or nearly so, much heat is produced, and the mixture rapidly solidifies into a mass of short needles, which, when expressed between folds of blotting-paper, closely resembles loaf-sugar. The salt is purified by solution in hot water, or better in boiling spirit. On cooling, it is deposited from the latter solution in fine needles, which can easily be obtained an inch long, even when operating on a very small scale. Sometimes it appears in short thick prisms. It is not deliquescent, but is extremely soluble in water, less so in alcohol, and not at all in ether. When heated in a retort it melts; and if the temperature be raised very gradually it sublimes as a white woolly mass; but, if briskly heated, it distils in the form of a thick oily fluid, which solidifies in the neck of the retort to a mass of acicular crystals. If the heat be carefully regulated it sublimes without undergoing the least change, but if rapidly distilled, a small quantity of red fumes are occasionally seen. Heated on a platinum knife it catches fire and burns with great brilliancy, and a rapidity almost approaching to deflagration. Analyses made on different preparations gave the following results:—

$$\text{I.}\ \left\{\begin{array}{lll} 5{\cdot}954 & \text{grains of nitrate dried at } 212^\circ \text{ gave} & \\ 9{\cdot}238 & \ldots & \text{carbonic acid and} \\ 2{\cdot}358 & \ldots & \text{water.} \end{array}\right.$$

II. { 6·036 grains of nitrate dried *in vacuo* gave
9·377 ... carbonic acid and
2·324 ... water.

III. { 4·455 grains obtained from the mother liquor of the previous crop gave
6·889 ... carbonic acid and
1·733 ... water.

IV. { 7·118 grains of nitrate dried at 212° gave
11·009 ... carbonic acid and
2·767 ... water.

| | Experiment. | | | | Calculation. | | |
|---|---|---|---|---|---|---|---|
| | I. | II. | III. | IV. | | | |
| Carbon, | 42·31 | 42·37 | 42·17 | 42·18 | 42.25 | $C_{10}$ | 60 |
| Hydrogen, | 4·40 | 4·28 | 4·32 | 4·31 | 4·22 | $H_6$ | 6 |
| Nitrogen, | ... | ... | ... | ... | 19·73 | $N_2$ | 28 |
| Oxygen, | ... | ... | ... | ... | 33·80 | $O_6$ | 48 |
| | | | | | 100·00 | | 142 |

These results correspond completely with the formula $C_{10}H_5N$ HO $NO_5$.

*Bisulphate of Pyridine.*—When sulphuric acid is supersaturated with pyridine, and evaporated in the water bath, a crystalline mass is left which is deliquescent, and soluble in all proportions in water and alcohol, but insoluble in ether. Its reaction is highly acid, and analysis showed it to be a bisulphate.

{ 21·012 grains bisulphate of pyridine, gave
27·867 ... sulphate of baryta = 45·50 per cent. of sulphuric acid.

The formula $C_{10}H_5N$ 2HO $SO_3$ requires 45·19.

## *Double Salts of Pyridine.*

The platinochloride of pyridine has been already described in the second part of this investigation.

*Aurochloride of Pyridine.*—This salt is immediately thrown down as a fine lemon-yellow crystalline powder when chloride of gold is added to a solution of hydrochlorate of pyridine. It dissolves readily in hot water, and is deposited, on cooling, in fine yellow needles, little soluble in cold water, and insoluble in alcohol. Analysis gave—

{ 8·525 grains aurochloride gave
46·40 ... carbonic acid and
1·450 ... water.

I. { 7·350 grains aurochloride gave
3·440 ... gold.

II. { 5·480 grains aurochloride gave
2·565 ... gold.

| | Experiment. I. | Experiment. II. | Calculation. | | |
|---|---|---|---|---|---|
| Carbon, | 14·84 | ... | 14·32 | $C_{10}$ | 60 |
| Hydrogen, | 1·89 | ... | 1·43 | $H_5$ | 6 |
| Nitrogen, | ... | ... | 3·34 | N | 14 |
| Chlorine, | ... | ... | 33·90 | $Cl_4$ | 142 |
| Gold, | 46·80 | 46·62 | 47·01 | Au | 197 |
| | | | 100·00 | | 419 |

Corresponding with the formula $C_{10}H_5N\ HCl\ Au\ Cl_3$.

When pyridine is added to a moderately dilute solution of sulphate of zinc in considerable excess, oxide of zinc is precipitated. And if a quantity of hydrochloric acid insufficient to neutralize the pyridine be then added, the fluid instantly becomes clear; but if it be stirred briskly, it rapidly fills with an abundant crystalline precipitate of a double salt. The salt dissolves with facility in boiling water, and is deposited, on cooling, in long, brilliant needles. Sulphate of copper, when treated in a similar manner, gives a pale greenish-blue precipitate, soluble in boiling water, from which it crystallizes in fine bluish needles. The salts of manganese and nickel, and protoxide of iron, appear also to form double salts, but they are very soluble, and have not been particularly examined.

### *Products of the Decomposition of Pyridine.*

Pyridine, like all its homologues, is an exceedingly stable base, and resists the action of oxidising agents. It may be boiled with the most concentrated nitric acid, or with chromic acid, without undergoing decomposition; and treatment with the former acid affords an invaluable means of freeing those bases from any empyreumatic matters with which they may be mixed.

*Action of Chlorine on Pyridine.*—The action of chlorine on pyridine depends upon the mode in which that agent is employed. When a current of the gas is passed through an aqueous solution of the base it is rapidly absorbed, the fluid acquires a dark brown colour, and evolves a peculiar pungent odour; and on the addition of potash, the smell of unchanged pyridine becomes apparent, while a quantity of a dark brown resinous matter is separated. But if an excess of pyridine be thrown into a large bottle of dry chlorine, and distributed over the sides as rapidly as possible, in order to prevent rise of temperature, it remains perfectly colourless, and is converted into a mass of radiated crystals. On the addition of water the crystals dissolve, leaving a quantity of a snow-white amorphous powder, and hydrochlorate of pyridine is found in the solution. The white powder has a faint smell, not unlike that of bleaching powder. It is insoluble in water, but dissolves in alcohol, and is precipitated again in white flocks on the addition of water. When boiled for some time with water it softens, but

does not thoroughly melt, and at the same time exhales a peculiar irritating vapour, due, apparently, to partial decomposition. It is insoluble in hydrochloric acid; strong nitric acid dissolves it, and the solution on boiling gives off red fumes; but on the addition of water the original substance is deposited apparently unchanged. Potash colours it brown, and on boiling dissolves it, giving a dark brown solution, from which acids precipitate brown flocks. Ammonia, and even carbonate of ammonia, produce a similar decomposition. When heated, it swells up, giving off a pungent smell, and leaving a bulky charcoal. This substance has not been analysed, but the corresponding product of the decomposition of picoline has been examined, and there can be no doubt that the two substances are of analogous constitution. I shall defer any observations on this point until I come to treat of the picoline compound.

*Action of Bromine on Pyridine.*—When bromine water is gradually added to a solution of pyridine, the fluid becomes muddy, and as the quantity of bromine increases, an abundant precipitate appears, and collects at the bottom of the vessel in the form of a reddish mass of a more or less resinous appearance. This substance is insoluble in water, but soluble in alcohol and ether. When boiled with water it melts and emits a pungent and irritating odour, resembling that of bromine. Hydrochloric acid decomposes it, dissolving pyridine, and liberating bromine, which collects at the bottom of the fluid. Potash likewise decomposes it, evolving pyridine, and combining with bromine. These characters lead to the conclusion that the substance is a direct compound of pyridine, with in all probability, several equivalents of bromine; but its properties were not so definite as to induce me to prepare it on a scale sufficiently large for a detailed examination and analysis. When dry pyridine is thrown into dry bromine vapour, it immediately solidifies into a crystalline mass, which dissolves in water, with the exception of a small quantity of a brownish flocky matter, probably analogous to the compound produced by similar treatment with chlorine. The solution in water becomes dark-coloured on evaporation, and yields a syrup which solidifies on standing, into a mass of minute crystals of hydrobromate of pyridine.

*Action of Iodine on Pyridine.*—When a mixture of pyridine and tincture of iodine is evaporated to dryness on the water-bath, a dark brown mass is left, which dissolves partially in water, leaving a quantity of brown crystals, too small in amount to admit of examination, and which are very easily decomposed. They appear to be a product similar to the iodine compounds of the fixed bases. The watery solution contains a quantity of a brown matter, removable by animal charcoal, and the fluid, on evaporation, yielded crystals which analysis proved to be the hydriodate of pyridine, although not quite pure.

$$\begin{cases} 5{\cdot}774 \text{ grains dried at } 212^\circ \text{ gave} \\ 7{\cdot}673 \quad \ldots \quad \text{iodide of silver.} \end{cases}$$

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, | ... | 29·01 | $C_{10}$ | 60 |
| Hydrogen, | ... | 2·90 | $H_6$ | 6 |
| Nitrogen, | ... | 6·74 | N | 14 |
| Iodine, | 62·43 | 61·35 | I | 127 |
| | | 100·00 | | 207 |

Corresponding with the formula $C_{10} H_5 N$ HI.

### *Picoline and its Compounds.*

The compounds of picoline have already been pretty fully described in my original paper on that base, but the possession of a larger quantity has induced me to examine more in detail some of the products of its decomposition, and to determine with greater exactitude certain of its physical properties. In the paper just referred to I fixed its boiling point at 272°; but an experiment made on a larger scale has convinced me that this is too low, and that when quite pure it boils at 275°. The specific gravity at 32° is 0·9613. The density of its vapour was determined by DUMAS's method with the following results:—

| | |
|---|---|
| Temperature of the air, | 13° cent. |
| ... vapour, | 166° ... |
| Excess of weight of the balloon, | 0·3490 gramme. |
| Capacity of do., | 288 c, c. |
| Barometer, | 762 m. m. |
| Residual air, | 22 c. c. |
| Specific gravity of vapour, | 3·29 |

The formula $C_{12} H_7 N$ requires—

$$\begin{array}{lll} 12 \text{ vol. carbon vapour} & = 0{\cdot}8290 \times 12 & = 9{\cdot}9480 \\ 14 \ldots \text{ hydrogen } \ldots & = 0{\cdot}0492 \times 14 & = 0{\cdot}9685 \\ 2 \ldots \text{ nitrogen } \ldots & = 0{\cdot}9713 \times \ \ 2 & = 1{\cdot}9626 \\ \hline & & 12{\cdot}8591 \end{array}$$

$$\frac{12{\cdot}8591}{4} = 3{\cdot}214$$

*Nitrate of Picoline.*—This salt has been already described as a deliquescent crystalline mass, but I have now succeeded in obtaining it in prismatic crystals of considerable size, which are formed when a quantity of the dry salt, covered with a saturated solution, is left for some weeks in a closely-stoppered bottle. At the end of that time the salt has been converted into a small number of four-sided prisms terminated by dihedral summits. Analysis gave—

$$\left\{\begin{array}{lll} 5{\cdot}080 & \text{grains dried at } 212^\circ \text{ gave} \\ 8{\cdot}580 & \ldots \text{ carbonic acid and} \\ 2{\cdot}395 & \ldots \text{ water.} \end{array}\right.$$

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, | 46·06 | 46·15 | $C_{12}$ | 72 |
| Hydrogen, | 5·23 | 5·12 | $H_8$ | 8 |
| Nitrogen, | ... | 17·96 | $N_2$ | 28 |
| Oxygen, | ... | 30·77 | $O_6$ | 48 |
| | | 100·00 | | 156 |

And its formula is $C_{12}H_7N\ HO\ NO_5$.

## *Products of the Decomposition of Picoline.*

*Action of Chlorine on Picoline.*—The action of a current of chlorine on picoline, both dry and dissolved in water, has been already described, and the results were not such as to induce further experiments in this way. But when an excess of picoline is projected into dry chlorine gas, it is rapidly converted into a more or less distinctly crystallized mass, which, when treated with water, leaves a quantity of an amorphous powder of dazzling whiteness. The properties of this substance are so like those of the corresponding pyridine compound, that the same words would almost serve to describe it. It is insoluble in water; but alcohol dissolves it easily, and the solution, when boiled, undergoes decomposition; an etherial odour, not unlike that of hydrochloric ether, and probably due to the formation of that substance, is first produced, and that is followed by a pungent vapour. It is insoluble in the dilute acids, but soluble in concentrated nitric acid. Potash decomposes it in the cold, and more rapidly if heated. Heated on platinum, it gives off a very pungent vapour and leaves a bulky charcoal. It is decomposed when heated in the water bath. The portion used for analysis is dried *in vacuo*. The results were,—

5·550 grains dried *in vacuo* gave
6·170 ... carbonic acid and
1·100 ... water.

4·445 grains burnt with lime gave
9·265 ... chloride of silver.

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, | 30·32 | 30·9° | $C_{12}$ | 72 |
| Hydrogen, | 2·20 | 2·14 | $H_5$ | 5 |
| Nitrogen, | ... | 5·02 | N | 14 |
| Chlorine, | 61·54 | 60·94 | $Cl_4$ | 142 |
| | | | | 233 |

This corresponds very closely with the formula

$$C_{12}\left.\begin{matrix}H_4\\Cl_3\end{matrix}\right\}N\ H\ Cl.$$

or that of the hydrochlorate of a base produced by the substitution of three

equivalents of chlorine for three of hydrogen in picoline, and which would be called trichloropicoline. This view derives confirmation, from the fact, that when exposed to 212°, the substance loses an equivalent of hydrochloric acid, as shown by the subjoined experiment.

$$\left\{\begin{array}{l}2\cdot815 \text{ grains heated to } 212^\circ \text{ gave} \\ 5\cdot960 \ \ldots \ \text{chloride of silver.}\end{array}\right.$$

This corresponds to 52·35 per cent. of chlorine, while the formula $C_{12}\left.\begin{matrix}H_4\\Cl_3\end{matrix}\right\}N$ requires 54·2. Although this is only a very distant approximation to the theoretical number, the discrepancy is not greater than might be expected when the properties of the substance, and the fact that it is coloured brown in the water bath, are taken into account.

### *Action of Sodium on Picoline.*

When sodium is thrown into picoline in the cold, it remains unchanged, and preserves its metallic lustre; but if the picoline be heated to its boiling point, an action begins to manifest itself, brown streaks are seen to appear on the surface of the sodium, and after continued boiling, the whole fluid becomes dark brown, and at length nearly black and viscid. In order to examine this change more minutely, picoline was introduced into a Florence flask, with a quantity of sodium, which varied in different experiments from a fourth to an eighth of its weight; and a long tube being fixed into the mouth of the flask, it was heated in the oil bath in such a manner that the picoline cohobated freely. The action requires some days for its completion, and at the end of that time the contents of the flask are converted into a dark brown hard resinous mass, containing lumps of unchanged sodium. The resinous matter contained sodium in some form of combination which could not be determined; the properties of the substance not being such as to induce an extended examination. It burnt with a smoky flame, leaving soda; and, when exposed to the air showed a tendency to deliquesce, and became sticky on the surface. The pieces of sodium having been carefully removed, the resinous matter was thrown into water, and on standing it was slowly converted into a thick viscid and very dark-coloured oil, much heavier than water, while soda was found in the solution. The oil smelt more or less distinctly of picoline, according to the length of time during which the action had been carried on. After having been carefully washed, so as to remove the soda, and then distilled with water, picoline passed over, and there was left behind a thick oily base, requiring a very high temperature for its distillation, and to which, for reasons to be afterwards explained, I give the name of parapicoline.

*Parapicoline.*—In the preparation of this base it was found not to be advantageous to push the action of sodium to the extreme; and the cohobation was

generally stopped at the end of the second day, when a considerable quantity of the picoline still remained unchanged, and the contents of the flask had acquired the consistence of treacle. The flask was then broken, the sodium removed as completely as possible, and the whole, along with the pieces of broken glass, to which a considerable quantity of thick matter adhered, was thrown into water. When the oil had collected at the bottom, which generally required some hours, the pieces of glass were removed, the supernatant fluid decanted, and the oily base washed with water, so as to remove the soda and the greater portion of the unchanged picoline. Occasionally a somewhat different process was adopted; the cohobating tube being replaced by another bent at right angles, and the heat continued, so as to distil off and recover the dry picoline; but this was found less convenient, as the increased viscidity of the contents of the flask rendered its after-treatment more troublesome. The well-washed oil was introduced into a small retort, and heat applied. At first a watery fluid containing picoline came over, then dry picoline appeared, and subsequently an oil insoluble in water, began to distil; while a thermometer, placed in the tubulature of the retort, rose at first very rapidly, afterwards more slowly, until, towards the end of the distillation, the temperature reached a point considerably beyond the range of the thermometer. Some crystals of carbonate of ammonia made their appearance in the neck of the retort; traces of pyrrol could be distinguished, and a quantity of charcoal was left. These experiments rendered it sufficiently obvious that the new base possessed a boiling point so high, and so near its point of decomposition as to render necessary the utmost precautions for its purification. The first portion of the distillate which contained unchanged picoline was therefore rejected, and the remainder was heated in a retort immersed in the oil-bath to the boiling point of picoline, while a current of dry hydrogen was passed through it. At first a small quantity of picoline and some crystals of carbonate of ammonia made their appearance; and when these ceased to increase, the temperature of the bath was raised until it reached 380°, when the receiver was changed, and the heat maintained as steadily as possible between that point and 400°, the current of hydrogen being continued all the time. The base was thus made to evaporate at a temperature considerably under its boiling point, and was obtained in a much more satisfactory state, but even then it was not absolutely pure, as it still gave faint indications of pyrrol, although the quantity must have been excessively small; and though wholly soluble in acids, the solution retained a distinctly empyreumatic smell. The small scale on which it was necessary to experiment rendered it impossible to adopt any very efficient means of removing these impurities; but by a second rectification in the current of hydrogen, when the first portions were again rejected, a considerable improvement took place.

Parapicoline is a pale yellow oil of the consistence of a fixed oil, which acquires a brown colour by exposure to the air. It is insoluble in water, although

it communicates its smell to that fluid when shaken with it. It dissolves in all proportions in alcohol, ether, the fixed and volatile oils. It has a highly characteristic empyreumatic smell, quite distinct from that of picoline, without pungency, and closely resembling that of the bases extracted from the portions of Dippel's oil of very high boiling point, and which not improbably contain it. Its smell adheres pertinaciously to the fingers. It fumes slightly when a rod dipped in hydrochloric acid is brought near it, and restores the blue colour of reddened litmus. Boiled with strong nitric acid, it gives off red fumes, and on dilution with water a small quantity of a resinous matter deposits, but the greater part of the base is separated unchanged on the addition of potash. It gives an emerald-green precipitate with sulphate of copper, which dissolves in hydrochloric acid, and forms a green solution, containing a double salt. Most of its compounds are uncrystallizable, and readily soluble in water. Its specific gravity is 1·077, and it boils between 500° and 600° Fahrenheit, and is partially decomposed. The portions employed for analysis were very carefully distilled for that purpose. Owing to the high boiling point, some difficulty was experienced in the combustion, and it was found convenient to weigh the substance in a small open tube, which was passed into the combustion tube. The results were,—

I. { 3·060 grains of parapicoline gave
8·730 ... carbonic acid and
2·158 ... water.

II. { 3·707 grains of parapicoline gave
10·601 ... carbonic acid and
2·655 ... water.

III. { 4·270 grains of parapicoline gave
12·195 ... carbonic acid and
3·072 ... water.

| | Experiment. | | | Calculation. | | |
|---|---|---|---|---|---|---|
| | I. | II. | III. | | | |
| Carbon, . . . . | 77·81 | 77·99 | 77·89 | 77·42 | $C_{12}$ | 72 |
| Hydrogen, . . . . | 7·83 | 7·96 | 7·99 | 7·53 | $H_7$ | 7 |
| Nitrogen, . . . . | ... | ... | ... | 15·05 | N | 14 |
| | | | | 10·000 | | 93 |

These numbers correspond almost exactly with those of picoline itself, as indicated by the calculation. The quantity of carbon in all the analyses is considerably above that required by theory; but it is easy to understand how a small quantity of empyreumatic matters formed during the decomposition may produce this effect; and itis sufficiently obvious that the base is isomeric with picoline. This is further confirmed by the analysis of its platinum salt, which is

immediately precipitated when bichloride of platinum is added to a solution of the hydrochlorate of parapicoline, as a pale yellow powder, almost insoluble in water. The results of the analysis were as follows:—

I. { 6·317 grains of platinochloride of parapicoline gave
2·059 ... platinum.

II. { 6·102 grains of platinochloride of parapicoline gave
1·970 ... platinum.

| | Experiment. I. | Experiment. II. | Calculation. | | |
|---|---|---|---|---|---|
| Carbon, | ... | ... | 24·07 | $C_{12}$ | 72 |
| Hydrogen, | ... | ... | 2·68 | $H_8$ | 8 |
| Nitrogen, | ... | ... | 4·67 | N | 14 |
| Chlorine, | ... | ... | 35·59 | $Cl_3$ | 106·5 |
| Platinum, | 32·59 | 32·28 | 32·98 | Pt | 98·7 |
| | | | 100·00 | | 299·2 |

These numbers correspond with the formula $C_{12}H_7N\ HCl\ PtCl_2$, which is that of the picoline salt; and the analysis would thus lead us to the conclusion that parapicoline is strictly isomeric with that base. But when its high boiling point and other properties are taken into consideration, it is impossible to resist the inference, that its real constitution must be different; and I believe it ought to be represented by the formula $C_{24}H_{14}N_2$, and that it is produced by the combination of two equivalents of picoline. Unfortunately the high boiling point of parapicoline precludes the determination of the specific gravity of its vapour; and as it is not possible in any other way to establish its true constitution, we are compelled to assume, as the most probable hypothesis, that it is produced by a species of reduplication, of which we have already numerous examples in the other classes of organic compounds, although this is the first instance in which it has been observed among the bases. The conversion of cyanic into cyanuric acid is a completely analogous case, the more especially as the three equivalents of cyanic acid which have combined retain their power of neutralizing as many equivalents of base. The simultaneous production of amilene, paramilene, and metamilene, during the action of sulphuric acid on amylic alcohol, may also be referred to as cases in which a somewhat similar reduplication occurs. It is very difficult to explain the mode in which the sodium produces the combination of the two equivalents of picoline, but it may possibly be due to a species of catalytic action, as a large quantity of the sodium employed is always recovered unchanged. A certain quantity of it, however, enters into some sort of combination with the picoline or parapicoline, to produce the resinous compound already mentioned; and it appears most likely that this substance is a sodiopicoline, represented by the formula $C_{12}H_6Na\ N$, in which an equivalent of hydrogen has been replaced by sodium. The action of water upon the resinous matter would then be represented by the following equation:—

$$2\,(C_{12}H_6Na\,N) + 2HO = C_{24}H_{14}N_2 + 2NaO.$$

If this be the case, hydrogen ought to be evolved during the action of sodium on picoline, but owing to the slow nature of the action which takes place, I have not been able to satisfy myself that such is the case.

Whatever be its nature, parapicoline must be considered a very remarkable base, and altogether unique in the mode of its production, but it is completely analogous in its constitution to nicotine, for the determination of the density of the vapour of that base has shown incontestably that its rational formula is $C_{20}H_{10}N_2$, and that of its platinum salt, $C_{20}H_{10}N\ 2HCl\ Pt_2Cl_4$. I think it can scarcely be doubted that nicotine, like parapicoline, has been formed by the combination of two equivalents of a base boiling at a temperature not greatly exceeding 212°, and which will some day be discovered. I have attempted to reconvert parapicoline into picoline, but without success; for though the change appears to be partially effected by rapid distillation, the process is not definite, much carbonate of ammonia being produced.

### *Salts of Parapicoline.*

The salts of parapicoline are chiefly uncrystallizable, and present but few points of interest. I have therefore submitted them to a very cursory examination.

*Sulphate of Parapicoline* is obtained as a gummy mass, very soluble in water, less so in alcohol. It shows no signs of crystallization.

*Nitrate of Parapicoline* is obtained by saturating nitric acid with the base, and evaporating. A syrupy fluid is left, which slowly solidifies on cooling into a mass of short needles. It is exceedingly soluble in water, less so in alcohol, and it does not deliquesce.

*Hydrochlorate of Parapicoline* is an amorphous resin, very soluble in water.

*Hydrargochloride of Parapicoline.* A solution of corrosive sublimate immediately gives an abundant curdy precipitate of this salt when added to an alcoholic solution of parapicoline. It is insoluble in alcohol and in water, but is instantly dissolved on the addition of a few drops of hydrochloric acid.

*Aurochloride of Parapicoline*, is a yellow insoluble amorphous substance, decomposed at the boiling heat.

The details now given, as well as those contained in the preceding parts of this investigation, may serve to illustrate with sufficient fulness the general characters of the bases of the pyridine series. It remains for me only to direct attention to their physical properties, which illustrate in a very striking manner the relations subsisting between the different members of a homologous series. The particulars of most of the experiments have been already given, and it is only necessary to add those by which the specific gravity of the vapour of lutidine was determined.

| | |
|---|---|
| Temperature of the air, . . . . | 17° cent. |
| ... vapour, . . . . | 201° „ |
| Excess of weight of the balloon, . . . | 0·4493 grammes. |
| Capacity of do. . . . . | 302 c. c. |
| Barometer, . . . . . | 776 m. m. |
| Residual air, . . . . . | 0 |
| Specific gravity of the vapour, . . . | 3·839 |

The formula $C_{14}H_9N$ requires

$$
\begin{array}{lrl}
\text{14 vol. carbon vapour,} & = & 0{\cdot}8290 \times 14 = 11{\cdot}6060 \\
\text{18 vol. hydrogen,} & = & 0{\cdot}0692 \times 18 = 1{\cdot}2456 \\
\text{2 vol. nitrogen,} & = & 0{\cdot}9713 \times 2 = 1{\cdot}9426 \\
\end{array}
$$

$$\frac{14{\cdot}7942}{4} = 3{\cdot}699$$

In the following table I have collected the whole of the data, all having been carefully redetermined with much purer materials than those used in my original experiments,—

| | Formula. | Boiling Point. | Specific Gravity. | | Specific Volume at 32°. | Diff. |
|---|---|---|---|---|---|---|
| | | | Vapour. | Liquid at 32° | | |
| Pyridine, . | $C_{10}H_5N$ | 242° | 2·916 | 0·9858 | 80·1 | ... |
| Picoline, . | $C_{12}H_7N$ | 275° | 3·290 | 0·9613 | 96·7 | 16·6 |
| Lutidine, . | $C_{14}H_9N$ | 310° | 3·839 | 0·9467 | 113·0 | 16·3 |
| Collidine, . | $C_{16}H_{11}N$ | 356° | ... | 0·9439 | 128·2 | 15·2 |

The boiling points of pyridine, picoline, and lutidine agree remarkably well with KOPP's law, but collidine differs very materially from it. Less reliance, however, is to be placed upon the boiling point of the last substance, as it was determined upon a very small quantity of material. The specific gravities of the vapours agree very closely with theory, while those of the fluids themselves, taken at 32°, illustrate also in a very remarkable manner the gradual diminution which is observed when we ascend through a series of homologous substances. To these experimental numbers have been added the specific volumes of the bases at 32°, calculated from the data they afford: but no determinations of the coefficient of expansion of these substances having been made, it is not possible to ascertain their specific volumes at the boiling points, although from the rapidity of their expansion, I believe it will be found that the difference must approach very closely to 22, which is that produced in non-nitrogenous substances by the addition of $C_2H_2$ to their atom.

### *Pyrrol.*

Reference has frequently been made throughout the course of this investigation to the substance discovered by RUNGE* in coal-tar, and called by him pyrrol. This substance he described as a gas, although he appears never to have prepared it in a pure state, but simply to have obtained its very singular reaction with fir-wood; and he mentions that it occurs in very small quantity, and accompanies the ammonia produced during destructive distillation. In the second part of this paper, when describing the preparation of the bases from crude bone oil, it was stated that the acid solution afforded on distillation a quantity of an oil possessing in a high degree the characteristic reaction of pyrrol, and which was decomposed when boiled with moderately concentrated acids, with the precipitation of a red resinous matter, while the fluid was found to contain different numbers of the pyridine series of bases. From these facts I was led to infer that this oil contained a series of bases in which pyridine and its homologues were coupled with some substance which was separated by acids, and converted into the red resin,—an opinion which further experiment has entirely refuted.

The oil collected during the distillation of the acid solution of the crude pyridine bases, had a peculiarly fetid and disagreeable smell, and was at first colourless, but soon acquired a reddish colour, and after a few days became nearly black. When freed from water it began to distil about 250°, and a thermometer placed in the tubulature of the retort gradually rose as the distillation proceeded, until at length it reached nearly 400°. The greater proportion of the oil passed between 280° and 310°, but large fractions were obtained at much higher temperatures. All the fractions had a characteristic smell different from that of the pyridine bases, and gave instantaneously the reaction of pyrrol. When treated with acids, the red resinous matter was deposited, and the filtered fluid, on treatment with potash, evolved the smell of different members of the pyridine series, according to the boiling point of the fraction selected for the experiment. The oil containing pyrrol was now subjected to a systematic fractionation; and it was found, after several rectifications, to manifest a decided tendency to concentrate itself towards a fixed point, the fractions collected between 270° and 280°, and 280° and 290°, greatly exceeding the others in bulk. The oil obtained at these temperatures was perfectly transparent and colourless when freshly distilled, but soon acquired a brown colour, though much less rapidly than the crude substance. When agitated with very dilute acids, a certain portion of it immediately dissolved, but the remainder was very slowly acted upon, and required a large excess of acid, and much shaking, in order to make it dissolve, which, however, it eventually did completely. This fact appearing to indicate that the substance

* Poggendorf's Annalen, vols. xxxi. and xxxii.

was a mixture, it was shaken up with a small quantity of dilute acid, and the watery solution withdrawn. On the addition of caustic potash to this solution, an oil separated, which had the smell of picoline mixed with that of pyrrol. For the purpose of separating this picoline, the whole of the larger fractions were mixed and shaken up with a small quantity of very dilute sulphuric acid, and the solution, after being siphoned off, was replaced by another quantity, and this was repeated a third time. The oil was thus diminished by about a third of its bulk, and the whole of the picoline or other bases appearing to have been removed, it was carefully dried by means of sticks of caustic potash, and again rectified, when its boiling point was found to have been materially reduced. It began to boil at much the same temperature as the crude oil, but the largest fraction was now collected between 270° and 280°, while that which boiled above 290° formed only a very small proportion of the whole; and after fifteen rectifications, it was obtained in such a state that it distilled almost entirely between 274° and 280°. In this condition it is a transparent and colourless oil, slowly acquiring a brown colour when exposed to air and light. It has a strong fetid smell, quite distinct from that of picoline, and a hot pungent taste. A piece of fir-wood, dipped in hydrochloric acid brought near its vapour, instantly acquires a fine red colour. When boiled with a dilute acid, it is immediately converted into a red resinous mass, which fills the fluid so completely, that the vessel containing it may be inverted without anything escaping. The fluid filtered from this substance is brown, and contains a small quantity of it in solution. After boiling for some time, so as to get rid of a peculiar smell which adhered to the fluid, and decompose the last traces of pyrrol, caustic potash was added, when the smell of ammonia, faintly contaminated with that of picoline, was evolved. The solution having been distilled, the ammonia was saturated with hydrochloric acid and bichloride of platinum added, when the platinochloride of ammonium was immediately precipitated, and the filtrate, on further evaporation, yielded an additional quantity of that salt, along with some indications of a more soluble platinum compound. For a long time I considered the oil prepared by the process now detailed to be pyrrol in a state of as great purity as it was possible to obtain it; and, as will be afterwards seen, it gave in different preparations, analytical results in perfect accordance with one another, and with its true formula; but in the course of examining the effect of different reagents upon it, it was found that caustic potash exerted a very singular and perfectly unique action, disclosed the presence of a small quantity of some impurity, and afforded the means of removing it, when the properties of the pyrrol underwent a very remarkable change. When pyrrol is mixed with five or six times its weight of caustic potash in coarse powder, and heated over the lamp in a flask fitted with a long tube, it at first cohobates very freely; but if the temperature be gradually raised, the fluid is found to distil up into the tube much less readily, and at

length the bottom of the flask may be heated nearly red hot, while a very insignificant quantity of oil distils up. In performing this process, glass flasks were corroded by the caustic potash long before the action was complete, and it was found very convenient to employ copper flasks made by the electrotype process. A plaster of Paris mould was taken from a glass flask of convenient size and shape; and from that a wax cast was made and electrotyped in the usual way. After about a week the copper was sufficiently thick for use. In such flasks pyrrol was boiled for a day or two with caustic potash, the heat being raised as high as an Argand or Bunsen's gas-lamp would bring it. A bent tube was then fitted into the mouth of the flask, and the heat again applied, so as to distil off all the oil that could be obtained. The distillate had the smell of pyrrol mixed more or less distinctly with that of picoline, and the preponderance of the latter smell depended on the quantity of potash having been sufficiently large to retain the true pyrrol, which, however, it was not possible to do entirely, even when a very large excess of potash was used. When the whole of this oil had distilled, the bent tube was removed from the mouth of the flask, and the still fluid potash poured out on a copper plate.

On cooling, it solidified into a hard white mass with a yellowish tinge, which, when perfectly dry had no smell, but it was only necessary to breathe upon it to cause it to exhale a delightful etherial and fragrant odour, not unlike that of chloroform, but softer and less pungent. When thrown into water the potash gradually dissolved, and a transparent and colourless oil collected on the surface of the solution, from which it was separated either by a pipette or by distillation. The potash solution on saturation with sulphuric acid evolved the smell of a fatty acid, and when distilled, yielded a fluid which reddened litmus strongly, and had a smell resembling that of valerianic acid. The distillate was saturated with carbonate of soda, and the solution evaporated to complete dryness and extracted with absolute alcohol. The alcoholic fluid was again evaporated, the residue dissolved in water, and a quantity of solution of nitrate of silver insufficient for complete precipitation added to it, and the precipitate was collected on a filter and washed. Another quantity of nitrate of silver was then added, and the precipitate collected, and finally, enough of the nitrate was used to throw the remainder of the fatty acids in the fluid. In this way three different silver salts were obtained, which were separately analysed. The first precipitate gave:—

$$\begin{cases} 6{\cdot}273 \text{ grains of silver salt gave} \\ 6{\cdot}510 \quad \ldots \quad \text{carbonic acid and} \\ 2{\cdot}452 \quad \ldots \quad \text{water.} \end{cases}$$

$$\begin{cases} 5{\cdot}192 \text{ grains of silver salt gave} \\ 2{\cdot}697 \quad \ldots \quad \text{silver.} \end{cases}$$

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, | 28·30 | 28·71 | $C_{10}$ | 50 |
| Hydrogen, | 4·34 | 4·31 | $H_9$ | 9 |
| Silver, | 51·94 | 51·67 | Ag | 108 |
| Oxygen, | ... | 15·31 | $O_4$ | 32 |
| | | 100·00 | | 209 |

which corresponds completely with the valerianate of silver. The second precipitate was manifestly a mixture, and gave variable quantities of silver, generally about 2 per cent. under that required by the valerianate. But the third precipitate consisted of propionate of silver, as shown by the subjoined analyses:—

I. 5·002 grains of the third precipitate gave 2·993 grains silver.
II. 4·796 ... another preparation gave 3·848 ...

| | Experiment. | | | Calculation. | | |
|---|---|---|---|---|---|---|
| | I. | II. | Mean. | | | |
| Carbon, | ... | ... | ... | 19·89 | $C_6$ | 36 |
| Hydrogen, | ... | ... | ... | 2·76 | $H_5$ | 5 |
| Silver, | 59·83 | 59·38 | 59·50 | 59·66 | Ag | 108 |
| Oxygen, | ... | ... | ... | 17·69 | $O_4$ | 32 |
| | | | | 100·00 | | 181 |

It thus appears that the crude pyrrol contained a small quantity of some substances yielding valerianic and propionic acids when acted on by potash. The exact nature of these compounds it was impossible to determine, as their quantity was extremely minute, and the silver salts obtained from a very considerable quantity of pyrrol, were no more than sufficient for the analyses just detailed.

The fragrant pyrrol separated from the potash solution by distillation is transparent and colourless, when freshly prepared, but acquires a brown colour by exposure to the air. Its taste is hot and pungent, and its smell pleasant and etherial, and recalls that of chloroform. It is sparingly soluble in water, but readily in alcohol, ether, and the oils. It is insoluble in alkaline solutions, but the acids dissolve it, although not very rapidly. Its specific gravity is 1·077, and it boils at 271°. It gives the remarkable reaction on fir-wood described by RUNGE in a very powerful manner. The reaction is best obtained by dipping a piece of fir-wood in concentrated commercial hydrochloric acid, and holding it near a vessel containing pyrrol, or in a current of its vapour; a pale pink colour immediately makes its appearance, and gradually deepens to an intense carmine. All kinds of fir-wood do not produce the reaction equally well, and it appears to depend in some way upon the resin, for if fir saw-dust be extracted by alcohol or ether, and bits of cotton or linen cloth dipped in the solution, they acquire the property of becoming red, when exposed to pyrrol vapour, after having been moistened with hydrochloric acid, although the colour is by no means so brilliant

as that developed on the wood itself. When agitated with cold dilute acids, pyrrol dissolves unchanged, but on heating the solution, it deposits a red flocky substance, and if not too dilute, the whole is converted into a gelatinous mass, so that the vessel may be inverted without anything escaping. The same change takes place in the cold, when the acid solution is kept for some days. When bichloride of platinum is added to a cold hydrochloric solution of pyrrol, it instantly becomes dark coloured, and in the course of a few minutes an abundant black precipitate, containing platinum, is deposited. When boiled with sesquichloride of iron, the solution becomes first green, and finally black. Bichromate of potash also decomposes it with the formation of an abundant black precipitate, and sulphate of copper, when heated with it for some time, acquires a green colour, and a small quantity of a black powder is deposited. It is rapidly oxidized by nitric acid, with the evolution of abundant red fumes, and formation of a dark-red solution, which, when diluted, permits a yellow resin to fall. By long-continued ebullition, oxalic acid is produced. An alcoholic solution of pyrrol gives white precipitates with corrosive sublimate and chloride of cadmium, but it does not precipitate the metallic oxides generally.

The combustion of pyrrol was very easily effected, and the results are subjoined. The first six analyses were those of crude or fetid pyrrol, and are all from different preparations except the first two. The last is that of the fragrant pyrrol.

| | | |
|---|---|---|
| I. | 5·805 | grains of pyrrol gave |
| | 15·250 | ... carbonic acid and |
| | 4·070 | ... water. |
| II. | 3·675 | grains of pyrrol gave |
| | 9·625 | ... carbonic acid and |
| | 2·550 | ... water. |
| III. | 5·250 | grains of pyrrol gave |
| | 13·830 | ... carbonic acid and |
| | 3·675 | ... water. |
| IV. | 4·033 | grains of pyrrol gave |
| | 10·590 | ... carbonic acid and |
| | 2·922 | ... water. |
| V. | 4·706 | grains of pyrrol gave |
| | 12·345 | ... carbonic acid and |
| | 3·307 | ... water. |
| VI. | 5·280 | grains of pyrrol gave |
| | 13·845 | ... carbonic acid and |
| | 3·676 | ... water. |
| VII. | 5·213 | grains of fragrant pyrrol gave |
| | 13·675 | ... carbonic acid and |
| | 3·649 | ... water. |

| | I. | II. | III. | IV. | V. | VI. | VII. | Mean. |
|---|---|---|---|---|---|---|---|---|
| Carbon, | 71·64 | 71·42 | 71·84 | 71·61 | 71·54 | 71·51 | 71·54 | 71·58 |
| Hydrogen, | 8·17 | 7·71 | 7·77 | 8·05 | 7·80 | 7·74 | 7·77 | 7·85 |
| Nitrogen, | ... | ... | ... | ... | ... | ... | ... | 20·57 |
| | | | | | | | | 100·00 |

These results correspond with the formula $C_8H_5N$, which requires the following numbers:—

| | | |
|---|---|---|
| 8 eq. carbon | = 48 | 71·64 |
| 5 ... hydrogen, | = 5 | 7·46 |
| 1 ... nitrogen, | = 14 | 20·90 |
| | 67 | 100·00 |

As none of the compounds of pyrrol are sufficiently definite to admit of their being used for fixing its atomic weight, recourse was had to the determination of the density of its vapour for this purpose, and three experiments were made at different stages of the investigation. The first was made after the pyrrol had received six rectifications and one treatment with acid, and its deviation from that required by theory showed that the material was not yet quite pure. The second, made after fourteen rectifications, and agitation with three successive portions of sulphuric acid, showed a close approximation to the theoretical number, while the third, made with the fragrant pyrrol, was as exact as could be desired. The details are as follow:—

| | I. | II. | III. |
|---|---|---|---|
| Temperature of the air, | 16° c. | 11° | 13° |
| ... vapour, | 198° | 186° | 201° |
| Excess of weight of the balloon, | 0·2285 grammes. | 0·2185 | 0·1610 |
| Capacity of do. | 324·5 c. c. | 328·5 | 303 |
| Barometer, | 767 m. m. | 744 | 764 |
| Residual air, | 0 | 1·5 | 4 |
| Density of the vapour, | 2·52 | 2·49 | 2·40 |

The formula $C_8H_5N$ requires:—

$$\begin{array}{lrl} 8 \text{ vol. carbon vapour,} & 0{\cdot}8290 \times 8 &= 6{\cdot}6320 \\ 10 \text{ ... hydrogen, ...} & 0{\cdot}0692 \times 10 &= 0{\cdot}6920 \\ 2 \text{ ... nitrogen, ...} & 0{\cdot}9713 \times 2 &= 1{\cdot}9426 \end{array}$$

$$\frac{9{\cdot}2666}{4} = 2{\cdot}31$$

Although the properties of the pyrrol now described are entirely distinct from those attributed to this substance by Runge, it cannot be doubted that they are really identical, although it is equally unquestionable that he never isolated his pyrrol, but merely obtained a small quantity of it held in solution by some gas, most probably a hydrocarbon. For this reason I think it right to retain his name,

although it is not formed in accordance with the received nomenclature of organic compounds, the more especially as it would be difficult, in the present state of our knowledge, to find another which would not be open to many objections. As far as its properties and chemical relations go, pyrrol approaches more nearly to the volatile organic bases than to any other class of nitrogenous compounds, but its basic properties are extremely weak, as it has no effect on test papers, and though soluble in dilute acids, can be expelled from the solution at the boiling heat. It forms, however, compounds with corrosive sublimate and chloride of cadmium, both of which are easily decomposed.

*Mercury Compound of Pyrrol.*—This substance is obtained by mixing alchoholic solutions of pyrrol and corrosive sublimate, when it is immediately precipitated as a white powder with a somewhat crystalline appearance, insoluble in water, and sparingly soluble in cold alcohol. It is more soluble on boiling, but is then partially decomposed. Excess of corrosive sublimate appears also to act upon it in some way, as the solution from which it has been deposited acquires, on standing, a dark red, and sometimes a fine purple colour, due, in all probability, to the oxidation of pyrrol. The substance employed for analysis was dried *in vacuo*, and was from different preparations:—

I. { 7·186 grains of mercury compound gave
2·079 ... carbonic acid.

II. { 8·906 grains of mercury compound gave
2·472 ... carbonic acid and
0·652 ... water.

III. { 7·131 grains of mercury compound gave
4·77 ... mercury.

| | Experiment. | | Calculation. | | |
|---|---|---|---|---|---|
| | I. | II. | | | |
| Carbon, | 7·89 | 7·57 | 7·88 | $C_8$ | 48 |
| Hydrogen, | ... | 0·81 | 0·82 | $H_5$ | 5 |
| Nitrogen, | ... | ... | 2·31 | N | 14 |
| Chlorine, | ... | ... | 23·31 | $Cl_4$ | 142 |
| Mercury, | ... | 66·89 | 65·68 | $Hg_2$ | 400 |
| | | | 100·00 | | 609 |

Corresponding with the formula $C_8H_5N + 2HgCl_2$.

*Cadmium Compound of Pyrrol* is obtained as a white crystalline powder, when alcoholic solutions of pyrrol and chloride of cadmium are mixed. It is insoluble in water, but dissolves readily in hydrochloric acid. It is rapidly decomposed when heated, either dry or in suspension in water or alcohol. Its analysis gave

{ 5·673 grains of cadmium salt gave
4·837 ... carbonic acid and
1·119 ... water.

| | Experiment. | Calculation. | | |
|---|---|---|---|---|
| Carbon, | 23·25 | 23·50 | $C_{16}$ | 96 |
| Hydrogen, | 2·19 | 2·44 | $H_{10}$ | 10 |
| Nitrogen, | ... | 6·87 | $N_2$ | 28 |
| Cadmium, | ... | 41·12 | $Cd_3$ | 168 |
| Chlorine, | ... | 26·07 | $Cl_3$ | 106·5 |
| | | 100·00 | | 408·5 |

This agrees pretty closely with the formula $2(C_8H_5N)+3CdCl$.

*Products of the Decomposition of Pyrrol.*

The decompositions of pyrrol have not led to results as definite as might have been anticipated; and I have therefore restricted myself to the examination of the red matter produced by the action of acids, and even that has been attended with no little trouble and difficulty.

*Pyrrol Red.*—This substance, as has already been frequently observed, is produced whenever pyrrol is boiled with an excess of acid; but notwithstanding the apparently definite nature of the change, it is extremely difficult to obtain it of uniform composition. This is due in part to its tendency to retain a small quantity of acid, and in part also to the fact that continued boiling produces a farther action, attended by the production of a dark colour in the acid liquid. When this occurs, the red matter gives very variable results when analysed, and hence, owing to the impossibility of ascertaining the exact length of time during which the fluid should be boiled to insure complete formation of the red matter, without going too far, the results of the analyses are by no means as concordant as might be desired. After a good many trials, it was found that the most successful results were obtained in the following manner:—Pyrrol was dissolved with the aid of brisk agitation in sulphuric acid diluted with from four to six parts of water, and the solution heated over the gas flame, while the flask was constantly shaken. As soon as the red matter had separated in distinct flocks, it was thrown on a filter and rapidly washed with boiling water, until the acid was almost entirely removed, during which process the pyrrol red acquired a slightly brown colour on the surface. A small quantity of diluted caustic potash was then poured upon the filter, when the product immediately became of a fine orange colour, which it retained after having been washed free of potash.

Pyrrol red is a fine, light, porous substance, with an orange-red colour, which becomes slightly brown by exposure to the air, especially when heated. It is insoluble in water, and is not readily moistened by that fluid. It is slightly soluble in cold, more so in boiling alcohol; and is again deposited on cooling in amorphous flocks. It is sparingly soluble in ether. Neither acids nor alkalies dissolve it, but if boiled with them for some time it is decomposed. Nitric acid oxidizes it, with the production of a resinous substance; and if the action be continued for a sufficient length of time, oxalic acid is found in the solution. When heated in

close vessels it yields an oil of an extremely offensive odour, and which gives the reactions of pyrrol, while a bulky charcoal is left in the retort. In the open air it catches fire, and burns readily. When exposed to 212° in the water-bath, it gains weight, owing to slow oxidation; and the portion used for analysis was therefore dried *in vacuo*. The results were—

I. { 6·381 grains of pyrrol red dried *in vacuo* gave
16·770 ... carbonic acid and
4·172 ... water.

II. { 6·440 grains of pyrrol red gave
0·846 ... nitrogen.

III. { 6·910 grains of pyrrol red gave
18·816 ... carbonic acid and
4·172 ... water.

IV. { 6·043 grains of pyrrol red gave
16·054 ... carbonic acid and
3·764 ... water

V. { 6·578 grains of pyrrol red gave
... nitrogen.

VI. { 6·132 grains of pyrrol red gave
16·248 ... carbonic acid and
3·793 ... water.

| | I. | II. | III. | IV. | V. | VI. | Mean. |
|---|---|---|---|---|---|---|---|
| Carbon, | 71·52 | ... | 71·77 | 72·45 | ... | 72·20 | 71·98 |
| Hydrogen, | 7·29 | ... | 6·70 | 6·66 | ... | 6·87 | 6·88 |
| Nitrogen, | ... | 13·14 | ... | ... | 14·05 | ... | 13·58 |
| Oxygen, | ... | ... | ... | ... | ... | ... | 7·56 |
| | | | | | | | 100·00 |

These results approximate most closely to the formula $C_{24} H_{14} N_2 O_2$, which requires

| | | |
|---|---|---|
| 24 eq. carbon, | 144 | 71·28 |
| 14 ... hydrogen, | 14 | 6·93 |
| 2 ... nitrogen, | 28 | 13·86 |
| 2 ... oxygen, | 16 | 7·93 |
| | 202 | 100·00 |

It is true that the numbers obtained by analysis do not accord well with this formula, and in particular the carbon is materially in excess, but this is undoubtedly due to a further decomposition produced by boiling; for if the heat be continued for some time during its preparation, the red matter acquires a dark-brown colour, and contains as much as 74 per cent. of carbon. The nature of the change by which the red matter is produced is readily intelligible, and is thus represented:—

| | |
|---|---|
| 3 equivalents pyrrol, | $C_{24} H_{15} N_3$ |
| − 1 eq. ammonia, | $H_3 N$ |
| | $C_{24} H_{12} N_2$ |
| + 2 eq. water, | $H_2 O_2$ |
| 1 eq. pyrrol red, | $C_{24} H_{14} N_2 O_2$ |

The formation of ammonia during this decomposition was demonstrated by distilling the acid filtrate from the red substance with potash. The distillate, which had the smell of ammonia contaminated with an empyreumatic odour, and sometimes with that of picoline, was saturated with hydrochloric acid, and evaporated with excess of bichloride of platinum to nearly complete dryness. Octahedral crystals of platinochloride of ammonium were deposited, which were examined under the microscope, and found to be free from any other salt.

The quantity of pyrrol contained in bone-oil is far from inconsiderable, and now that its properties have been investigated, it is easy to see that a great deal must have been destroyed during the treatment by which the crude bases were extracted. As my previous investigation of the picoline from coal-tar had shown that its neutral sulphate is converted into bisulphate by boiling, I took care to add to the crude sulphates extracted by agitating bone-oil with sulphuric acid, a large excess of acid before boiling it for the purpose of separating pyrrol; and in this way large quantities of the red matter in an impure state were produced during the early part of the investigation. It was only after I had advanced some way in the investigation that the cause of its formation became intelligible, and the crude sulphates were then distilled without the addition of acid, and the pyrrol mixed with empyreumatic oils and bases of the picoline series was obtained in quantity sufficient for investigation. The difficulty experienced in removing the last traces of pyrrol from the bases was very great, and it was necessary to boil the solution for several days; but I have now found that oxidizing agents, such as nitric acid, or, still better, bichromate of potash, offer invaluable means of purification, as they decompose the pyrrol without producing the slightest effect on the bases.

In the present and preceding parts of this investigation, I have directed attention to the basic constituents of bone-oil. In the next part, I propose to treat of its non-basic constituents, in the investigation of which some progress has already been made. In particular, it has been found that, by repeated rectifications, a fine volatile fluid, boiling as low as 150° Fahr., is obtained. This oil consists of at least two different substances, separable by means of a freezing mixture, which causes the fluid to divide into two perfectly distinct strata, with a well-marked line of separation. The higher fractions do not present this peculiarity, but they are also complex, containing benzene, and apparently some of its homologues, along with the alcohol radicals of the fatty series, and also nitrogenous compounds decomposable by alcoholic solution of potash and by sodium.

# XXXVI.—*On the Application of the Theory of Probabilities to the Question of the Combination of Testimonies or Judgments.* By GEORGE BOOLE, LL.D., Professor of Mathematics in Queen's College, Cork. Communicated by Bishop TERROT.

(Read 19th January 1857.)

1. The method for the solution of questions in the theory of probabilities applied in this paper, is that which was developed by the author in a treatise entitled, "An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities." The practical object of the paper is to deduce from that method certain conclusions relating to the combination of testimonies or judgments. Beside this, however, it will have a speculative reference to some more general questions connected with the theory of probabilities; and especially to the following question, viz.: To what extent the different modes in which the human mind proceeds, in the estimation of probability, may be considered as mutually confirming each other,—as manifestations of a central unity of thought amid the diversity of the forms in which that unity is developed.

The special problems relating to the combination of testimonies or judgments which are considered in this paper are the following: 1*st*, That in which the testimonies to be combined are merely differing numerical measures of a physical magnitude, as the elevation of a star, furnished by different observations taken simultaneously; 2*dly*, That in which the testimonies or judgments to be combined relate not to a numerical measure, but to some fact or hypothesis of which it is sought to determine the probability,—the probabilities furnished by the separate testimonies or judgments constituting our data.

2. I have, in the treatise to which reference has been made, described the method which will be practically applied in this paper as a general one. It will, I think, ultimately appear that there is a true and real sense in which the propriety of the description may be maintained. But at present I am anxious to qualify the appellation, and to speak of the method as general only with respect to problems which have been resolved into purely logical elements, or which are capable of such resolution. A more thorough analysis of the mental phenomena of expectation will, I think, tend to establish the position that all questions of probability, in the mathematical sense, admit of being resolved into primary elements of this nature, or, to speak more strictly, admit of being adequately represented by other problems whose elements are logical only. Postponing the consideration of this question, I will first endeavour to explain what is meant by the logical elements of a problem, and how the consideration of such elements affects the mode of its solution.

I regard the elements of a problem relating to probability as logical, when its data and its quæsitum are the probabilities of *events*. The reason for this appellation will shortly be seen. In *expression*, events may be distinguished as simple or compound. A simple event, *i.e.*, an event simple in expression, is one which is expressed by a single term or predication; a compound event, one which is formed by combining the expressions of simple events. "It rains,"—"it thunders," would be simple events; "it rains and thunders,"—"it either rains or thunders," &c., would be compound events. The constructions by which such combinations are expressed, although they belong to language, have their foundations in Logic. Thus the conjunctions *and*, *either*, *or*, &c., express merely certain operations of the faculty of Conception, the entire theory of which belongs to the science of Logic. The calculus of Logic, to which I shall have occasion to refer, is a development of that science in mathematical forms, in which letters represent things, or events, as subjects of Conception, and signs connecting those letters represent the operations of that faculty, the laws of the signs being the expressed laws of the operations signified. It is simply a mistake to regard that calculus as an attempt to reduce the ideas of Logic under the dominion of number. Such are the grounds upon which the class of problems to which I have referred are said to involve logical elements. The description is, however, not entirely appropriate, for the problems, as they are concerned with probabilities, in the mathematical acceptation of that term, involve numerical as well as logical elements; but it is by the latter that they are distinguished, and of them only is account taken in the nomenclature.

Thus, as an illustration of what has been said, that problem would be composed of logical elements, which, assigning for its numerical data the probabilities of the throwing an ace or six with each single die, should propose to determine the probability that the issue of a throw with two dice should be two aces, or that it should be an ace and a six, or that it should be either two aces or an ace and a six; and so on for any conceivable throw with any number of dice.

3. In the above example, the events whose numerical probabilities are given are simple events, of which the event whose probability is sought is a logical combination. But it might happen that the former events were themselves combinations of simple events. For instance, the data might be the probabilities that certain meteorological phenomena, as rain, thunder, hail, &c., would occur in certain definite combinations, and the object sought might be the probability that they would occur in certain other combinations; all these combinations being, such as it is within the province of language to express by means of conjunctions, and of the adverb *not*. Now this would still be a problem whose elements are logical.

4. But there are questions universally recognised as belonging to the theory of probabilities, whose elements cannot, in their direct significance, be regarded

as logical. The problem of the reduction of astronomical observations belongs to this class. Two observers, equally trustworthy, take an observation at the same place and time of the altitude of a star. One of them declares that it is 50° 20′, the other that it is 50° 22′. From these data, what shall we regard as the most probable altitude? We cannot, in this case, directly affirm that the numerical data are measures of probability at all. They are conflicting measures of a physical magnitude. And that which is sought is not the measure of a probability, but the most probable measure of the same magnitude. This is a problem evidently of a different kind from the one which we last considered. And accordingly it will be found that the principles of solution which have been actually applied to it are different from, perhaps we ought rather to say supplementary to, those which have sufficed for the solution of the others. In the problem of the dice, we have only to apply, and that directly, such principles as the following, viz., that when the probability of the occurrence of an event is $p$, that of its non-occurrence is $1-p$; that if the probabilities of two independent events are $p$ and $q$, that of their concurrence is $pq$; and so on. In the reduction of the conflicting elements of the observers' problem, another and quite distinct principle is usually employed, viz., the principle of the arithmetical mean, which affirms that if two different values are, on equal authority, assigned to a magnitude which is in itself single and definite, the mind is led to consider the arithmetical mean of those values as more likely to be its true measure than any other value. This is not the only principle which has been employed for the reduction in question. We shall refer to others. But it may justly be regarded as the most obvious of all which have been employed; and there is ground for considering it, as some eminent writers have expressly done, as primary and axiomatic in its nature.

5. The following is the typical form of problems whose elements are logical. If we represent the simple events involved in their expression by $x, y, z$, &c., then may all their data (we will suppose the number of data to be $n$) be expressed, in accordance with the principles of the calculus of Logic, under the general forms

$$\text{Prob. } \phi_1(x, y, z\,..)=p_1, \quad \text{Prob. } \phi_2(x, y, z\,..)=p_2, \;..\; \text{Prob. } \phi_n(x, y, z\,..)=p_n,$$

and the quæsitum, or object sought, will be the value of

$$\text{Prob. } \psi(x, y, z\,..),$$

where $\phi_1, \phi_2, .. \phi_n$ and $\psi$ denote different but given logical functions of $x, y, z$.

Although the method for the solution of questions in the Theory of Probabilities whose elements are logical has been developed at considerable length in a special chapter of the *Laws of Thought*, yet much that is essential for its proper and distinctive exhibition, has only been discovered since the publication of that work. For this reason it will be proper to offer some account here of the principles upon which the method rests.

6. I define the mathematical probability of an event as the ratio which the number of distinct cases or hypotheses favourable to that event bears to the whole number of distinct cases possible, supposing that to none of those cases the mind is entitled to give any preference over any other. Fundamentally, this definition agrees with that of LAPLACE. "La théorie des hazards consiste," he remarks, "à réduire tous les évènements du même genre a un certain nombre de cas également possibles c'est à dire tels que nous soyons également indécis sur leur existence et à déterminer le nombre de cas favorables à l'évènement dont on cherche la probabilité. Le rapport de ce nombre a celui de tous les cas possibles est la mesure de cette probabilité."—*Essai Philosophique sur les Probabilités*.

It is implied in this definition, that probability is relative to our actual state of information, and varies with that information. Of this principle LAPLACE gives the following illustration:—"Let there be three urns, A, B, C, of which we are only informed that one contains black and the other white balls; then, a ball being drawn from C, required the probability that the ball is black. As we are ignorant which of the urns contains black balls, so that we have no reason to suppose it to be the urn C rather than the urn A or the urn B, these three hypotheses will appear equally worthy of credit, but as the first of the three hypotheses alone is favourable to the drawing of a black ball from C, the probability of that event is $\frac{1}{3}$. Suppose now that, in addition to the previous data, it is known that the urn A contains only white balls, then our state of indecision has reference only to the urns B and C, and the probability that a ball drawn from C will be black is $\frac{1}{2}$. Lastly, if we are assured that both A and B contain white balls only, the probability that a black ball will issue from C rises into certitude."—*Essai Philosophique sur les Probabilités*, p. 9.—(*Phil. Mag.*, p. 433.) Our estimate of the probability of an event varies not absolutely with the circumstances which actually affect its occurrence, but with our knowledge of those circumstances.

7. When the probabilities of simple events constitute our only data, we can, by virtue of the above definition, determine the probability of any logical combination of those events, and this either, 1*st*, absolutely; or, 2*dly*, conditionally. The reason why we can, in this case, more immediately apply the definition is, that not only is no connection expressed among the events whose probabilities are given, but none is implied, nor is any restraint imposed upon their possible combinations. This, as we shall see, is not the case when the data are the probabilities of compound events.

As an example, let us suppose that the probability of the conjunction of two events, $x$ and $y$, is required, the data being simply that the probability of the event $x$ is $p$, and that of the event $y$ is $q$. Or, to express the problem in a form which we shall hereafter generally employ:

| | | |
|---|---|---|
| Given | Prob. $x=p$, | Prob. $y=q$, |
| Required | Prob. $xy$. | |

Let $a$ be the number of distinct cases favourable to the event $x$, out of $m$ distinct cases equally possible, from the comparison of which the probability $p$ has been assigned to the event $x$. In like manner let $b$ be the number of distinct cases favourable to the event $y$, out of $n$ distinct cases equally possible, from the comparison of which the probability $q$ has been assigned to the event $y$. Then,

$$\frac{a}{m}=p \quad \text{and} \quad \frac{b}{n}=q.$$

Now the conjunction $xy$ can only come to pass through the combination of some one of the $a$ cases in which $x$ happens, with some one of the $b$ cases in which $y$ happens, at the same time that we have an equal right to suppose that any one of the $m$ cases in which $x$ happens or fails may combine with any one of the $n$ cases in which $y$ happens or fails. To none of these combinations is the mind entitled to attach any preference over any other, Hence there exist $ab$ distinct cases favourable to the conjunction of $x$ and $y$ out of a total of $mn$ distinct and equally possible cases. Thus, by the definition, the probability of the conjunction of $x$ and $y$ will be represented by the product $\frac{ab}{mn}$ or $pq$.

Here the question may be asked,—Does, then, no difference exist between the case in which the events $x$ and $y$ are known to be independent, and that in which we are simply ignorant of the existence of any connection between them? I reply that there is none, so far as the numerical estimation of probability is concerned. There is, however, an important difference as respects the practical value of the numerical result. If the events $x$ and $y$ are known to be independent, and to have probabilities $p$ and $q$, we know that, in the long run, the conjunction $xy$ will tend to recur with a frequency which will be proportional to the magnitude of the fraction $pq$, We do not know that this will be the case if we are simply ignorant of any connection between $x$ and $y$. This is the difference referred to, and it is an important one. But it does not affect the calculation of probability as flowing from the definition of its numerical measure.

8. As from the data Prob. $x=p$, Prob. $y=q$, we deduce Prob. $xy=pq$, so from the same data we should have, adopting the language of the calculus of Logic,

$$\text{Prob. } x(1-y)=p(1-q) \qquad \text{Prob. } (1-x)(1-y)=(1-p)(1-q),$$

and so on. Here $x(1-y)$ denotes the compound event which consists in the occurrence of $x$ conjointly with the non-occurrence of $y$; $(1-x(1-y)$, the compound event which consists in the joint non-occurrence of both $x$ and $y$.

Extending this mode of investigation, we arrive at the theorem

$$\text{Prob. } \phi(x, y, z \ldots) = \phi(p, q, r \ldots) \quad . \quad . \quad . \quad . \quad (1)$$

where $x$, $y$, $z$, &c., denote any simple events whose probabilities (our only data) are $p$, $q$, $r$ . ., and $\phi(x, y, z \ldots)$ denotes any event which can be expressed by

means of the simple events $x$, $y$, $z$, &c., in accordance with the notation of the calculus of Logic.

By the above theorem the probability of any compound event is determined absolutely, when the probabilities of its simple components are given.

9. And by the same mode of investigation, the probability of any combination may be determined conditionally, *i.e.*, the probability which the combination will have under a given condition consisting in the happening of some other combination. Thus, if our data are as before,

$$\text{Prob. } x=p, \qquad \text{Prob. } y=q, \qquad \text{Prob. } z=r, \ \&c.$$

and if we require the probability that if the event $\phi(x, y, z \,. \,.)$ present itself, the event $\psi(x, y, z \,. \,.)$ will be present at the same time, we may demonstrate the following result, viz.:—

Prob. that if $\phi(x, y, z \,. \,.)$ happen, $\psi(x, y, z \,. \,.)$ will be present also

$$=\frac{\chi(p, q, r \,. \,.)}{\phi(p, q, r \,. \,.)} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where the form of the function $\chi$ is determined by multiplying together, according to the principles of the calculus of Logic, the functions $\phi(x, y, z \,. \,.)$ and $\psi(x, y, z \,. \,.)$, and representing the result by $\chi(x, y, z \,. \,.)$.—(*Laws of Thought*, p. 258, Prop. I.)

10. I postulate that when the data are not the probabilities of simple events, we must, in order to apply them to the calculation of probability, regard them, not as primary, but as derived from some anterior hypothesis, which presents the probabilities of simple events as its system of data, and exhibits our actual data as flowing out of that system, in accordance with those principles which have already been shown to be involved in the very definition of probability.

The ground of this postulate is, that to begin with the simple and proceed to the complex, seems to be, in all questions involving combinations such as we are here concerned with, a necessary procedure of the understanding. The calculation of probability depends upon combinations subject to a peculiar condition, viz., that they shall always present to us a series of cases or hypotheses, to none of which the mind is entitled to attach any preference over any other. We cannot, in endeavouring to ascend from the complex to the simple, secure the maintenance of this condition; but we can do so in descending from the simple to the complex. We have had an illustration of this truth in the reasoning by which we deduced the expression for the probability of the complex event $xy$ from the probabilities of the simple events $x$ and $y$, supposed to be given. And the methods which have been actually employed in the solution of problems whose immediate data were not the probabilities of simple events, have in fact rested upon the postulate above referred to. Thus in questions relating to juries, the immediate data are the probabilities, founded upon continued observation, that a decision will be unanimous, or that it will be pronounced by a given majority, &c.

But it is usual, in solving these problems, to regard such events as compound, and to derive them from a hypothesis which presents as its scheme or system of data, the probabilities of individual correctness of judgment in the members of the jury; the correctness of judgment in any such members being regarded as a simple event. And this mode of procedure is a very natural and very obvious one. For the degree of unanimity of a decision will so far depend upon the correctness of judgment in the members, that, if we knew what the probability of correctness in each member was, we could determine *à priori* the probability of any proposed measure of agreement in the body.

The only question which arises, indeed, is not concerning the necessity of the postulate, but concerning the mode in which it may be lawfully applied. How shall we lawfully construct the hypothesis by which the solution of a problem shall be made to depend upon the consideration of simple events. In answering this question, I will endeavour to show, 1*st*, upon what the construction of the hypothesis does not depend; 2*dly*, upon what it does depend.

11. The legitimate construction of the hypothesis in question cannot depend upon the accidents of language, or causes deeper than accident, which have led us to express particular things or events by simple terms, thus regarding them as simple events; and other events by combinations of these simple terms, thus presenting them as compound. The solution of a question in the theory of probabilities must depend upon the *information* conveyed in the data, not upon the peculiar elements and constructions of the language which is the vehicle of that information. Languages differ widely in these respects. Objects and events which in one language are expressed by simple terms, are in another expressed by combinations of simple terms. It is affirmed that a perfectly general method of solution must be independent of, and superior to, differences like these.

I will endeavour to illustrate this principle by an example. Let the problem to be resolved be the following. The probability of the concurrence of rain and snow is $p$, of the concurrence of snow and wind $q$ and of the concurrence of wind and rain $r$; required the probability of the concurrence of wind, rain, and snow.

Now suppose that we had to interpret the problem into a language in which there were no simple terms corresponding to the simple terms "wind," "rain," "snow," but in which there were simple terms for the three first of the concurrences above described.

We may, for simplicity, suppose that language to be a dialect of English, and the concurrence of rain and snow to be represented in it by the term "sleet," the concurrence of snow and wind by the term "drift," and the concurrence of wind and rain by the term "storm."

The event whose probability is sought, viz., the concurrence of rain, snow, and wind, would, in such a language, be represented by the combination either of two of the terms above defined (as of sleet with drift), or of all the terms together,

since the presence of any two of the phenomena "sleet," "drift," "storm," implies that of the third, and involves the conjunction of the phenomena of rain, snow, and wind.

The data of the problem we are considering might then, in the imagined dialect, assume the following form :

The probability of sleet is $p$, of drift $q$, and of storm $r$; required the probability of the concurrence of the phenomena of sleet, drift, and storm.

But in this form the problem would not, in its data, express all the knowledge which the person using such language must possess of the connection of the events to which it related. He must know that it was impossible that any two of the events sleet, drift, and storm, should occur without the third, so that the problem, if so stated as to embody the same amount of actual knowledge as is conveyed in the previous statement of it, would assume the following form :

The probability of sleet is $p$, of drift $q$, of storm $r$, and these events are so connected, that no two of them can occur without the third occurring, What is the probability of their concurrence?

Now the principle affirmed declares that the solution of the problem must be the same whichsoever of these forms of statement we adopt.

As languages increase in affluence, the number of their simple terms becomes augmented, partly through the necessity of giving expression to new ideas, partly through the wish to give more convenient expression to definite and oft-recurring combinations of the old ones. With every term invented in subserviency to the latter purpose, a definition must be introduced. A dictionary, setting aside its philological portion (and even this not wholly), is a record of such definitions. As a consequence of such definitions of terms, spring up also propositions innumerable connecting these terms—propositions which in no degree add to the amount of our absolute knowledge, which are quite distinct from the discovered facts and laws of nature and of human history, but are merely logical deductions from the definitions. We might conceive of a language in which all possible combinations of ideas should be expressed by simple terms, with connecting definitions and propositions *ad infinitum*. The realization of such a conception is neither practicable nor desirable; but it is, nevertheless, the *limit* toward which all languages, which are not dead or decaying, do actually tend. The progressive action of this tendency does not affect the laws of expectation, neither, therefore, can it affect any consistent and scientific theory which is founded upon those laws.

We are not, therefore, permitted to assume that any events which, in the language of the problem, may be presented as simple events, must therefore be adopted as such into the hypothesis which is to form the basis of our method of solution. Nor, on the other hand, are we forbidden to employ transformations (sanctioned by the rules of Logic) which will have the effect of introducing an

entirely new scheme of simple events as the elements of the hypothesis in question.

12. To what conditions, then, must the hypothesis be subject? This question I now proceed to answer.

The hypothesis must be such that it may be consistently applied, without imposing upon the data any other conditions than those of possibility, *i.e.*, of accordance with a possible experience.

This principle is so obviously true, that it will only be needful to show how the conditions of possible experience are discovered. I shall subsequently show how their discovery limits and determines the hypothesis upon which the solution of questions in the theory of probabilities, whose elements are logical, depends.

The data of such problems are the probabilities of events. The object sought is also the probability of an event. The numerical values of these probabilities must be expressible by positive proper fractions. At any rate, they must not transcend the limits 0 and 1. This is one condition to which they are subject. Generally, however, there will exist other conditions dependent upon the mutual relations of the events whose probabilities are given.

Thus, if $p$ were the probability that an event $x$ will happen, $q$ the probability that $x$ and $y$ will both happen, we have, as a necessary condition,

$$p \geqq q.$$

Again, if $p$ were the probability that $x$ and $y$ will both happen, $q$ the probability that they will both fail, we must have the condition,

$$p + q \leqq 1,$$

a condition which does not hold in the previous case.

I have, in the *Laws of Thought*, treated of these conditions, and of the principles by which they may be determined, in a special chapter (Cap. xix., *On Statistical Conditions*). A more simple, and at the same time perfectly general method, for their determination was afterwards discovered by me, and published in the *Philosophical Magazine*, Aug. 1854. As the method is of fundamental importance, I shall here illustrate it by an example, at the same time introducing a slight change in the mode of treatment, which leaves nothing to be desired in point of simplicity. The conditions to the discovery of which the method is applicable will be termed, in accordance with the language employed in the *Philosophical Magazine*,—the "Conditions of Possible Experience;" inasmuch, as whenever the numerical data of a problem are derived from actual experience these conditions will be satisfied, and whenever in data professing to be thus derived they are not satisfied, the presence of mistake or fraud may with certainty be affirmed.

*Determination of the Conditions of Possible Experience.*

13. To explain the method of effecting this object, by an example, I will first symbolically express the problem of Art. 11.

Let us then represent rain by $x$, snow by $y$, and wind by $z$. The problem in question then takes the following form:—

Given Prob. $xy=p$, Prob. $yz=q$, Prob. $xz=r$ . . . (1)

Required Prob. $xyz$ . . . . . . (2)

The value required we shall represent by $u$. It is our present object, not to solve this problem, but to ascertain the conditions which must connect $p$, $q$, and $r$, in order that the data may be possible, with the corresponding limitations of $u$. For if $u$ were itself determined by experience, it would be subject to conditions of possibility similar to those which govern $p$, $q$, and $r$.

Now let us write, resolving the events in the problem into the possible alternations out of which they are formed,

$$\text{Prob. } xyz=u, \quad \text{Prob. } xy\bar{z}=\lambda, \quad \text{Prob. } xz\bar{y}=\mu, \quad \text{Prob. } yz\bar{x}=\nu.$$

We have then

$$u+\lambda=p, \quad u+\nu=q, \quad u+\mu=r \quad . \quad . \quad . \quad (3)$$

The first of these equations only expresses that the probability of the concurrence of $x$ and $y$ is equal to the probability of the concurrence of $x$, $y$, and $z$, and the probability of the concurrence of $x$ and $y$ without $z$. To the equations (3) we must now add the inequations

$$\left.\begin{array}{c} u \geqq 0, \quad \lambda \geqq 0, \quad \mu \geqq 0, \quad \nu \geqq 0, \\ u+\lambda+\mu+\nu \leqq 1 \end{array}\right. \quad . \quad . \quad . \quad (4)$$

expressing the conditions to which $u$, $\lambda$, $\mu$, $\nu$, *1st*, as probabilities, and, *2dly*, as probabilities which do not altogether make up certainty, are subject.

First, we will eliminate $\lambda$, $\mu$, and $\nu$. Their values found from (3) are

$$\lambda=p-u \qquad \mu=r-u \qquad \nu=q-u.$$

Substituting these in (4) we have

$$u \geqq 0 \qquad p-u \geqq 0 \qquad q-u \geqq 0 \qquad r-u \geqq 0$$
$$p+q+r-2u \leqq 1,$$

Whence,

$$\left.\begin{array}{l} u \leqq p, \quad u \leqq q, \quad u \leqq r, \\ u \geqq 0, \quad u \geqq \dfrac{p+q+r-1}{2} \end{array}\right\} \quad . \quad . \quad . \quad (5)$$

Such are the conditions to which the quantity $u$ is subject, conditions which the value of Prob. $xyz$ must *à priori* satisfy.

To determine the conditions connecting $p$, $q$, and $r$, we must from (5) eliminate $u$. Now, if we have any two inequations of the form

$$u \leqq a \qquad u \geqq b$$

the only condition connecting $a$ and $b$ which they establish is,

$$a \geqq b.$$

Applying this principle to (5), we have

$$p \geqq 0 \,. \qquad p \geqq \frac{p+q+r-1}{2}\,.$$

$$q \geqq 0 \,. \qquad q \geqq \frac{p+q+r-1}{2}\,.$$

$$r \geqq 0 \,. \qquad r \geqq \frac{p+q+r-1}{2}\,.$$

These may be reduced to the somewhat simpler form

$$\left.\begin{array}{lll} p \geqq 0 & q \geqq 0 & r \geqq 0 \\ p \geqq q+r-1 & & \\ q \geqq r+p-1 & & \\ r \geqq p+q-1 & & \end{array}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

Such are the conditions of possible experience in the data.

Suppose, for instance, it was affirmed as a result of medical statistics, that in two-fifths of a number of cases of disease of a certain character, two symptoms, $x$ and $y$, were observed; in two-thirds of all the cases, the symptoms $y$ and $z$ were observed; and in four-fifths of all the cases, the symptoms $x$ and $y$ were observed; so that the number of cases observed being large, we might, on a future outbreak of the disease, consider the fractions two-fifths, two-thirds, and four-fifths, as the probabilities of recurrence of the particular combinations of the symptoms $x$, $y$, and $z$, observed. The above formulæ would show that the evidence was contradictory. For representing the respective fractions by $p$, $q$, and $r$, the condition

$$p \geqq q+r-1$$

is not satisfied.

It is an evident consequence of the principle enunciated in Art. 11, that in determining the conditions of possible experience and of limitation, we may employ any *translated* form of the problem, just as well as the form in which it is originally expressed. Thus, if we take the translated form of the problem of that article, and represent sleet by $s$, drift by $t$, storm by $u$, we shall have as the data

$$\text{Prob. } s=p, \qquad \text{Prob. } t=q, \qquad \text{Prob. } u=r$$

with the conditions

$$s\bar{t}u=0, \qquad ut\bar{s}=0, \qquad us\bar{t}=0 \quad . \quad . \quad . \quad . \quad (7)$$

the quæsitum being Prob. $stu$, which, as before, we shall represent by $u$.

Now if we write

$$\text{Prob. } stu=u, \qquad \text{Prob. } s\bar{t}u=0, \qquad \text{Prob. } sut=0, \qquad \text{Prob. } \bar{s}\bar{u}t=\lambda$$

$$\text{Prob. } tu\bar{s}=0, \qquad \text{Prob. } t\bar{u}\bar{s}=\mu, \qquad \text{Prob. } u\bar{s}\bar{t}=\nu \quad . \quad . \quad . \quad (8)$$

we have the following equations:—

$$\left.\begin{array}{l} u+\lambda=p \\ u+\mu=q \\ u+\nu=r \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

with the inequations

$$u \gtreqless 0, \quad \lambda \gtreqless 0, \quad \mu \gtreqless 0, \quad \nu \gtreqless 0, \quad u+\mu+\nu+\lambda \quad \lesseqgtr 1 \quad . \quad (10)$$

Determining from the equations $\lambda$, $\mu$, $\nu$, and substituting in the inequations, we get

$$u \gtreqless 0 \quad p-u \gtreqless 0, \quad q-u \gtreqless 0, \quad r-u \gtreqless 0, \quad p+q+r-2u \quad \lesseqgtr 1 \quad . \quad (11)$$

a system which agrees with that obtained by the previous investigation (5) Art. 13.

14. The general rule for the determination of the conditions of possible experience and of limitation in a question of probability may be thus stated.

Resolve the events whose probabilities are either given or sought, into the mutually exclusive alternatives which they involve. If the calculus of Logic is employed, this is done by development.

Represent the probabilities of these alternations by $\lambda$, $\mu$, $\nu$, &c., and express the probabilities given and sought by the corresponding sums of these quantities. This will furnish a series of *equations*, which we will suppose to be $n$ in number.

Determine from these equations any $n$ of the quantities $\lambda$, $\mu$, $\nu$, in terms of the others.

Substitute the values thus obtained in the *inequations*

$$\lambda \gtreqless 0 \quad \mu \gtreqless 0 \quad \nu \gtreqless 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\lambda+\mu+\nu \,.\,. \quad \lesseqgtr 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eliminate in succession such of the quantities $\lambda$, $\mu$, $\nu$, . . as are left in the above inequations after the substitution.

The elimination of any quantity as $\tau$ from the inequations, is effected by reducing each inequation to the form $\tau \lesseqgtr a$, or to the form $\tau \gtreqless b$, and observing that two such forms as the above give $a \gtreqless b$.

If the "alternations" into which the events whose probabilities are given or sought are resolved, extend to all possible combinations of the simple events out of which they are formed, the inequations (2), must be replaced by the equation

$$\lambda+\mu+\nu \,.\,. =1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The rest of the process will be the same as before.

In the form of the above method developed in the *Philosophical Magazine* the quantities $\lambda$, $\mu$, $\nu$, . . represent the probabilities, not of those alternations alone, which are contained in the events whose probabilities are given or sought, but of all possible alternations which can be formed, by combining the simple events $x$, $y$, $z$ . . In this form, therefore, we have always an equation of the form (3), in the place of an inequation of the form (2). But though the result is the same, the form given to the method in this section is to be preferred, as it requires the elimina-

tions of a smaller number of symbols, except when the condition referred to in (3) is fulfilled, in which case, the methods are identical.

15. It remains to show how the conditions of possible experience as above determined, restrict us in the choice of the hypothesis, by the aid of which the final solution is to be obtained.

Taking for example the above problem of Art. 13, let us inquire whether it would be lawful to assume $x$, $y$, and $z$ as the primary simple events of the problem.

If we make this assumption and then write

$$\text{Prob. } x=\alpha \qquad \text{Prob. } y=\beta \qquad \text{Prob. } z=\gamma$$

we find

$$\text{Prob. } xy=\alpha\beta \qquad \text{Prob. } yz=\beta\gamma \qquad \text{Prob. } zx=\gamma\alpha$$

whence comparing with (1) Art. 13—

$$\alpha\beta=p \qquad \beta\gamma=q \qquad \gamma\alpha=r$$

solving which equations we have

$$\alpha=\sqrt{\frac{qr}{p}} \quad \beta=\sqrt{\frac{rp}{q}} \quad \gamma=\sqrt{\frac{pq}{r}} \therefore \text{ Prob. } xyz=\alpha\beta\gamma=\sqrt{pqr} \quad . \quad (4)$$

Now, $\alpha$, $\beta$, $\gamma$, being by assumption probabilities, and therefore, lying numerically between the limits 0 and 1, we must have

$$qr \leqq p \qquad rp \leqq q \qquad pq \leqq r \quad . \quad . \quad . \quad . \quad (5)$$

as the conditions to which $p$, $q$, and $r$ (beside being fractional) must be subject. These conditions do not, however, agree with, and are not involved in, the conditions of possible experience, determined in (6) Art. 13. We may conclude, therefore, that the hypothesis upon which our solution is founded, involves elements, the introduction of which is unwarranted, and that the value of Prob. $xyz$ determined is erroneous.

We may show, in fact, that the conditions (5) imply the conditions of possible experience, and something more. If $qr \leqq p$ then, *à fortiori*, $qr \leqq p+(1-q)(1-r)$ since $(1-q)(1-r)$ is essentially positive. Therefore,

$$qr \leqq p+1-q-r+qr$$

$$\text{whence} \qquad p \geqq q+r-1$$

which is one of the conditions (6) Art 13. In the same way the other conditions in that article may be deduced from (5). The reverse reduction is, however, impossible.

16. The hypothesis upon which the method developed in the *Laws of Thought*, cap. xvii., for the solution of questions in the theory of probabilities whose elements are logical, is founded, seems to be the only one which satisfies the requirement referred to in Art. 12. It was not, however, upon such considerations as this,

that the method was founded. As presented in the *Laws of Thought*, it rests upon principles which, to my own mind, have something of an axiomatic character. Viewed in this light, its perfect accordance with the requirement above explained may be considered as a verification of it *à posteriori*. In itself, however, this accordance affords a sufficient ground of confidence in the legitimacy of the hypothesis. On the proof of this accordance I shall say something hereafter. At present I will only state the hypothesis, and show in what the accordance consists.

The hypothesis is the following:—Translating our problem by the aid of the calculus of logic into a language in which the events whose probabilities are given, appear as simple events subject to conditions founded on their definitions, Art. 11, we ascend above these simple events to another scheme of simple events, which are free, and which, when actually subjected to the conditions to which the before-mentioned simple events are necessarily subject, shall have the same probabilities, and shall in every respect take their place. The unknown probabilities of the free simple events, which form the elements of this hypothesis, must be so determined as to render the substitution possible, and to permit a formal construction of the problem, both in its data and by its quæsitum, out of those new elements.

The unknown probabilities being thus determined, the problem assumes a form in which its elementary data are the probabilities of simple events unrestricted by any condition. In this form the solution of the problem is possible by mere consequence of the fundamental definition of probability. The ground upon which this hypothesis was presented in the *Laws of Thought* was its intrinsic reasonableness. On this point I will only refer to my observations in the original work. The ground upon which, in the present essay, I wish to rest the hypothesis is, that it is the only one which does not impose upon the data other conditions than those of conformity with a possible experience. The conditions which must be fulfilled in order that $p'$, $q'$, &c., in the substituted and hypothetical data, may be measures of probability at all,—*i.e.*, may be positive proper fractions,—are precisely the conditions of possible experience in the original data. (See Appendix.)

17. The application of this hypothesis is so fully explained in the *Laws of Thought*, cap. xvii., that I shall here only describe the general method for the solution of questions in probabilities to which it leads, and show the connection which exists between the several parts of that method and the foregoing doctrine.

*General Method.*

Representing the problem to be solved under the form—

Given Prob. $\phi_1(x, y, z, \ldots) = p$ Prob. $\phi_2(x, y, z \ldots) = q$, &c.

Required Prob. $\psi(x, y, z \ldots)$

and expressing the unknown value of Prob. $\psi(x, y, z \ldots)$ by $u$, we form the logical equations:—

$$\phi_1(x, y, z \ldots) = s \qquad \phi_2(x, y, z \ldots) = t, \text{ \&c.}$$
$$\psi(x, y, z \ldots) = w$$

and hence, determining $w$ as a developed logical function of $s, t \ldots$ we have a result of the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D \quad . \quad . \quad . \quad . \qquad (1)$$

Here A, B, C, D are logical combinations of the simple events, $s$, $t$, &c., and the connection in which they stand to the event $w$ and to each other is the following: A expresses those combinations of $s$, $t$, &c., which are entirely included in $w$,—*i.e.*, which cannot happen without our being permitted to say that $w$ happens. B represents combinations which may happen, but are not included under $w$; so that when they happen, we may say that $w$ does not happen. C represents those combinations, the happening of which leaves us in doubt whether $w$ happens or not. D, those combinations, the happening of which is wholly interdicted.

Thus far we have only translated our problem into a language in which its data are the probabilities of simple events, viz.:—

$$\text{Prob. } s = p \qquad \text{Prob. } t = q, \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The condition, founded on definition, to which these simple events are subject is,

$$D = 0$$

or, which amounts to the same thing,

$$A + B + C = 1$$

indicating that the combinations expressed by A, B, and C can alone happen. If we represent $A+B+C$ by V, we have

$$w = A + \frac{0}{0}C \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

with the condition

$$V = 1 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Of these equations, the latter expresses the conditions to which the simple events, $s$, $t$, &c., are subject; the former expresses $w$ as a logical combination of those events.

We now, in accordance with the hypothesis, ascend to a new scheme of simple events, $s'$, $t'$, &c., unrestricted by any condition, and possessed of unknown probabilities, $p'$, $q'$, which are to be so determined that when $s'$, $t'$ .. are subjected to the same condition (4) to which $s, t$ .. are subject, they will have the same probabilities as $s, t$ .. The system of equations to which we are thus led, and which contains the implicit solution of the problem, is the following (*Laws of Thought*, cap. xvii., p. 267):—

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = \frac{A+C}{u} = V \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

$V_s$ being formed by selecting those terms from V, which contain $s$ as a factor; V

those, which contain $t$ as a factor, &c.; and then regarding $s$, $t$, &c., as algebraic quantities. From the system thus formed, we must determine $u$ as a function of $p, q$ . . and the arbitrary constant $c$, should it be present. This will be the solution of the problem.

The quantities $s, t$ . . are the same as $p', q'$ . . and represent the probabilities of the hypothetical simple events, represented by $s', t'$ . . Accordingly, as probabilities, they must admit of being determined as positive proper fractions, and that the solution may not be ambiguous, they must admit of only one such determination. These conditions will be fulfilled whensoever the problem represents a possible experience, and it will be then only fulfilled. And in this way, or by directly investigating the conditions of possibility by the rule of Art. 14, a solution is made determinate.

The arbitrary constant $c$ does not, as has been intimated, always present itself. When it does, it represents the unknown probability, that if the event C occur, $w$ will occur. It indicates, therefore, the new experience which would be necessary in order to make the solution definite.

18. I will, for the sake of illustration, apply the method to the problem of Art. 11, and in so doing I will limit the solution by the conditions relative to $s$, $t$, &c.

The problem, as symbolically expressed in Art. 13, is as follows:—

$$\left.\begin{array}{llll} \text{Given} & \text{Prob. } xy=p & \text{Prob. } yz=q & \text{Prob. } zx=r \\ \text{Required} & \text{Prob. } xyz & & \end{array}\right\} \quad . \qquad (1)$$

Translating the problem as directed in the first part of the rule, we write

$$\left.\begin{array}{ccc} xy=s & yz=t & zx=v \\ & xyz=w & \end{array}\right\} \quad . \quad . \quad . \qquad (2)$$

whence, by the calculus of logic,

$$w=stv+0\,(s\bar{t}\,\bar{v}+t\bar{s}\,\bar{v}+v\bar{s}\,\bar{t}+\bar{s}\,\bar{v}\,\bar{t})$$
$$+\frac{1}{0}\,(s\,t\,\bar{v}+s\,\bar{v}\,t+t\,\bar{v}\,s) \quad . \quad . \quad . \quad . \qquad (3)$$

Hence we find

$$\mathrm{V}=stv+s\bar{t}\,\bar{v}+t\bar{s}\,\bar{v}+v\bar{s}\,\bar{t}+\bar{s}\,\bar{t}\,\bar{v} \quad . \quad . \quad . \qquad (4)$$

and are led to the algebraic system of equations

$$\frac{stv+s\bar{t}\bar{v}}{p}=\frac{stv+t\bar{s}\bar{v}}{q}=\frac{stv+v\bar{s}\bar{t}}{r}$$
$$=\frac{stv}{u}=stv+s\bar{t}\bar{v}+t\bar{s}\bar{v}+v\bar{s}\bar{t}+\bar{s}\bar{t}\bar{v} \quad . \quad . \quad . \qquad (5)$$

These equations may be simplified by dividing every term by $\bar{s}\,\bar{t}\,\bar{v}$, and then assuming

$$\frac{s}{\bar{s}}=s' \qquad \frac{t}{\bar{t}}=t' \qquad \frac{v}{\bar{v}}=v' \quad . \quad . \quad . \quad . \qquad (6)$$

They thus give

$$\frac{s't'v'+s'}{p}=\frac{s't'v'+t'}{q}=\frac{s't'v'+v'}{r}$$
$$=\frac{s't'v'}{u}=s't'v'+s'+t'+v'+1 \quad . \quad . \quad . \quad . \qquad (7)$$

The condition to which $s'\,t'\,v'$ are subject obviously is, that they shall be positive quantities, for this is equivalent to the condition that $s, t, v$ shall be positive fractions.

From (7) we readily find

$$\frac{s'}{p-u} = \frac{t'}{q-u} = \frac{v'}{r-u} = \frac{s't'v'}{u} = s't'v'+s'+t'+v'+1 \quad . \quad (8)$$

Whence

$$\left.\begin{aligned} s' &= \frac{p-u}{2u-p-q-r+1} \\ t' &= \frac{q-u}{2u-p-q-r+1} \\ v' &= \frac{r-u}{2u-p-q-r+1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

Substitute these values in the equation

$$\frac{s'}{p-u} = \frac{s't'v'}{u}$$

and reducing we get

$$(p-u)\,(q-u)\,(r-u) = u\,(2\,u-p-q-r+1)^2 \quad . \quad . \quad (10)$$

an equation for determining $u$.

And now let us inquire into the consequences which flow from the condition that $s'\,t'\,v'$ are positive quantities.

In the first place, the last member of (8), and therefore each other member of that system will be positive. This requires that the denominators, $p-u$, $q-u$, $r-u$, and $u$, should be positive, whence we have

$$\left.\begin{aligned} &u \overline{\overline{>}} 0 \\ u \overline{\overline{<}} p,\ &u \overline{\overline{<}} q,\ u \overline{\overline{<}} r \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (11)$$

Again, $p-u$, $q-u$, and $r-u$ being positive, the common denominators, $2u-p-q-r+1$ in (9) must be positive, whence

$$u \overline{\overline{>}} \frac{p+q+r-1}{2} \quad . \quad . \quad . \quad . \quad (12)$$

Such are the conditions relative to $u$. They agree in all respects with those assigned in the previous investigation, in (5), Art. 13; and, as in that article, the elimination of $u$ leads to the conditions of possible experience,

$$\left.\begin{array}{lll} p \overline{\overline{>}} 0 & q \overline{\overline{>}} 0 & r \overline{\overline{>}} 0 \\ p \overline{\overline{>}} & q+r-1 & \\ q \overline{\overline{>}} & p+r-1 & \\ r \overline{\overline{>}} & p+q-1 & \end{array}\right\} \quad . \quad . \quad . \quad . \quad (13)$$

It may be well to notice, that these conditions involve the necessity of $p$, $q$, and $r$ being fractional, though of course this does not exhaust their significance.

19. It remains to show that when the above conditions are satisfied, the system (7) will admit of but one solution in positive values of $s'$, $t'$, $v'$, and that (10) will furnish but one value of $u$ satisfying the conditions (11) and (12).

Let us write 10 in the form

$$u\,(2\,u-p-q-r+1)^2-(p-u)\,(q-u)\,(r-u)=0 \quad . \quad (14)$$

or, for simplicity, in the form

$$U=0.$$

The lower limit of $u$ is, by (11) and (12), either 0 or $\frac{p+q+r-1}{2}$, according as the latter quantity is positive or negative; the upper limit of $u$ is the least of the quantities $p$, $q$, $r$; suppose it $p$. First, let $\frac{p+q+r-1}{2}$ be positive, then, making $u$ equal to this quantity, the value of U, as given in the first member of (14) becomes negative. Again, let $u=p$, then U becomes positive. Thus, as $u$ varies from $\frac{p+q+r-1}{2}$ up to $p$, U changes from negative to positive. Now

$$\frac{dU}{du}=(2u-p-q-r+1)^2+4u(2u-p-q-r+1)+(p-u)(q-u)+(q-u)(r-u)+(r-u)(p-u) \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

which within the supposed limits is always positive. Hence U varies by continuous increase, and once only in its variation becomes equal to 0.

Secondly, let $\frac{p+q+r-1}{2}$ be negative, then $u$, varying from 0 up to $p$, U as before will vary by continuous increase from a negative to a positive value. See the first member of (14). Whence U, changing by continuous increase from a negative to a positive value, will still only once become equal to 0.

Wherefore, in either case, one root only of (10) will lie within the limits assigned to $u$ in (11) and (12). And this one value substituted in (9) will give one set of values for $s'$, $t'$, $v'$.

20. The solutions which we have now obtained of the same problem on different hypotheses with respect to the selection of the simple events, set in clear light the principles upon which the due selection of such hypotheses depends. The hypothesis which seems most readily to present itself utterly fails, while the other, based quite as much upon an apparently remote speculation on language, as upon the study of the laws of expectation as usually conceived, finds a support and confirmation within the realm of pure mathematics which is of the most remarkable kind.

21. A practical simplification of the general method is suggested by that step of the preceding solution, which reduces (5) to the form (7). If we remove the traces (′) from the letters in the latter system (and they do not at all affect the solution), we obtain what (5) would become if we replaced each of the symbols $\bar{s}$, $\bar{t}$, $\bar{v}$, by unity. Practically, therefore, we may modify the general rule in the following manner:—Having obtained V, replace each of the symbols $\bar{s}$, $\bar{t}$, &c., by unity, and proceed with the reduced value of V just as before, *i.e.*, let $V_s$ represent that portion of V of which $s$ is a factor, &c., then form the system of equations

$$\frac{V_s}{p}=\frac{V_t}{q} \cdot \cdot \cdot =\frac{A+cC}{u}=V \quad . \quad . \quad . \quad . \quad (1)$$

and hence determine $u$ as a function of $p, q$ . . and $c$. The conditions of possible experience and of limitation will be found by supposing $s, t$, to admit of a single determination in positive values. Or as before, they may be found independently, and then applied to limit the solution.

22. We now proceed to the consideration of the problems referred to in Art. (1.) We shall first examine the problem which has for its object the determination of the most probable measure of a physical magnitude, two conflicting measures of which have been assigned by observation. The problem is not, as has been said, Art. 2, in its immediate presentation, one whose elements are logical, but it admits, as we shall see, of being so represented as to give it this character.

### Problem 1.

*Two simultaneous observations of a physical magnitude, as the elevation of a star, assign to it the respective values* $p_1$ *and* $p_2$. *The probability, when the first observation has been made, that it is correct, is* $c_1$, *the corresponding probability for the second observation is* $c_2$. *Required the most probable value of the physical magnitude hence resulting.*

#### *First Solution.*

23. The numerical elements which are not, in their immediate presentation, probabilities, are $p_1$ and $p_2$. But these become such if we contemplate the problem under another aspect. Let a quadrant be taken as the unit of magnitude, then $p_1$ and $p_2$ are proper fractions; $p_1$ actually expressing the probability afforded by the first observation, $p_2$ that afforded by the second observation, that a pointer, directed at random to that quadrant of elevation in which the star, regarded as a physical point, is situated, will point below the star. The problem thus regarded contains the following logical elements, which we shall express by appropriate symbols, viz.

The event which consists in the first observation, such as it is, being made $=x$.

The event which consists in the second observation, such as it is, being made $=y$.

The event which consists in the first observation being correct, $=w$.

The event which consists in the second observation being correct, $=v$.

The event which consists in a pointer, directed at random to the quadrant in which the star is situated, pointing below that star, $=z$.

We must now express symbolically the data, including therein whatever logical connections we can establish among the events, $x$, $y$, $w$, $v$, and $z$.

The probability that the first observation, when made, is correct, is $c_1$. This is a conditional probability; or, to adopt a well-known form of expression, it is a probability *à posteriori*. Viewed from a point of time anterior to the observa-

tion, it is the probability that if the observation be made under its actual circumstances of care, personal fitness, instrumental accuracy, &c., it will be absolutely correct. Symbolically, it is the probability that if the event $x$ take place, the event $w$ will take place. The only mode of expressing this is by writing for the probability of $x$ an arbitrary constant $a_1$ we have then

$$\text{Prob. } x=a_1 \quad \text{Prob. } x\,w=a_1\,c_1 \quad . \quad . \quad . \quad (1)$$

The events $x$ and $w$ are not, however, independent. If we can affirm that a given observation is correct, we can affirm that that observation has been made. Symbolically, the occurrence of the event $w$ implies the occurrence of the event $x$. Expressing this proposition in the language of the calculus of Logic we have the equation.

$$w\,\bar{x}=o \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This forms a part of our data. It permits us to change also the form of one of the previous data, and instead of (1) to substitute

$$\text{Prob. } x=a_1 \quad \text{Prob. } w=a_1\,c_1 \quad . \quad . \quad . \quad (3)$$

In like manner, representing the arbitrary probability of the event $y$ by $a_2$, we have

$$\text{Prob. } y=a_2 \quad \text{Prob. } yv=a_2\,c_2 \quad . \quad . \quad . \quad (4)$$

With the connecting condition

$$v\,\bar{y}=0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which would permit us to substitute for (4) the system

$$\text{Prob. } y=a_2 \quad \text{Prob. } v=a_2\,c_2 \quad . \quad . \quad . \quad (6)$$

Again, when it is known that the first observation is a correct one, the probability that an indicator directed at random to the quadrant in which the star is situated will point below the star is $p_1$. This, too, is a conditional probability. Symbolically, it is the probability that if the event $w$ occur, the event $z$ will occur. Hence, as the probability of the occurrence of $w$ as $a_1\,c_1$, we have

$$\text{Prob. } w\,z=a_1\,c_1\,p_1 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In like manner we find

$$\text{Prob. } vz=a_2\,c_2\,p_2 \quad . \quad . \quad . \quad . \quad (8)$$

Lastly, it is supposed that the values $p$ and $q$ are different. This involves the condition that the observations cannot both be correct. Whence we have the logical equation.

$$w\,v=0 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This completes the analysis of the logical elements involved in the data of the problem. We now proceed to analyse those involved in its quæsitum or object proposed.

That object is to determine the probability of the event $z$, when the occurrence

of the events $x$ and $y$ is known. Symbolically expressed, it is the value of the fraction.

$$\frac{\text{Prob. } x\,y\,z}{\text{Prob. } x\,y}$$

or, as it may, by resolving the denominator, be written,

$$\frac{\text{Prob. } x\,y\,z}{\text{Prob. } x\,y\,z + \text{Prob. } x\,y\,\bar{z}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To effect this object, we shall determine the value of Prob. $x\,y\,z$ and Prob. $x\,y\,\bar{z}$ separately.

Collecting the elements furnished by the preceding analysis, the first of the partial problems herein involved may be thus stated:—

$$\text{Given} \quad \left.\begin{array}{ll} \text{Prob. } x=a_1 & \text{Prob. } y=a_2 \\ \text{Prob. } w=a_1c_1 & \text{Prob. } v=a_2c_2 \\ \text{Prob. } wz=a_1c_1p_1 & \text{Prob. } vz=a_2c_2p_2 \end{array}\right\} \quad . \quad . \quad . \quad (11)$$

with the conditions, $w\bar{x}=0, \quad v\bar{y}=0, \quad wv=0, \quad . \quad . \quad . \quad . \quad (12)$

Required $u$, the value of Prob. $xyz$.

In selecting the above, I have chosen to employ (3) in place of (1), and (6) in place of (4). It makes no difference in the final result.

In accordance with the rule, let us write

$$wz=s, \quad vz=t, \quad xyz=\phi \quad . \quad . \quad . \quad (13)$$

we must then from (12) and (13) determine $\phi$ as a developed logical function of $x$, $y$, $w$, $v$, $s$, and $t$.

This problem admits of perfectly definite solution on the principles of the calculus of Logic. I shall here merely give the result, and point out a method by which it may be independently verified. We find

$$\phi=xyws\bar{v}\bar{t}+xyvt\bar{w}\bar{s}+0\,(xws\bar{y}\bar{v}\bar{t}+yvt\bar{x}\bar{w}\bar{s}+x\bar{y}\bar{w}\bar{v}\bar{s}\bar{t}$$

$$+y\bar{x}\bar{w}\bar{v}\bar{s}\bar{t}+\bar{x}\bar{y}\bar{w}\bar{v}\bar{s}\bar{t}+xyw\bar{v}\bar{s}\bar{t}+xw\bar{y}\bar{v}\bar{s}\bar{t}+xyv\bar{w}\bar{s}\bar{t}+yv\bar{x}\bar{w}\bar{s}\bar{t})$$

$$+\frac{0}{0}\,xy\bar{w}\bar{v}\bar{s}\bar{t}+\text{ terms whose coefficient is } \frac{1}{0} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We may verify this expansion by substituting for $s$ and $t$ their values $wz$ and $vz$, paying attention to the conditions (12), and then comparing the result with the value of $\phi$, viz., $xyz$.

Thus the term $xyws\bar{v}\bar{t}$ becomes, on substitution

$$xyw\bar{v}\times wz\times(1-vz)=xyzw\bar{v}$$

by the calculus of Logic. Now this represents a class entirely included in the class $xyz$, whence the coefficient of the term is unity.

The term $xws\bar{y}\bar{v}\bar{t}$ reduces to $xwz\bar{y}\bar{v}$, and represents a class no part of which is included in $xyz$, whence the coefficient is 0.

The term $xy\bar{w}\bar{v}\bar{s}\bar{t}$, reduces to $xy\bar{w}\bar{v}$, and represents a class some part of which is included, and some part not included under the class $xyz$, whence the coefficient $\frac{0}{0}$; for an event included under the former class may or may not be included under the latter.

Lastly, any term whose coefficient in the expansion is $\frac{1}{0}$ would, on effecting the above-named substitutions, become 0, indicating the absolute non-existence of the class which it represents.

Resuming the value of $\phi$, and adopting the simplification of Art. 21, we find for V the value

$$\begin{aligned} V &= xyws + xyvt + xws + yvt + x + y + 1 + xyw + xw + xyv + yv + xy \\ &= (x+1)(y+1) + yv(x+1)(t+1) + xw(y+1)(s+1) \quad . \quad . \quad (15) \end{aligned}$$

And hence we have the following system of algebraic equations:

$$\begin{aligned} &\frac{x(y+1)+xyv(t+1)+xw(y+1)(s+1)}{a_1} = \frac{y(x+1)+yv(x+1)(t+1)+xwy(s+1)}{a_2} \\ &= \frac{xw(y+1)(s+1)}{a_1 c_1} = \frac{yv(x+1)(t+1)}{a_2 c_2} = \frac{xw(y+1)s}{a_1 c_1 p_1} = \frac{yv(x+1)t}{a_2 c_2 p_2} \\ &= \frac{xyws+xyvt+cxy}{u} = (x+1)(y+1)+yv(x+1)(t+1)+xw(y+1)(s+1) \quad (16) \end{aligned}$$

From these equations, if we assume

$$(x+1)(y+1)+yv(x+1)(t+1)+xw(y+1)(s+1)=\lambda,$$

$\lambda$ being a subsidiary quantity introduced for convenience, we readily deduce

$$u=\frac{xyws+xyvt+cxy}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$a_1 c_1 p_1=\frac{xws(y+1)}{\lambda}$$

$$a_2(1-c_2)=\frac{y(x+1)+xwy(s+1)}{\lambda}$$

$$1-a_2 c_2=\frac{(x+1)(y+1)+xw(y+1)(s+1)}{\lambda}$$

Hence

$$\frac{a_1 c_1 p_1 \times a_2(1-c_2)}{1-a_2 c_2} = \frac{xyws}{\lambda} \quad . \quad . \quad . \quad . \quad (18)$$

In like manner

$$\frac{a_2 c_2 p_2 \times a_1(1-c_1)}{1-a_1 c_1} = \frac{xyvt}{\lambda} \quad . \quad . \quad . \quad . \quad (19)$$

Again we have

$$a_1(1-c_1)=\frac{x(y+1)+xyv(t+1)}{\lambda}$$

$$a_2(1-c_2)=\frac{y(x+1)+xwy(s+1)}{\lambda}$$

$$1-a_1c_1-a_2c_2=\frac{(x+1)(y+1)}{\lambda}$$

$$1-a_1c_1=\frac{(x+1)(y+1)+yv(x+1)(t+1)}{\lambda}$$

$$1-a_2c_2=\frac{(x+1)(y+1)+xw(y+1)(s+1)}{\lambda}$$

Whence we find

$$\frac{a_1(1-c_1)a_2(1-c_2)(1-a_1c_1-a_2c_2)}{(1-a_1c_1)(1-a_2c_2)}=\frac{xy}{\lambda} \quad . \quad . \quad . \quad (20)$$

By means of (18), (19), and (20), we reduce (17) to the form

$$u=\frac{a_2(1-c_2)}{1-a_2c_2}a_1c_1p_1+\frac{a_1(1-c_1)}{1-a_1c_1}a_2c_2p_2+c\frac{a_1(1-c_1)a_2(1-c_2)(1-a_1c_1-a_2c_2)}{(1-a_1c_1)(1-a_2c_2)}$$

therefore effecting a slight reduction

$$\text{Prob. } xyz=\frac{a_1(1-c_1)a_2(1-c_2)}{(1-a_1c_1)(1-a_2c_2)}\left\{\frac{1-a_1c_1}{1-c_1}c_1p_1+\frac{1-a_2c_2}{1-c_2}c_2p_2+c(1-a_1c_1-a_2c_2)\right\} \quad (21)$$

The arbitrary constant $c$, interpreted according to the rule, is the probability that if the event $xy\bar{w}\,\bar{v}\,\bar{s}\,\bar{t}$ take place, $xyz$ will take place. Putting for $\bar{s}$ and $\bar{t}$ their values, and reducing as before, we find that $c$ is the probability that if $xy\bar{w}\,\bar{v}$ take place, $xyz$ will take place. In the end this amounts to the following statement.

$c$ = probability that if both observations are incorrect, a pointer directed at random to the quadrant in which the star is situated will point below the star.

The value of Prob. $xy\bar{z}$ will be obtained from that of Prob. $xyz$ by changing $p_1\,p_2$ and $c$ into $1-p_1$, $1-p_2$, and $1-c$. If we effect this change, and then substitute the expressions above found, in the formula,

$$\frac{\text{Prob. } xyz}{\text{Prob. } xyz+\text{Prob. } xy\bar{z}}$$

We shall find

$$\left.\begin{aligned}\frac{\text{Prob. } xyz}{\text{Prob. } xy}&=\frac{\frac{1-a_1c_1}{1-c_1}c_1p_1+\frac{1-a_2c_2}{1-c_2}c_2p_2+c(1-a_1c_1-a_2c_2)}{\frac{1-a_1c_1}{1-c_1}c_1+\frac{1+a_2c_2}{1-c_2}c_2+1-a_1c_1-a_2c_2}\\&=\frac{\frac{1-a_1c_1}{1-c_1}c_1p_1+\frac{1-a_2c_2}{1-c_2}c_2p_2+c(1-a_1c_1-a_2c_2)}{1+\frac{1-a_1}{1-c_1}c_1+\frac{1-a_2}{1-c_2}c_2}\end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

This expression involves an arbitrary constant $c$ which we have no means of determining. This circumstance indicates that those principles of probability which relate to the combination of *events* do not *alone* suffice to enable us to combine into a definite result the conflicting measures of an astronomical observation.

The arbitrary character of the final solution might have been inferred from

the appearance of the symbol $\frac{0}{0}$ in (14). I have thought it better to complete the investigation, especially as it will serve as a model for the one which follows.

24. Before proceeding to the second solution of the problem, I will endeavour to explain the principle on which it will be founded. It is involved in the following definition.

*Definition.* The mean strength of any probabilities of an event which are founded upon different judgments or observations is to be measured by that supposed probability of the event *à priori* which those judgments or observations following thereupon would not tend to alter.

Thus, suppose we were considering the question of the suitableness of a newly discovered island for the growth of a particular plant, and that the probability of its suitableness, as dependent upon general impressions of the climate were $r$; but that added special observations,—such as analysis of the soil, determination of allied species growing in the locality, &c., had some of them the effect of raising, others that of depressing, the general expectation before entertained. Now we might suppose that expectation to have had such a measure, that the added observations should, when united, leave the mind in the same state as before. I call that measure the *mean value* of the testimonies—the value about which, to adopt (for illustration, not for argument) a mechanical analogy, they balance each other. I conceive that in thus doing, I am only giving a scientific meaning to a term which has been hitherto used in a vague sense. I shall show that the formula of the arithmetical mean is a special determination applicable only to particular problems, of the more general mean of which I here speak, and that other determinations of it exist, applicable to other problems, but possessing, in common, certain definite characteristics.

To apply this principle to the problem under consideration, we must add to the data a new element, viz., the *à priori* value of Prob. $z$, *i.e.*, the value which the mind is supposed to attach to it before the evidence furnished by the observations. We will suppose this value $r$. We must then seek, as before, the *a posteriori* value of Prob. $z$, *i.e.*, its value after the observations, and, equating the two expressions, determine thence the value of $r$.

I shall, in referring to the above principle, speak of it as the "principle of the mean."

*Special solution of Problem I. founded upon the principle of the mean.*

Assigning to $z$ the *à priori* probability $r$, our data are the following:

$$\text{Prob. } x=a_1 \qquad \text{Prob. } y=a_2 \qquad \text{Prob. } z=r$$
$$\text{Prob. } w=a_1 c_1 \qquad \text{Prob. } v=a_2 c_2$$
$$\text{Prob. } wz=a_1 c_1 p_1 \qquad \text{Prob. } vz=a_2 c_2 p_2.$$

with the conditions $\quad w\bar{x}=0 \qquad v\bar{y}=0 \qquad wv=0$

and hence we are to seek, as before, the value of

$$\frac{\text{Prob. } xyz}{\text{Prob. } xyz + \text{Prob. } xy\bar{z}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Assuming then as before,

$$wz = s, \qquad vz = t, \qquad xyz = \phi$$

we find, by the calculus of Logic, the following expression for $\phi$ as a developed logical function of $x$, $y$, $w$, $v$, $s$, $t$, and $z$, viz.:

$$\begin{aligned}\phi = {} & xywsz\bar{v}\bar{t} + xyvtz\bar{w}\bar{s} + xyz\bar{w}\bar{v}\bar{s}t \\ & + 0\{xyv\bar{w}\bar{s}\bar{t}\bar{z} + yvzt\bar{x}\bar{w}\bar{s} + yv\bar{x}\bar{w}\bar{s}\bar{z}\bar{t} + xyws\bar{z}\bar{v}\bar{t} + xwzs\bar{y}\bar{v}\bar{t} \\ & + xw\bar{y}\bar{z}\bar{v}\bar{s}\bar{t} + xy\bar{z}\bar{v}\bar{w}s\bar{t} + xz\bar{y}\bar{v}\bar{w}\bar{s}\bar{t} + yz\bar{x}\bar{s}\bar{v}\bar{w}\bar{t} + x\bar{y}\bar{z}\bar{v}\bar{w}\bar{s}\bar{t} \\ & \quad y\bar{x}\bar{z}\bar{v}\bar{w}\bar{s}\bar{t} + z\bar{x}\bar{y}\bar{v}\bar{w}\bar{s}\bar{t} + \bar{x}\bar{y}\bar{z}\bar{v}\bar{w}\bar{s}\bar{t}) \\ & \qquad + \text{terms with coefficient } \frac{1}{0} \quad . \quad . \quad . \quad . \quad . \qquad (2)\end{aligned}$$

Hence, adopting the simplification of Art. 21, we have

$$\begin{aligned}V = {} & xywsz + xyvtz + xyz + xyv + yvzt + yv + xyw + xwzs \\ & + xw + xy + xz + yz + x + y + z + 1 \\ = {} & xw(y+1)(zs+1) + yv(x+1)(zt+1) + (x+1)(y+1)(z+1) \quad . \qquad (3)\end{aligned}$$

whence we form the algebraic system

$$\begin{aligned} & \frac{xw(y+1)(zs+1) + xyv(zt+1) + x(y+1)(z+1)}{a_1} \\ = & \frac{xwy(zs+1) + yv(x+1)(zt+1) + (x+1)y(z+1)}{a_2} \\ = & \frac{xw(y+1)(zs+1)}{a_1 c_1} = \frac{yv(x+1)(zt+1)}{a_2 c_2} \\ = & \frac{xw(y+1)zs}{a_1 c_1 p_1} = \frac{yv(x+1)zt}{a_2 c_2 p_2} \\ = & \frac{xw(y+1)zs + yv(x+1)zt + (x+1)(y+1)z}{r} \\ = & \frac{xyzws + xyzvt + xyz}{u} \\ = & \frac{xw(y+1)(zs+1) + yv(x+1)(zt+1) + (x+1)(y+1)(z+1)}{1} = \lambda \quad . \qquad (4)\end{aligned}$$

$\lambda$ being a subsidiary quantity introduced for convenience.

From the above we find

$$a_2 - a_2 c_2 = \frac{xyw(zs+1) + y(x+1)(z+1)}{\lambda}$$

$$a_1 c_1 p_1 = \frac{xwzs(y+1)}{\lambda}$$

$$1 - a_2 c_2 = \frac{xw(y+1)(zs+1) + (x+1)(y+1)(z+1)}{\lambda}$$

whence

$$\frac{a_2(1-c_2)}{1-a_2 c_2} a_1 c_1 p_1 = \frac{xyzws}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In like manner we find

$$\frac{a_1(1-c_1)}{1-a_1 c_1} a_2 c_2 p_2 = \frac{xyzvt}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Again, since we have from the above

$$\frac{a_2-a_2 c_2}{1-a_2 c_2} = \frac{y}{y+1} \quad \text{and} \quad \frac{a_1-a_1 c_1}{1-a_1 c_1} = \frac{x}{x+1}$$

we have

$$\frac{a_1(1-c_1)\, a_2(1-c_2)}{(1-a_1 c_1)(1-a_2 c_2)} = \frac{xy}{(x+1)(y+1)}$$

moreover,

$$r-a_1 c_1 p_1-a_2 c_2 p_2 = \frac{(x+1)(y+1)z}{\lambda}$$

Multiplying the two last equations together we find

$$\frac{a_1(1-c_1)\, a_2(1-c_2)}{(1-a_1 c_1)(1-a_2 c_2)} (r-a_1 c_1 p_1-a_2 c_2 p_2) = \frac{xyz}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now,

$$u = \frac{xyzws+xyzvt+xyz}{\lambda}$$

Substituting in this expression the values found for its several terms in (5), (6), and (7), we have

$$u = \frac{a_2(1-c_2)}{1-a_2 c_2} a_1 c_1 p_1 + \frac{a_1(1-c_1)}{1-a_1 c_1} a_2 c_2 p_2 + \frac{a_1(1-c_1)\, a_2(1-c_2)}{(1-a_1 c_1)(1-a_2 c_2)} (r-a_1 c_1 p_1-a_2 c_2 p_2)$$

This is the value of Prob. $xyz$. That of Prob. $xy\bar{z}$ will be found by simply changing in the above expression $p_1$, $p_2$, and $r$, into $1-p_1$, $1-p_2$, and $1-r$ respectively. These expressions admit of some reductions, and give

$$\text{Prob. } xyz = \frac{a_1(1-c_1)\, a_2(1-c_2)}{(1-a_1 c_1)(1-a_2 c_2)} \left\{ \frac{1-a_1}{1-c_1} c_1 p_1 + \frac{1-a_2}{1-c_2} c_2 p_2 + r \right\} \quad . \quad . \quad . \quad (8)$$

$$\text{Prob. } xy\bar{z} = \frac{a_1(1-c_1)\, a_2(1-c_2)}{(1-a_1 c_1)(1-a_2 c_2)} \left\{ \frac{1-a_1}{1-c_1} c_1 (1-p_1) + \frac{1-a_2}{1-c_2} c_2 (1-p_2) + 1 - r \right\} \quad . \quad (9)$$

whence we find for the *à posteriori* value of Prob. $z$,

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy} = \frac{\frac{1-a_1}{1-c_1} c_1 p_1 + \frac{1-a_2}{1-c_2} c_2 p_2 + r}{\frac{1-a_1}{1-c_1} c_1 + \frac{1-a_2}{1-c_2} c_2 + 1}$$

Equating this to $r$ we have

$$\frac{1-a_1}{1-c_1} c_1 p_1 + \frac{1-a_2}{1-c_2} c_2 p_2 + r = \left( \frac{1-a_1}{1-c_1} c_1 + \frac{1-a_2}{1-c_2} c_2 + 1 \right) r$$

Whence

$$r = \frac{\frac{1-a_1}{1-c_1} c_1 p_1 + \frac{1-a_2}{1-c_2} c_2 p_2}{\frac{1-a_1}{1-c_1} c_1 + \frac{1-a_2}{1-c_2} c_2} \quad . \quad . \quad . \quad . \quad (10)$$

26. Such is the final general expression for the probable altitude of the star. The following observations may throw light upon its real nature:—

1*st*, In the analysis by which this expression was obtained, $p_1$ and $p_2$ are the observed altitudes of the star, a quadrant of the celestial arc being taken as unity. Considered, however, as the expression, not of a *probability*, but of the most probable measure of a physical magnitude, the truth of the formula will of course be independent of the unit of magnitude.

2*dly*, The formula is independent of mechanical analogy. We may place it in the well-known form

$$r = W_1 p_1 + W_2 p_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which, as the subject is usually treated $W_1$ and $W_2$ are called the *weights* of the observations. Here, however, these quantities are determined as functions of the initial data—these data being probabilities. We have

$$W_1 = \frac{\frac{1-a_1}{1-c_1}c_1}{\frac{1-a_1}{1-c_1}c_1 + \frac{1-a_2}{1-c_2}c_2} \qquad W_2 = \frac{\frac{1-a_2}{1-c_2}c_2}{\frac{1-a_1}{1-c_1}c_1 + \frac{1-a_2}{1-c_2}c_2} \quad . \quad (2)$$

3*dly*, The initial probabilities, of which $W_1$ and $W_2$ are functions, are neither foreign nor imaginary elements. They may be difficult to determine, but theoretically their determination rests upon considerations which are entirely proper to the subject. When an observation has been made, the question whether it is correct or not is a question of probability. We can never predicate absolute correctness. We can seldom affirm absolutely that an observation is incorrect. Our knowledge of the circumstances of the observation, Art. 22, leads us to regard the probability in question as sometimes greater, sometimes less. To suppose it capable of a numerical value, as we have done, by the introduction of the constants $c_1\, c_2$, is then perfectly legitimate. It has been said that an estimate of the correctness of the observation rests upon the circumstances by which it was accompanied. These circumstances, taken in the aggregate, are themselves a subject of probability. This we express by the introduction of the constants $a_1\, a_2$. The probability after an observation is made that it is correct, and the probability before it is made that the state of things shall be such as to give to the result that particular probability of correctness, are quite different things.

4*thly*, In the same course of observations made by the same individual with consciously uniform regard to personal and instrumental accuracy the values of $a_1$ and $a_2$ would be sensibly equal. The formula (10) would thus reduce to the following, viz.:—

$$r = \frac{\frac{c_1}{1-c_1}p_1 + \frac{c_2}{1-c_2}p_2}{\frac{c_1}{1-c_1} + \frac{c_2}{1-c_1}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here $$W_1 = \frac{\frac{c_1}{1-c_1}}{\frac{c_1}{1-c_1} + \frac{c_2}{1-c_2}} \qquad W_2 = \frac{\frac{c_2}{1-c_2}}{\frac{c_1}{1-c_1} + \frac{c_2}{1-c_2}} \quad . \quad . \quad . \quad (4)$$

If $c_1 = 1$, we have
$$W_1 = 1 \qquad W_2 = 0$$
and
$$r = p_1.$$

This accords with the condition that if either of the observations is believed to be correct, the value which it furnishes for the altitude of the star must be taken as the true one.

*5thly,* If $c_1 = c_2$, *i.e.*, if we have no right to give preference to one observation over the other, we have

$$r = \frac{p_1 + p_2}{2} \quad . \quad . \quad . \quad . \quad (5)$$

the formula of the arithmetical mean.

*6thly,* From the form of $W_1$, $W_2$ in (4), it is evident that the weights, so to speak, of the observations vary in a higher ratio than that of the simple probabilities of correctness of the observations. The practical lesson to be drawn from this is, that we ought to attach a greater weight to good observations, and a smaller to bad ones, than, according to usual modes of consideration, we should be disposed to do.

The above are the most important observations suggested by the formula to which the last investigation has led. One or two remarks remain to be offered upon the analysis by which it was obtained.

Although the two forms of investigation which we have exhibited differ, there is nothing inconsistent in the results to which they lead. If we compare corresponding formulæ in the two, *e.g.*, the values of Prob. $xyz$, or those of Prob. $xy\bar{z}$, we shall find that the one investigation assigns a definite but consistent value to what the other left arbitrary. Either comparison gives

$$c = \frac{r - a_1 c_1 p_1 - a_2 c_2 p_2}{1 - a_1 c_1 - a_2 c_2}$$

We may prove, either by the "conditions of possible experience," or independently, that this value is necessarily a proper positive fraction, and this accords with the interpretation of $c$ as a probability. Art. 23.

27. But a much more important consideration is the following. It is a plain consequence of the logical theory of probabilities, that the state of expectation which accompanies entire ignorance of an event is properly represented, not by the fraction $\frac{1}{2}$, but by the indefinite form $\frac{0}{0}$. And this agrees with a conclusion at which Bishop TERROT, on independent, but as I think just grounds, has arrived.* Now this shows, why, if the consideration of the *à priori* probability of $z$ is, from

* Transactions of the Royal Society of Edinburgh, vol. xxi. p. 375.

the insufficiency of the remaining data, necessary in order to give to the *à posteriori* probability of $z$ a definite value, the solution obtained when that *à priori* value is neglected should involve the symbol $\frac{0}{0}$. The presence of this symbol in a solution always indicates insufficiency in the data. And herein, as it seems to me, consists the reason why the mind, impatient of incertitude even while dealing with the very science of uncertain knowledge, is led to seek escape from its doubts, by calling in the aid, in some form or another, of that adventitious principle which I have denominated the principle of the mean. I say in some form or another: for I can conceive of another form of the same principle connected more directly with the idea of a *limit* than with that of a mean. Thus as testimonies which are insufficient of themselves to produce a definite expectation may definitely modify a definite expectation previously formed, we have suggested to us the idea of that limiting state to which perpetual and independent repetition of the same series of testimonies would cause the mind, whatever its starting point of expectation might be, to tend. And as this limiting state would be one which a further repetition would not alter, we should thus arrive in effect at the same solution as is indicated by the principle of the mean, in its direct expression.

28. I have extended the preceding analysis to the case in which three observations are to be combined, a case which, in connection with the previous one, is sufficient to determine the general law. The result is what the preceding analysis suggests, and may be expressed in the following theorem:—

If $n$ conflicting observations assign to the altitude of a star the respective values $p_1 p_2 \ldots p_n$; if, moreover, $a_1 a_2 \ldots a_n$ are the antecedent probabilities that the observations will be such as they prove to be with respect to those circumstances which determine their relative accuracy, and $c_1 c_2 \ldots c_n$ their respective probabilities of correctness to a mind acquainted with these circumstances, *i.e.*, to the mind of the observer after the observations have been made, then the most probable altitude of the star will be

$$\frac{\frac{1-a_1}{1-c_1}c_1 p_1 + \frac{1-a_2}{1-c_2}c_2 p_2 \ldots + \frac{1-a_n}{1-c_n}c_n p_n}{\frac{1-a_1}{1-c_1}c_1 + \frac{1-a_2}{1-c_2}c_2 \ldots + \frac{1-a_n}{1-c_n}c_n}$$

This expression admits of the same deductions as the one before obtained for the case in which the observations are two in number, and in particular it leads, when the circumstances of the observations are judged to be in all respects the same, to the principle of the arithmetical mean expressed by the formula

$$\frac{p_1 + p_2 \ldots + p_n}{n}$$

29. I have remarked that the principle of the arithmetical mean has some

claim to be regarded as axiomatic. In the preceding sections it presents itself as a special result of a very complex analysis founded upon the logical theory of probabilities. Now I wish to observe, that there is nothing in these circumstances which we have a right to regard as denoting inconsistency. Of the theory of probabilities it is eminently true that modes of investigation, which to our present conceptions must appear fundamentally different, habitually lead us to the same result. A profounder acquaintance with the laws of the human mind, and a deeper insight into the relations of things, might perhaps show us that principles which appear to us to have nothing in common may yet have a necessary connection with each other,—may possibly spring up from a common origin. I will endeavour to make my meaning clear by two illustrations, which will present this question in somewhat different lights.

30. An idea which seems naturally to suggest itself in connection with the theory of probabilities is that of mechanical analogy. Evidence of this we see in the language, already referred to, which attributes *weight* to observation. The complete and scientific development of the idea will be found in a memoir by Professor DONKIN,* who, establishing a kind of metaphysical statics on proofs of the same nature as those which are employed in deducing *à priori* the laws of ordinary statics, has arrived, by legitimate deduction, at the remotest consequences of GAUSS's theory of the combination of observations. The mind, in the developed analogy, is compared to a lever acted upon by different weights, or to a mechanical system subject to given forces, and seeking, under this action, a position of equilibrium. Now it is at least a very remarkable circumstance, that an analogy of this kind should not only admit of exact scientific expression, but should, through a long train of analytical consequences, present the same laws and results, and suggest the same methods, as the principle of the arithmetical mean already referred to. All the abstract terms by which mental states and emotions are expressed, derive, if philology be of any value, their origin from outward and material things. And hence, though it might be impossible to ascend historically to the first employment of those expressions which describe the mind under the action of *forces*, and speak of the *balancing* of opinions, we cannot doubt that a perceived analogy was their source. But it could hardly have been anticipated that this analogy should remain complete and unimpaired through so lengthened a range of scientific deductions.

To what I have said above I will only add, that it is as instruments of expression and communication, rather than of thought, that material symbols, and the analogies which they furnish, seem to me to possess importance. Even the analogy which we have been considering cannot of itself occupy the place of a first principle, but seems to be a particular manifestation of that deeper

* Sur la Theorie de la Combinaison des Observations. LIOUVILLE's Journal de Mathematiques, tom. xv., 1850.

truth of which LEIBNITZ had a glimpse when he spoke of the principle of fitness and congruity—"*principe de la convenance*,"*—the ground of rational mechanics. Of course, I do not contemplate this or any subjective principle whatever, as affording us the slightest ground for affirming that the constitution of nature must, *à priori*, possess such and such a character. But it does seem to be a fact that the material system has been constituted in a certain degree of accordance with our rational faculties. The study of this accordance, *à posteriori*, is a perfectly legitimate object; and I think it the more interesting, when it brings before our view the scientific form of any of those analogies which commended themselves to the minds of the fathers of our race, which are embodied in our common speech, and without which we could apparently never hold converse with our fellows, except upon material objects.

31. The second illustration which I have to offer is the following. Many of the most important applications of the theory of probabilities, the method of least squares, for example, rest upon what has been termed the law of facility of error. This consists in the position, that in seeking to determine by observation a physical magnitude, as the elevation of a star, the probability that any measure will deviate by a quantity $x$ from the true value, will vary directly as the function $\epsilon^{-k^2x^2}$, where $k$ is a constant quantity. The probability that our measure will fall between the limits $x$ and $x+dx$ being expressed by the function

$$\frac{k}{\sqrt{\pi}}\epsilon^{-k^2x^2}dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

GAUSS has shown that this is the only "law of facility" consistent with the assumption that, in a series of observations of the same magnitude, the arithmetical mean of the several measures obtained is the most probable value. It may even be shown, that whatever the actual "law of facility," under given circumstances, may be, and it is plain that it must vary with circumstances, such as the constitution of the instrument and the character of the observer, &c., the probability that the arithmetical mean of a very large number of values determined by observation will deviate from some fixed value by a quantity $x$, will vary directly as $\epsilon^{-k^2x^2}$, $k$ being a constant dependent upon the nature of the observations.† Such, at least, is the limiting form of the function to which the law of deviation approaches as the number of obervations is increased. Now it is remarkable that considerations of a totally different kind, and founded mainly upon our conceptions of space, lead to a similar result. The probability of linear deviation (measured in a given direction) of a ball from a mark at which it is aimed, seems to obey the same law; the principle upon which that law is deter-

* ERDMANN's Edit., p. 716.

† For some very interesting illustrations of this doctrine, see the letters of M. BRAVAIS, published in the notes to QUETELET's Letters on the Theory of Probabilities.

mined being, not that of the arithmetical mean, but rather a principle of geometrical consistency, intimately connected with our ideas of the composition of motion.

The principle was first stated in a popular and somewhat inexact form, by Sir JOHN HERSCHEL, I believe, in the Edinburgh Review.* It was afterwards made the subject of an adverse criticism in the Philosophical Magazine, by Mr LESLIE ELLIS.† There is no living mathematician for whose intellectual character I entertain a more sincere respect than I do for that of Mr ELLIS; and even while stating the grounds upon which I differ from him, with respect to the value of Sir JOHN HERSCHEL's principle, I avail myself of his labours, in giving to that principle a more scientific form and expression, and in developing its consequences. The language adopted in the following statement, will be, as far as possible, that of the author of the principle,—the analysis will be that of Mr ELLIS.

Suppose a ball dropped from a given height, with the intention that it shall fall on a given mark. Now, taking the mark as the origin of two rectangular axes, let it be assumed, that the actual deviation observed is a compound event, of which the two components are the corresponding deviations measured along the rectangular axes. Grant, also, that the latter deviations are independent events. Further, let us represent by $f(x^2)$, $f(y^2)$, the probabilities of the respective component deviations measured along the axes $x$ and $y$,—we give to them this form, because, positive and negative deviations being equally probable, the function expressing probability must be an *even* one, *i.e.*, must not change sign with the error. Hence the probability of the actual deviations observed will be $f(x^2)$ $f(y^2)$. Let it be observed that this is not the probability of a deviation to the extent $\sqrt{x^2+y^2}$ from the mark, but of a deviation to that extent *in a particular line of direction.* Now, let the principle be assumed, that this expression is independent of the position of the axes, *i.e.*, that we may regard component deviations along any two rectangular axes as independent events, by the composition of which the actual deviation is produced. We have then $x'$ and $y'$ representing two new component deviations,

$$f(x^2)\,f(y^2)=f(x'^2)\,f(y'^2) \quad . \quad . \quad . \quad . \qquad (2)$$

If $y'=\sqrt{x^2+y^2}$ then $x'=0$ and we have

$$f(x^2)\,f(y^2)=f(0)\,f(x^2+y^2) \quad . \quad . \quad . \quad . \qquad (3)$$

An equation of which the complete solution is,

$$f(x^2)=\mathrm{A}\,\epsilon^{hx^2}$$

A and $h$ being constants. The condition that the probability of the error must

* Vol. xcii. p. 17, Art. QUETELET on Probabilities.

† Vol. xxxvii. p. 321, "Letter addressed to J. D. FORBES, Esq., Professor of Natural Philosophy in the University of Edinburgh, on an alleged proof of the Method of Least Squares."

diminish as the amount of the error increases, requires that $h$ should be negative. We may therefore write,$-k^2$, for $h$. Whence

$$f(x^2)=A\,\epsilon^{-k^2x^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To apply this result to the case in which the ball is supposed at some point on the plane which, projected on the axis of $x$, will fall between $x$ and $x+\delta x$, we must give to A the form $C\,\delta x$.

Thus we get the expression,

$$C\,\epsilon^{-k^2x^2}\,\delta x$$

Lastly, the *certainty* that the ball must fall at some point for which the value of $x$ lies between $-\infty$ and $\infty$ gives us the equation

$$\int_{-\infty}^{\infty} C\,\epsilon^{-k^2x^2}\,\delta x=1$$

whence $C\frac{\sqrt{\pi}}{k}=1$ and $C=\frac{k}{\sqrt{\pi}}$. Thus, the probability of a deviation from the axis $y$ to a distance lying between $x$ and $x+\delta x$ will be given by the formula

$$\frac{k}{\sqrt{\pi}}\,\epsilon^{-k^2x^2}\,\delta x \quad . \quad . \quad . \quad . \quad (5)$$

an expression which agrees with (1).

In like manner, the probability that the ball will deviate to a distance greater then $y$ and less then $y+\delta y$ from the axis $x$ will be

$$\frac{k}{\sqrt{\pi}}\,\epsilon^{-k^2y^2}$$

whence the probability that it will actually fall upon the elementary area $\delta x\,\delta y$ will be

$$\frac{k^2}{\pi}\,\epsilon^{-k^2(x^2+y^2)}\,\delta x\,\delta y$$

Now, this result admits of a remarkable confirmation. For it is manifest that the probability that the ball will fall *somewhere* between the distances $x$ and $x+\delta x$ from the axis $y$, ought to be equal to the above expression integrated with respect to $y$ between the limits $-\infty$ and $\infty$. But that probability has been already determined to be $\frac{k}{\sqrt{\pi}}\,\epsilon^{-k^2x^2}\,\delta x$; we ought therefore to have

$$\int_{-\infty}^{\infty}\left(\frac{k^2}{\pi}\,\epsilon^{-k^2(x^2+y^2)}\,\delta x\right)\delta y=\frac{k}{\sqrt{\pi}}\,\epsilon^{-k^2x^2}\,\delta x \quad . \quad . \quad (6)$$

an equation which is actually true.

Mr Ellis considers this as showing, that the principle from which the demonstration sets out, viz., that the actual deviation of the ball from the mark may be regarded as a compound event, of which the two independent components are the deviations from the axes, involves either a mistake or a *petitio principii*. But consistency of results can never be a proof of mistake in the principles from which they are deduced; and *alone*, it offers no adequate ground for the suspicion of a

*petitio principii.* It is to be observed, that it is only the probability of deviation from a fixed axis which follows, according to the above investigation, the law expressed by GAUSS's function. The probability of deviation in *any* direction to a distance between $r$ and $r+\delta r$ from the mark, is expressed by a different function. This would be fatal to any hypothesis which should represent GAUSS's function as determining, *à priori*, the actual law of deviation. There are indeed few cases in which it can be determined what the law is, and writers on probability have been far too anxious to interpret nature in accordance with their formulæ. No one has shown this more clearly than Mr ELLIS. The precise value of Sir JOHN HERSCHEL's principle, as corrected by him, I conceive to be this,—that it establishes an identity between the law of facility of error expressed by GAUSS's function and the law which in a special problem, involving the consideration of space and motion, seems to accord with our most elementary conceptions of these things; and this identity I apprehend to be, not an accidental thing, but a very distinct expression of that harmonious relation which binds together the different spheres of thought and existence.

33. We proceed next to the consideration of the second general problem,—that in which it is proposed to determine the combined force of two testimonies or judgments in support of a fact, the strength of each separate testimony being given.

The problem has a material as well as a formal aspect. Thus oral testimonies differ from the judgments which are furnished by the immediate personal observation of facts. And although no definite general laws have, so far as I am aware, been assigned concerning the mode in which the material character of the evidence affects expectation, it is not to be doubted that an influence does proceed from this source. As respects testimony alone, there are cases in which we feel that it is cumulative,—there are cases in which we feel that it is not so; and this difference we also feel depends upon the nature of the testimony itself. But in the majority of cases, we should probably feel that the elements upon which this difference of character depends are blended together, some decided preponderance being due to the one or to the other. Testimony will be chiefly or entirely cumulative which is given quite independently by different persons, and is at the same time based upon different grounds. In proportion as these conditions fail of being satisfied, the testimony partakes less and less of the cumulative character. Still this possession of cumulative character may be regarded as the standard by which the distinctive qualities of testimonies, as affecting belief or expectation, may be estimated. In judgments founded upon the personal observation of facts, though this character may be observed, the standard seems to be different. When different modes of considering a subject—different courses of experiment or inquiry—lead to different probabilities of a fact, some making it more probable, some less, we generally feel that a kind of mean ought to be taken among them. Perhaps the most succinct general statement would be,

that it belongs to testimony, in its normal character, to be cumulative,—to judgment, to require the application, in some form or other, of the principle of means or averages; but that all departures from these normal states involve the blending of the two elements together, in proportions determined by the degree of the deflection.

Now, although it does not belong to the theory of probabilities, in its formal and scientific character, to pronounce upon the material character of a problem, and to say whether its data are in their own nature cumulative or not, yet the results to which the theory leads are, in a very remarkable degree, accordant with the distinctions which have just been pointed out. I shall show that the solution of the problem of the combination of testimonies, when the data are presented in a purely formal character, and without any adventitious principle, involves arbitrary constants, and is therefore indefinite,—being capable, however, under certain circumstances, of assuming a definite form. I shall show that such a form is assumed when the circumstances are such as to give to the testimonies the highest degree of cumulative character. I shall then solve the problem a second time, introducing that adventitious principle which I have already exemplified in the problem of the reduction of astronomical observations, and which appears to me to contain the true theory of means or averages. The form of the solution thus obtained, which is also perfectly definite, will apply to the case, in which it is our object, not to combine testimonies, in the ordinary sense of the term, but to determine the mean of expectations founded upon the issues of conflicting judgments. To one point of importance I must again, before entering upon the analytical investigation, ask the attention of the reader. It is, that in the present subject, the question of the right application of a formula is quite distinct from that of the validity of the processes by which that formula is derived from its data. The latter is a question of formal science, the former involves considerations which belong rather to the philosophy of the human mind.

I will first express the problem which we have to consider in a general form, equally applicable to the combination of testimonies or of judgments. I shall consider the fact of a testimony having been borne, or an observation made, as a circumstance or event affecting our expectation of the event to which it has reference.

## Problem II.

34. *Required the probability of an event* z, *when two circumstances* x *and* y *are known to be present,—the probability of the event* z, *when we only know of the existence of the circumstance* x *being* p,—*and its probability when we only know of the existence of* y *being* q.

Here we are concerned with three events, $x$, $y$, and $z$. For convenience and uniformity I shall, in the solution of the problem, speak of $x$ and $y$ as *events*, as

well as of $z$. A circumstance is an event—a state of things which comes to pass, or has come forth—*evenit*.

The data leave wholly arbitrary the probabilities of the event $x$ and $y$. Thus $p$ and $q$ are *conditional* probabilities; $p$ is the probability that if the event $x$ occur, the event $z$ will occur; $q$ is the probability that if the event $y$ occur, the event $z$ will occur. Hence

$$p=\frac{\text{Prob. } xy}{\text{Prob. } x}, \quad q=\frac{\text{Prob. } yz}{\text{Prob. } y} \quad . \quad . \quad . \quad . \quad (1)$$

Our object is to determine the probability that if the events $x$ and $y$ both occur, the event $z$ will occur. We have therefore to seek the value of the fraction

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy}$$

or, as for our present purpose it is more convenient to say, of

$$\frac{\text{Prob. } xyz}{\text{Prob. } xyz+\text{Prob } xy\bar{z}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In seeking the value of Prob. $xyz$, which we shall represent by $u$, the formal statement of our *data* and *quæsitum* will therefore be

$$\text{Given} \begin{cases} \text{Prob. } x=c, & \text{Prob. } y=c' \\ \text{Prob. } xz=cp, & \text{Prob. } yz=c'q \end{cases} \quad . \quad . \quad . \quad . \quad (3)$$

Required Prob. $xyz$.

$c$ and $c'$ being arbitrary constants expressing the unknown probabilities of the events $x$ and $y$.

A misconception may here arise respecting the meaning of Prob. $x$, Prob. $y$, which it is worth while to anticipate. In the case of testimony, Prob. $x$ would not mean the probability that a testimony would be borne, but the probability that the particular kind of testimony actually recorded considered with reference to its object, credibility, &c., would be borne. Testimonies differ, not merely as to their degree of credibility, but as to their *unexpectedness*—as to the *surprise* which they occasion. And it is, I think, matter of personal experience that this unexpectedness is in itself an element affecting the strength of that expectation which combined testimonies produce. So, too, if $x$ and $y$ are facts of observation, *e.g.*, observed symptoms of a disease $z$, the probability of that disease, when both symptoms present themselves, is not determined by the strength of the separate presumptions merely, but is consciously increased by our knowledge of the rarity of the symptoms themselves. And thus the elements Prob. $x$ Prob. $y$, which have been introduced by a formal necessity of the statement of the problem are seen to belong to the very matter of its solution.

Making

$$xz=s, \qquad yz=t, \qquad xyz=\phi,$$

we find, by the calculus of logic,

$$\phi = xyst + 0\,(xs\bar{y}\bar{t} + yt\bar{x}\bar{s} + xy\bar{s}\bar{t} + x\bar{y}\bar{s}\bar{t} + y\bar{x}\bar{s}\bar{t} + \bar{x}\bar{y}\bar{s}\bar{t})$$

$$+ \text{ terms whose coeficients are } \frac{1}{0} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

a result which may be verified by the method applied to (14) in Art. 23.

Hence we find, adopting the simplification of Art. 21,

$$V = xyst + xs + yt + xy + x + y + 1,$$

and since we have

$$\left.\begin{array}{c} \text{Prob. } x = c, \quad \text{Prob. } y = c', \quad \text{Prob. } s = cp, \quad \text{Prob. } t = c'q \\ \text{Prob. } xyz = u, \end{array}\right\} \quad . \quad . \quad . \quad (5)$$

we find, as an algebraic system of equations,

$$\left.\begin{array}{l} \dfrac{xyst + xs + xy + x}{c} = \dfrac{xyst + yt + xy + y}{c'} \\ = \dfrac{xyst + xs}{cp} = \dfrac{xyst + yt}{c'q} \\ = \dfrac{xyst}{u} = xyst + xs + yt + xy + x + y + 1 \end{array}\right\} \quad . \quad . \quad . \quad (6)$$

This system is easily reduced to the form

$$\left.\begin{array}{l} \dfrac{xs}{cp - u} = \dfrac{yt}{c'q - u} = \dfrac{xy + x + y + 1}{1 + u - cp - c'q} \\ = \dfrac{x + 1}{1 + u - cp - c'} = \dfrac{y + 1}{1 + u - c - c'q} = \dfrac{xsyt}{u} \end{array}\right\} \quad . \quad . \quad . \quad (7)$$

And if we equate the respective products of the first three and of the last three members of the above, we find

$$(cp - u)\,(c'q - u)\,(1 + u - cp - c'q) = (1 + u - c' - cp)\,(1 + u - c - c'q)u \quad . \quad . \quad (8)$$

a quadratic equation by which the value of $u$ must be determined.

If, in like manner, we assume

$$\text{Prob. } xy\bar{z} = t$$

we shall find

$$(c\overline{1-p} - t)\,(c'\overline{1-q} - t)\,(1 + t - c\overline{1-p} - c'\overline{1-q}) = (1 + t - c' - c\overline{1-p})\,(1 + t - c - c'\overline{1-q})t \quad (9)$$

From these equations the values of $u$ and $t$ being determined, we have finally

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy} = \frac{u}{u + t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Before we can apply this solution, we must determine the conditions of possible experience, and the conditions limiting the values of $u$ and $t$. For this purpose writing

$$\text{Prob. } xyz = u, \quad \text{Prob. } xy\bar{z} = t, \quad \text{Prob. } xz\bar{y} = \mu, \quad \text{Prob. } x\bar{y}\bar{z} = \nu,$$
$$\text{Prob. } yz\bar{x} = \rho, \quad \text{Prob. } y\bar{x}\bar{z} = \sigma,$$

we have, from the data, the equations,

$$\begin{aligned} u + t + \mu + \nu &= c \\ u + t + \varrho + \sigma &= c' \\ u + \mu \quad &= \quad cp \\ u + \varrho \quad &= \quad c'q, \end{aligned}$$

to which must be added the inequations

$$u \geqq 0 \qquad t \geqq 0 \qquad \mu \geqq 0 \qquad \nu \geqq 0 \qquad \varrho \geqq 0 \qquad \sigma \geqq 0$$
$$u+t+\mu+\nu+\varrho+\sigma \leqq 1$$

Proceeding as in Art. 14, we find ultimately as the conditions of possible experience

$$cp \leqq 1-c'(1-q) \qquad c'q \leqq 1-c(1-p) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

together with the usual condition that $c$, $c'$, $p$ and $q$ must be positive proper fractions, or at any rate must not transcend the values 0 and 1. We find, too, as the conditions limiting $u$ and $t$,

$$\left.\begin{array}{lll} u \leqq cp & u \leqq c'q & \\ u \geqq c+c'q-1 & u \geqq c'+cp-1 & u \geqq 0 \\ t \leqq c\,\overline{1-p} & t \leqq c'\,\overline{1-q} & \\ t \geqq c+c'\,\overline{1-q}-1 & t \geqq c'+\overline{c-p}-1 & t \geqq 0 \end{array}\right\} \quad . \quad . \quad . \quad , \quad (12)$$

The solution of the problem assumes, therefore, the following form and character:—

1*st*, It involves two constants $c$ and $c'$, which are arbitrary, except in that they are subject to the conditions (11).

2*ndly*, The values of $u$ and $t$, determined from (8) and (9), in subjection to the conditions (12), are to be substituted in the formula (10).

3*dly*, In the absence of any means of determining $c$ and $c'$, the value obtained will be indeterminate, except for particular values of $p$ and $q$. Some general conclusions may nevertheless be deduced from its expression indicating the manner in which expectation is influenced by circumstances insufficient of themselves to give to it a definite amount of strength. This will appear from the following analysis.

*Analysis of the Solution.*

35. The solution is contained in the numbered results, from (8) to (12) inclusive, of the preceding Article. Of these, (11) expresses the conditions of possible experience, (12) the conditions limiting $u$ and $t$. From (8) and (9), these quantities are to be determined in accordance with (12), and the resulting values substituted in (10).

By a proper reduction of (8) and (9), the solution may also be put in the following form:—

$$a'u^2 + (cc'm - aa')u - acc'pq = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$at^2 - (cc'm + aa')t - a'cc'(1-p)(1-q) = 0 \quad . \quad . \quad . \quad (2)$$

where $\quad a = cp + c'q - 1 \qquad a' = c(1-p) + c'(1-q) - 1 \qquad m = p + q - 1.$

The values of $u$ and $t$ hence found, in accordance with the limitations expressed by (12), are to be substituted in the equation

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy} = \frac{u}{u+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The following special deductions may now be noted.

1*st*, If either of the quantities $p$ and $q$ is equal to 1, the probability sought is equal to 1, whatever the values of $c$ and $c'$ may be.

Thus let $p=1$. Then (2) gives $t=0$; the only value which satisfies the conditions (12), in connexion with (11). The equation (1) is not satisfied by $u=0$, whence

$$\frac{u}{u+t} = \frac{u}{u} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This result is obviously correct. If, for example, of two symptoms which are present, and which furnish ground of inference respecting a particular disease, one be of such a nature as to make the existence of the disease a matter of certainty, the fact of that existence is established, however adverse to such a conclusion the presumption furnished by the other symptom, supposing it our only ground of inference, would be.

So, too, the verdict of an authority deemed infallible is consistently held to annul and make void all opposing testimony or argument, however powerful such testimony or argument, considered in itself, may be.

2*ndly*, If either of the quantities $p$ and $q$ is equal to 0, the probability sought reduces to 0, as it evidently ought to do.

3*dly*, If $p=\frac{1}{2}$ and $q=\frac{1}{2}$, the equations for determining $u$ and $t$ become identical. Hence $u=t$, and

$$\text{Probability sought} = \frac{u}{u+u} = \frac{1}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This result is quite independent of the values of $c$ and $c'$. And it is obviously a correct result. If the causes in operation, or the testimonies borne, are, separately, such as to leave the mind in a state of equipoise as respects the event whose probability is sought, united they will but produce the same effect, whatever the *à priori* probability may be that such causes will come into operation, or that such testimonies will be borne.

4*thly*, If $c=1$, and at the same time $c'$ is not equal to 0, we find, for the equations determining $u$ and $t$,

$$(p-u)\,(c'q-u)\,(u-p-c'q+1)=u(u-c'-p+1)\,(u-c'q)$$

$$(1-p-t)\,(c'\overline{1-q}-t)\,(t-c'\overline{1-q}+p)=t(t-c'+p)\,(t-c'\overline{1-q})$$

These give

$$u=c'q \qquad t=c'(1-q)$$

as the only values which satisfy (12). Hence

$$\text{Probability sought} = \frac{c'q}{c'q + c'\overline{1-q}} = q \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This result is evidently correct. The probability that such an event will take place when two other events, $x$ and $y$, are present, is the same as the probability that it will take place when the event $y$ is present, if it is known that the other event $x$ is never absent.

*5thly*, If $c=c'$ and $q=1-p$, we find in like manner $u=t$, whence

$$\text{Probability sought} = \frac{1}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This result is evidently correct. If the events or testimonies $x$ and $y$ are equally likely to happen, and if the first yields the same presumption *in favour* of that event whose probability is sought as the other yields *against* it, the chances are equally balanced, and the probability required is $\frac{1}{2}$.

*6thly*, But if $q=1-p$, while $c$ and $c'$ are *not* equal, then the value of the probability sought is no longer $\frac{1}{2}$. It may be shown, by a proper discussion of the formulæ, that the presumption afforded by the event $x$, whether favourable or unfavourable, is stronger than the *opposite* presumption afforded by the event $y$, whenever $c$ is less than $c'$, and *vice versa*. And hence it follows, that if there be two events which, by themselves, afford equal presumptions, the one for and the other against some third event, of whose probability nothing more is known, then, if the said two events present themselves *in combination*, that one will yield the stronger presumption, which is itself, of the more rare occurrence. This, too, is agreeable to reason. For in those statistical observations by which probability is determined, we can only take account of co-existences and successions. We do not attempt to pronounce whether the presence of the event $z$ in conjunction with the event $x$ is due to the efficient action of the event $x$, or whether it is a product of some other cause or causes. The more frequent the occurrence of $x$, the less entitled are we to assert that those things which accompany or follow it derive their being from it, or are dependent upon it. If, for instance, $x$ were a standing event, or a state of things *always* present, the probability that any event $z$ would occur when $x$ and $y$ were jointly present, would be the same as the simple probability of that event $z$ when $y$ was present, and it would be wholly uninfluenced by the presence of $x$. This is the limiting case of the general principle.

*7thly*, The case in which $c=c'$ and $p=q$, is a very interesting one. A careful analysis leads to the following results.

If there be two events $z$ and $y$, which are in themselves equally probable, the probability of each being $c$, and if when the event $x$ is known to be present, while

it is not known whether $y$ is present or not, the probability of $z$ is $p$, the same probability being assigned to $z$, when it is known that $y$ is present, but not known whether $x$ is present or not; then, considering $p$ as a presumption for or against $z$, according as $p$ is greater or less than $\frac{1}{2}$.

1. That presumption is strengthened if the events $x$ and $y$ are known to be jointly present, *i.e.*, the probability of $z$ is greater than $p$, if $p$ is greater than $\frac{1}{2}$, but less than $p$ in the contrary case.

2. The strengthening of the presumption is greatest when $c$ is least. In other words, the less likely the events $x$ and $y$ are to happen, the more does their actual concurrence strengthen the presumption, favourable or unfavourable, which either of them alone must afford.

8*thly*, If we suppose $c$ and $c'$ both to approximate to 0, the values of $u$ and $t$ also approximate to 0, and the ratio $\frac{u}{u+t}$ assumes at the limit the form $\frac{0}{0}$. It may, however, be shown that its actual value at the limit is

$$\frac{pq}{pq+(1-p)(1-q)} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

This is most readily obtained from (1) and (2), by rejecting the terms $a'u^2$ and $at^2$, which we may do when $u$ and $t$ are infinitesimal. We thus find that $u$ and $t$ tend to assume the values $cc'pq$ and $cc'(1-p)(1-q)$, whence

$$\frac{u}{u+t} = \frac{pq}{pq+(1-p)(1-q)}$$

It is interesting here to inquire whether the appearance of the limiting value $\frac{pq}{pq+\overline{1-p}\,\overline{1-q}}$ is due merely to the *smallness* of $c$ and $c'$. In studying this question, it occurred to me that it is generally not the mere improbability of events, or the mere unexpectedness of testimonies considered in themselves, but the improbability of the *concurrence* of such events or testimonies which gives to their *union* the highest degree of force. I therefore anticipated, that, if I should introduce among the primary data of the problem, the probability of the concurrence of the events $x$ and $y$, assigning to it a value $m$, it would appear that, whenever $m$ approached to 0, the presumptions with reference to the event $z$, founded upon $x$ and $y$, would receive strength, whatever the values of $c$ and $c'$ might be. And this expectation was verified. On taking for the data

$$\text{Prob. } x=c, \quad \text{Prob. } y=c', \quad \text{Prob. } xy=m, \quad \text{Prob. } xz=cp, \quad \text{Prob. } yz=c'q$$

and representing the sought value of $\frac{\text{Prob. } xyz}{\text{Prob. } xy}$ by $w$, I found, for the determination of $w$, the equation

$$(cp-mw)(c'q-mw)(1-w)=w(c\overline{1-p}-m\overline{1-w})(c'\overline{1-q}-m\overline{1-w}) \quad . \quad . \qquad (9)$$

the conditions of possible experience being that $c$, $c'$, $p$, $q$, and $m$, should be positive proper fractions, subject to the relation

$$c+c' \leqq 1+m \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

that root of (9) being taken, which satisfies the conditions

$$w \leqq \frac{cp}{m}, \quad w \leqq \frac{c'q}{m}, \quad w \leqq 1$$

$$1-w \leqq \frac{c\,\overline{1-p}}{m}, \qquad 1-w \leqq \frac{c'\,\overline{1-q}}{m}.$$

Now if in (9) we suppose $m$ to vanish, we find

$$cp\,c'q\,(1-w) = wc\,\overline{1-p}\,c'\,\overline{1-q}$$

$$\therefore\ pq\,\overline{1-w} = w\,\overline{1-p}\,\overline{1-q}$$

whence
$$w = \frac{pq}{pq+(1-p)\,(1-q)} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

The condition (10) becomes simply $c+c' \leqq 1$. The remaining conditions are all satisfied by the value of $w$.

The formula (11), which in the present investigation appears as a kind of limiting value, applicable only to cases in which the presumption for or against the event $z$ increases most by the combination of the testimonies given, is usually regarded as expressing the *general* solution. The reasoning by which it is supposed to be established is the following.

Let $p$ be the general probability that A speaks truth, $q$ the general probability that B speaks truth; it is required to find the probability, that if they agree in a statement they both speak truth. Now, agreement in the same statement implies that they either both speak truth, the probability of which beforehand is $pq$, or that they both speak falsehood, the probability of which beforehand is $(1-p)\,(1-q)$. Hence the probability beforehand that they will agree is $pq+(1-p)\,(1-q)$, and the probability that if they agree, they will agree in speaking the truth, is accordingly expressed by the formula (11).* In the case of $n$, testimonies whose separate probabilities are $p_1\, p_2 \ldots p_n$, the corresponding formula is

$$\frac{p_1 p_2 \,.\,.\, p_n}{p_1 p_2 \,.\,.\, p_n + (1-p_1)\,(1-p_2)\,.\,.\,(1-p_n)} \qquad . \quad . \quad . \quad . \quad . \qquad (12)$$

In applying which, it is usual to regard one of the testimonies as the initial testimony of the mind itself.† Substantially the same reasoning is applied to determine the probability of correctness of a decision pronounced unanimously by a jury, the probabilities of a correct decision by each member of the jury being given.

In this reasoning there is no recognition that it is to the *same fact* that the several testimonies are borne. Take the case of two testimonies, and the problem

* Cournot Exposition de la Theorie des Chances, p. 411. De Morgan, Formal Logic, p. 191.

† Formal Logic, p. 195.

which is substituted for the true one is the following. The probability that A speaks truth is $p$, that B speaks truth is $q$; what is the probability that, if they both make assertions, and these assertions are both true or both false, they are both true? Whether A and B make the same assertion or not is assumed to be a matter of indifference. But this assumption is, in point of fact, as erroneous as it is unwarranted. The problem which we have solved in the preceding sections, interpreted in relation to testimony, is the following. Two witnesses, A and B, assert a fact. The probability of that fact, if we only knew of A's statement, would be $p$, if we only knew of B's, would be $q$; what is its probability when we know of both? The formal expression of this problem will be seen in Art. 34. The most complete formal expression of the problem which has been substituted for it, taking into account all its elements, is as follows. Let $x$ and $y$ represent the testimonies of A and B, $w$ and $z$ the facts to which these testimonies respectively relate. Observe that no hypothesis is here made as to the connection, by sameness or difference, of $w$ and $z$. And the simple absence of any such hypothesis is properly signified by expressing the events by different symbols, unaccompanied by any logical equation connecting these symbols.

If we wish to indicate that the events $w$ and $z$ are identical, we must write as a connecting logical equation,

$$w=z$$

though it must be simpler to express the identity by the employment of a single symbol as before. Any other definite relation may be expressed in a similar way.

The Problem now stands thus:—

$$\text{Given} \quad \left.\begin{array}{ll} \text{Prob. } x=c, & \text{Prob. } xw=cp, \\ \text{Prob. } y=c', & \text{Prob. } yz=c'q, \end{array}\right\} \quad . \quad . \quad . \quad . \qquad (13)$$

$$\text{Required} \quad \frac{\text{Prob. } xywz}{\text{Prob. } xywz+\text{Prob. } xy\bar{w}\bar{z}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

First, we will seek the value of Prob. $xywz$.

Let $\quad xw=s, \qquad yz=t, \qquad xywz=v$

From these logical equations we must now determine $v$ as a developed logical function of $x$, $y$, $s$, and $t$. The result is

$$v=xyst+0(xys\bar{t}+xyt\bar{s}+xy\bar{s}\bar{t}+xs\bar{y}\bar{t}+x\bar{y}\bar{s}\bar{t}$$
$$+yt\bar{x}\bar{s}+y\bar{x}\bar{s}\bar{t}+\bar{x}\bar{y}\bar{s}\bar{t})$$
$$+\text{ terms whose coefficients are } \frac{1}{0}.$$

Let $u$ be the value of Prob. $v$. Then, by the simplification of Art. 21, we have

$$\frac{xyst+xys+xyt+xy+xs+x}{c}=\frac{xyst+xys+xyt+xy+yt+y}{c'}$$

$$=\frac{xyst+xys+xs}{cp}=\frac{xyst+xyt+yt}{c'q}$$

$$=\frac{xyst}{u}=xyst+xys+xyt+xy+xs+x+yt+y+1$$

Equating the product of the third and fourth to that of the fifth and sixth members of the above system, we have

$$u=cc'pq=\text{Prob. } xywz$$

whence $$cc'(1-p)(1-p)=\text{Prob. } xy\bar{w}\bar{z}$$

And hence

$$\frac{\text{Prob. } xywz}{\text{Prob. } xywz+\text{Prob. } xy\bar{w}\bar{z}}=\frac{pq}{pq+(1-p)(1-q)} \quad . \quad . \quad . \quad (15)$$

Here it will be noted, that although the arbitrary constants $c$ and $c'$ were necessarily introduced into the expression of the data of problem, they have no place in its solution. The result, it will also be seen, agrees with (8); and it thus shows that that formula would express the true solution of the problem originally proposed, if it were permitted to neglect the circumstance that it is to the *same* fact that the testimonies have reference, and so to regard their agreement as merely an agreement in being true or in being false, but not in being true or in being false about the same thing.

*Special Solution of Problem II. founded upon the principle of the limit.*

36. In the present investigation we employ the principle stated in Art. 24, our object being to determine the mean between $p$ and $q$, when they represent probabilities founded upon different judgments, just as in Art. 25 we have determined the mean between $p$ and $q$, when they represent different observed values of a physical magnitude.

To the previous data, viz.,

$$\text{Prob. } x=c, \quad \text{Prob. } y=c', \quad \text{Prob. } xz=cp, \quad \text{Prob. } yz=c'q \quad . \quad . \quad (1)$$

we now add, as the supposed *à priori* value of Prob. $z$,

$$\text{Prob. } z=r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From these collective data we determine the fraction

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy} \text{ or } \frac{\text{Prob. } xyz}{\text{Prob. } xyz+\text{Prob. } xy\bar{z}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

representing the *à posteriori* value of Prob. $z$, and, equating the *à priori* and *à posteriori* values, determine $x$. The principle upon which the investigation proceeds, is, that we attribute to the mean strength of the probabilities $p$ and $q$ such a value, that if the mind had previously to the evidence been in the state of expectation which that value is supposed to measure, the evidence would not have

tended to alter that state. By the evidence I mean, of course, that which forms the basis of the judgments.

Making, as before,

$$xz=s \qquad yz=t \qquad xyz=v$$

and determining $v$ as a developed logical function of $x$, $y$, $z$, $s$, and $t$, we find

$$v=xyzst+0\,(xy\bar{z}\bar{s}\bar{t}+xzs\bar{y}\bar{t}+yzt\bar{x}\bar{s}+x\bar{y}\bar{z}\bar{s}\bar{t}+y\bar{x}\bar{z}\bar{s}\bar{t}$$
$$+z\bar{x}\bar{y}\bar{s}\bar{t}+\bar{x}\bar{y}\bar{z}\bar{s}\bar{t})$$

$$+\text{ terms whose coefficients are } \frac{1}{0}.$$

Hence, availing ourselves of the simplification of Art. 21, we have

$$\frac{xyzst+xy+xzs+x}{c}=\frac{xystz+xy+ytz+y}{c'}$$
$$=\frac{xystz+xsz}{cp}=\frac{xystz+ytz}{c'q}$$
$$=\frac{xystz+xsz+ytz+z}{r}=\frac{xystz}{u}$$
$$=xystz+xy+xsz+ytz+x+y+z+1$$

If we equate the product of the third and fourth to that of the fifth and sixth members of the above system, we find

$$\text{Prob. } xyz=\frac{cc'pq}{r}$$

whence by symmetry,

$$\text{Prob. } xy\bar{z}=\frac{cc'(1-p)\,(1-q)}{1-r} \quad . \quad . \quad . \quad (4)$$

Substituting these values in (3), we have

$$\frac{\text{Prob. } xyz}{\text{Prob. } xy}=\frac{pq\,(1-r)}{pq(1-r)+(1-p)\,(1-q)\,r} \quad . \quad . \quad . \quad (5)$$

Before proceeding further, it will be well to note that in this formula $p$ and $q$ represent, not the general probabilities which the testimonies or evidences upon which our judgments are founded would give to the event $z$, but the probabilities which they would separately produce in a mind embued with a previous expectation of the event $z$, the strength of which is measured by $r$. And there are some curious confirmations of the truth of the theorem, two of which I shall notice.

If we represent the *à posteriori* value of Prob. $z$ by R, and accordingly make

$$\frac{pq\,(1-r)}{pq\,(1-r)+(1-p)\,(1-q)\,r}=\mathrm{R} \quad . \quad . \quad . \quad . \quad (6)$$

we find, on solving the equation relatively to $r$,

$$\frac{pq\,(1-\mathrm{R})}{pq\,(1-\mathrm{R})+(1-p)\,(1-q)\,\mathrm{R}}=r \quad . \quad . \quad . \quad . \quad (7)$$

from which it appears, that if $r$ is the *à priori* expectation of an event $z$, and if evidences are presented which severally would change $r$ to $p$ and $q$, and unitedly would change it to R; then, reciprocally, if R measured the *à priori* expectation of the event, and evidences were received which would severally change it to $p$ and $q$, unitedly they would reduce it to $r$. Now this is evidently what ought to be the case, since testimonies simply countervailing those by which $r$ was changed to R, would simply undo what was done, and again reduce R to $r$.

We see that $p$ and $q$ being the same, R is greater when $r$ is less, and less when $r$ is greater; and this, though it is contrary to what we might at first expect, is agreeable to reason. For the effect of evidence is to be measured, not by the state of expectation which exists after it has been offered, but by the degree in which the previous state of expectation has been changed by it. Suppose $p$ and $q$ much greater than $r$, which we will conceive to be a small quantity, then the separate evidences greatly increase the probability of an event which was before very improbable; and unitedly they do this in a much higher degree than if the separate evidences had merely been such as to raise to the measures $p$ and $q$, an expectation which was before not much below these measures.

Now, introducing the principle of the mean already explained, Art. 24, let us in (6) make $R=r$, we have

$$\frac{pq(1-r)}{pq(1-r)+(1-p)(1-q)r}=r$$

and solving this equation relatively to $r$, we find

$$r=\frac{\sqrt{pq}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}} \quad . \quad . \quad . \quad . \qquad (8)$$

the formula required.

37. Upon this result, the following observations may be made:—

In the first place it may be shown, from the formula itself, that it always expresses a value intermediate between the values $p$ and $q$. Thus we have

$$r-p=\frac{\sqrt{pq}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}}-p$$

$$=\frac{\sqrt{q(1-p)}-\sqrt{q(1-q)}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}}\times\sqrt{p(1-p)} \quad . \qquad (9)$$

on reduction. In like manner we have

$$r-q=\frac{\sqrt{p(1-q)}-\sqrt{q(1-p)}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}}\times\sqrt{q(1-q)}$$

$$=-\frac{\sqrt{q(1-p)}-\sqrt{p(1-q)}}{\sqrt{pq}+\sqrt{\overline{1-p}\,\overline{1-q}}}\times\sqrt{q(1-q)} \quad . \quad . \qquad (10)$$

As $p$ and $q$ are positive fractions, the values of $r-p$ and $r-q$, given in (9) and (10), are clearly of opposite signs, whence $r$ must lie between $p$ and $q$.

In the second place, it may be shown, that $r$ approaches more nearly to that one of the two values $p$ and $q$, which most nearly approaches either of the limits 0 and 1. To show this, let us suppose $q$ greater than $p$, and let us first inquire, under what circumstances $r$ approaches more nearly to $q$ than to $p$.

We must then assume

$$q-r<r-p$$

Substituting the values of these members from (9) and (10) we have

$$\frac{\sqrt{q(1-p)}-\sqrt{p(1-q)}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}}\times\sqrt{q(1-q)}<\frac{\sqrt{q(1-p)}-\sqrt{p(1-q)}}{\sqrt{pq}+\sqrt{(1-p)(1-q)}}\times\sqrt{p(1-p)}$$

Now, $q$ being by hypothesis greater than $p$, it is evident that $\sqrt{q(1-p)}-\sqrt{p(1-q)}$ will be positive. Rejecting, then, the common positive factor on both sides of the inequation we have

$$\sqrt{q(1-q)}<\sqrt{p(1-p)}$$

$$q-q^2<p-p^2$$

$$q-p<q^2-p^2$$

and dividing both sides by the positive factor $q-p$

$$1<p+q$$

$$\therefore 1-q<p$$

a condition which shows that $q$ must be nearer to 1 than $p$ is to 0.

On the other hand, as would appear from the very same analysis, changing only the signs $<$ into $>$, the condition that $r$ may approach more nearly to $p$ than to $q$, is that $p$ may be nearer to 0 than $q$ is to 1.

Now, 1 and 0, as limiting the measures of probability of the event $z$, indicate, the one that it certainly will, the other that it certainly will not occur. And the approach of any measure of probability to these limits indicates the approach of the probability to certainty. We see, then, that when $p$ and $q$ are measures of the probability of an event founded on different judgments, the *mean* between these measures, as determined by (8), will not be the usual arithmetical mean, but will always fall nearer to that one of the two values $p$ and $q$ which expresses a probability the most nearly approaching to certainty.

Now, this seems to be in accordance with reason. Evidence of any kind which enables us to pronounce a judgment with certainty, entirely preponderates over that which only enables us to affirm a probable judgment, Art. 35. And the more of the character of certainty that is possessed, the greater is the weight which is due to the evidence to which it belongs.

38. By an analysis similar to that which is applied in the previous sections, I have determined the general value of $r$, when the number of judgments is $n$, and

the values which they respectively give to the probability of the event $z$ are $p_1$ $p_2$ . . $p_n$. The result is

$$r=\frac{\left(p_1 p_2 \ldots p_n\right)^{\frac{1}{n}}}{\left(p_1 p_2 \ldots p_n\right)^{\frac{1}{n}}+\left((1-p_1)(1-p_2) \ldots (1-p_n)\right)^{\frac{1}{n}}} \quad . \quad . \quad (1)$$

This is the general formula of the mean in reference to judgments, and much as it differs from the formula of the mean, in reference to the observations of a physical magnitude, some remarkable points of analogy exist. I will notice but one. The arithmetical mean is not altered if to the quantities among which it is taken we add another equal to the previous mean. Thus we have

$$\frac{p_1+p_2 \ldots +p_{n+1}}{n+1}=\frac{p_1+p_2 \ldots +p_n}{n}$$

provided that $p_{n+1}=\frac{p_1+p_2 \ldots +p_n}{n}$. Or representing $\frac{p_1+p_2 \ldots +p_n}{n}$ by $\mathrm{P}_n$ we have

$$\mathrm{P}_{n+1}=\mathrm{P}_n$$

provided that

$$p_{n+1}=\mathrm{P}_n \quad . \quad . \quad . \quad . \quad (2)$$

The same relation may readily be shown to hold also, if $\mathrm{P}_n$ represent the mean of judgment, as expressed in (1).

39. The following is a brief summary of the conclusions established in this paper.

1*st*, The solution of the problem of astronomical observations by the logical theory of probabilities is, in its general form, indefinite.

2*ndly*, It becomes definite, if we introduce the general principle of means. The result is in accordance with the usual formulæ, but expresses the so-called *weights* of the observations as determinate functions of certain probabilities relating to the correctness of the observations, and the character of the observers.

3*dly*, When, as respects the two last elements, the observations are considered equal, the formula is reduced to the expression of the arithmetical mean.

4*thly*, The complete solution of the problem of the combination of two probabilities of an event founded upon different testimonies or judgments is indefinite, but admits, in various cases, of being reduced to a definite form.

5*thly*, This indefiniteness is due to the circumstance indicated by the formula, that the strength of the probabilities in combination is due, not to the strength of the separate probabilities alone, but also to the degree of unexpectedness of the testimonies or judgments themselves.

6*thly*, Combined presumptions, whether for or against an event, are generally strengthened by the unexpectedness of the combination.

7*thly*, When probabilities as $p_1, p_2, \ldots p_n$ are in a high degree cumulative, owing

to the exceeding improbability *à priori* of their combination, the expression for their united force tends to assume the form

$$\frac{p_1 p_2 \ldots p_n}{p_1 p_2 \ldots p_n + (1-p_1)(1-p_2) \ldots (1-p_n)}$$

commonly assumed to express the *general* solution.

8*thly*, When the probabilities are so far from being cumulative that we feel that we ought to take a mean between them, the above formula is replaced by the following, viz.:—

$$\frac{\left\{p_1 p_2 \ldots p_n\right\}^{\frac{1}{n}}}{\left\{p_1 p_2 \ldots p_n\right\}^{\frac{1}{n}} + \left\{(1-p_1)(1-p_2) \ldots (1-p_n)\right\}^{\frac{1}{n}}}$$

9*thly*, This formula takes, in reference to ordinary judgments, the place of the arithmetical mean, with relation to the problem of astronomical observations, both being expressions of a more general principle.

40. It will probably appear to some of the readers of this paper, that I have dwelt more upon questions of philosophy and of language, than it is usual to do in mathematical treatises, and that I have also, in various parts, assumed the office of a critic, rather than that of an expositor of original views. Respecting the first of these points, I will only express a hope, that I have nowhere in this paper entered into discussions that are not strictly relevant to the subject. Upon the second, I have to observe, that the theory of probabilities is one in which as it seems to me, the critical office is especially needed. I do not think that it is likely to gain much advance from mere analysis. As respects the original portions of this paper, it is my strongest wish, that they should be regarded chiefly as materials for future judgment. Thus it is possible that the theory which I have developed with reference to problems of which the elements are logical, may be found to involve inconsistencies as a scientific theory, though I do not think this likely to be the case. But whether that theory shall finally be accepted or not, it is, I conceive, of some present importance, to establish the necessary dependence of *any* theory, professing to deal with the same class of problems, upon what I have termed the conditions of possible experience,—to show how those conditions may be determined, and how they are to be applied. As respects the so-called principle of the mean, applied in certain portions of this paper, it is open to inquiry whether it in all cases leads to results possessing the characteristic property, noted in Art. 38, and the decision of this question would materially affect our estimate of its value. Lastly, it is, I think, highly probable, that conditions which we do not yet know of may be discovered, affecting, not the *possibility* of the data of a problem as discussed in this paper, but their *adequacy*, and the principles which, in statistical research especially, ought to guide us in their selection. I am so conscious how limited, imperfect, and in some cases fluc-

tuating my own views upon important questions connected with this subject are, that I should regret having engaged in inquiries so lengthened and laborious as those of which I now take leave, if I did not think that as materials for future judgment, they may possess value and importance. And although the interest attaching at present to these inquiries is chiefly speculative, it may be that they will yet be found to possess a practical utility. The vast collections of modern statistics seem to demand some kind of reduction. I am sure that all who read this paper will feel that even towards this end I regard the labours of the mathematician as contributing only in a secondary degree.

---

## APPENDIX A.

The following proposition in Algebra is of extreme importance in connection with the theory of probabilities. It was originally published by me in the *Philosophical Magazine* for March 1855; but the present paper would be incomplete without some notice of it.

### Proposition.

If V be a rational and integral function of $n$ variables $x, y, z\,.\,.$, involving no power of these variables higher than the first, and having all its coefficients positive, and being complete in all its terms, then if $V_x$ represent that part of V which contains $x$, $V_y$ that part which contains $y$, and so on; the system of equations

$$\frac{V_x}{p}=\frac{V_y}{q}\,.\,.=V \qquad (1)$$

$p$, $q$, &c. being positive fractions, admits of one solution, and of only one solution, in positive values of $x, y, z\,.\,.$

To exemplify this proposition, let us suppose

$$V=axy+bx+cy+d$$

$a$, $b$, $c$, and $d$ being all greater than 0; then it is affirmed that the system of equations

$$\frac{axy+bx}{p}=\frac{axy+cy}{q}=axy+bx+cy+d \qquad (2)$$

$p$ and $q$ being positive, admits of one, and only one solution, in positive values of $x$ and $y$.

The proposition is true when $n=1$. For then $V=ax+b$ and the system (1) is reduced to the single equation

$$\frac{ax}{p}=ax+b$$

Whence we have

$$x=\frac{bp}{a\,(1-p)}$$

and this value is positive if $a$ and $b$ are positive, and $p$ a positive fraction.

The general proof consists in showing, that if the proposition is true for a particular value of $n$, it is true for the next greater. Whence, being true for the case of $n=1$, it is true universally. I will exemplify the method by showing how the truth of the proposition, when $n=2$ is dependent upon its truth when $n=1$.

Let $n=2$, then we have to consider the system (2), which may be reduced to the form

$$\frac{axy+bx}{axy+bx+cy+d}=p \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$$\frac{axy+cy}{axy+bx+cy+d}=q \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Let us represent by Y the variable value of the first member of (4), when $x$ and $y$ are supposed to vary in subjection to the single condition (3). We have then

$$\mathrm{Y}=\frac{axy+cy}{axy+bx+cy+d} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Now differentiating (3) and (5) relatively to $x$ and $y$, we find, after slight reductions,

$$(ay+b)\,(cy+d)\,dx+(ad-bc)\,xdy=0 \quad . \quad . \quad . \quad . \qquad (6)$$

$$d\mathrm{Y}=\frac{(ad-bc)y}{\mathrm{V}^2}dx \quad +\frac{(ax+c)\,(bx+d)}{\mathrm{V}^2}dy \quad . \quad . \quad . \qquad (7)$$

where, as before, $\mathrm{V}=axy+bx+cy+d$. Substituting in (7) the value of $dx$ found from (6), we have

$$d\mathrm{Y}=\frac{(ax+c)\,(bx+d)\,(ay+b)\,(cy+d)-(ad-bc)^2xy}{(ay+b)\,(cy+d)\,\mathrm{V}^2}dy$$

The numerator of this expression may be reduced to the form

$$\mathrm{V}\,(abcxy+abdx+acdy+bcd)$$

whence

$$\frac{d\mathrm{Y}}{dy}=\frac{abcxy+abdx+acdy+bcd}{(ay+b)\,(cy+d)\,\mathrm{V}} \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

This represents the differential coefficient of Y taken with respect to $y$ as independent variable, $x$ being regarded as a function of $y$ determined by (3). The expression is always positive, if $x$ and $y$ are positive.

Now let $y$ vary from 0 to $\infty$ through the whole range of positive magnitude. Writing (3) in the form

$$\frac{\mathrm{A}x}{\mathrm{A}x+\mathrm{B}}=p \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

where $\mathrm{A}=ay+b$, $\mathrm{B}=cy+d$, the quantity $x$ must, by reference to the case of $n=1$, have a positive value, since A and B are positive and $p$ fractional. Whence, as $y$ varies from 0 to $\infty$, the value of $\frac{d\mathrm{Y}}{dy}$ is always positive.

Now when $y=0$, $\mathrm{Y}=0$, and when $y=\infty$, $\mathrm{Y}=1$, as is evident from (5). Therefore, as $y$ increases from 0 to $\infty$, Y continuously increases from 0 to 1. In this variation

it must once, and only once, become equal to $q$. Wherefore the system (3) (4) admits of one, and only one, solution in positive values of $x$ and $y$.

The reasoning might also be presented in the following form. The condition of Y having a maximum or minimum value is expressed by the equation

$$abcxy+abdx+acdy+bcd=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It is obvious that this, as all the terms in the first member are positive, can never be satisfied by positive values of $x$ and $y$. Hence Y has no maximum or minimum, consistently with (3) being satisfied, and thus it never resumes a former value, and is only once, in the course of its variation, equal to $q$.

In the case of $n=3$, we have

$$V=axyz+byz+cxz+dxy+ex+fy+gz+h$$

and the system to be considered is

$$\frac{axyz+cxz+dxy+ex}{V}=p \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{axyz+byz+dxy+fy}{V}=q \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{axyz+byz+cxz+gz}{V}=r \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Let the first number of the last equation, considered as a variable function of $x$, $y$, $z$ be represented by Z, and suppose $x$, $y$, and $z$ to vary in subjection to the conditions (11) (12). Just as before, it may be shown that Z increases continuously with $z$. The condition of Z having a maximum or minimum value, will be expressed by the following equation:

$$\begin{aligned}&(D+H+E+F)\,(ABC+ACG+ABG+BCG)\\&+(A+B+C+G)\,(DHE+DHF+DEF+HEF)\\&+(AC+BG)\,(DF+DH+EF+EH)\\&+(AG+BC)\,(DF+EH+DE+FH)\\&+(AB+CG)\,(DE+DH+FE+FH)\\&+4\,AGFE+4\,BCDH=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)\end{aligned}$$

Wherein

$$\begin{array}{llll}A=axyz & B=byz & C=czx & D=dxy\\ E=ex & F=fy & G=gz & H=h\end{array}$$

And as this equation has positive values only in its first member, it cannot be satisfied by positive values of $x$, $y$, $z$; whence, by the same reasoning as before, the system (11), (12), (13) cannot have more than one solution in positive values of $x$, $y$, $z$.

To show that it will have one such solution, let $z$ vary from 0 to $\alpha$, then Z continuously increases from 0 to 1, and once becomes equal to $r$. At every stage of its variation we may give to (11) and (12) the form

$$\frac{Axy + Bx}{V} = p$$

$$\frac{Axy + Cy}{V} = q$$

which corresponds with the form of the general system (3) (4) in the case of $n=2$. Whence, for each positive value of $z$, one positive set of values of $x$ and $y$ will be found. The system (11), (12), (13) admits, therefore, of one solution in positive values of $x$, $y$, $z$, and of only one.

To prove the proposition generally, it ought to be shown that the function exemplified in the first members of (10) and (14), for the cases of $n=2$ and $n=3$ possesses universally the same property of consisting only of positive terms. I have proved that it does for the case of $n=4$, and the analysis was such as to leave no doubt whatever of its general truth.

I will now offer a few remarks on the application of the above proposition.

The system of equations for determining $s$ and $t$ . ., Art, 21, is of the form

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = V \quad \ldots \quad (15)$$

V being a function of the same general character as the one discussed in the foregoing proposition, but with this difference, that its coefficients, if we regard it as a complete function, are all equal either to 1 or to 0.

Thus in Art. 18, we have

$$V = stv + s + t + v + 1$$

Here the terms $st$, $tv$, and $vs$, must be considered as present, but with the coefficient 0.

This limitation does not affect the essentially positive character of the determining function exemplified in (10) and (14). Whence the system (15) cannot have more than one solution in positive values of $s$, $t$, &c. *This shows that the solution of the system of equations furnished by the general method can never be ambiguous.*

The vanishing of some of the coefficients of V does, however, affect the reasoning by which it has been shown, that for the general form of V discussed in the last proposition, one solution of the algebraic system in positive values will exist. Thus Y in (5) does not vanish with $y$, if both $b$ and $d$ vanish. And generally this vanishing of coefficients in V entails conditions among the quantities $p$, $q$, $r$ . ., in addition to that of their being fractional, in order that the derived algebraic system may admit of a solution in positive values.

Thus if we take, as in (7) Art. 18,

$$V = stv + s + t + v + 1$$

with the derived algebraic system

$$\frac{stv + s}{V} = p \qquad \frac{stv + t}{V} = q \qquad \frac{stv + v}{V} = r$$

it is evident that if $s$, $t$, and $v$ are positive quantities, and if we write

$$\frac{stv}{\mathrm{V}}=u, \qquad \frac{s}{\mathrm{V}}=\lambda, \qquad \frac{t}{\mathrm{V}}=\mu, \qquad \frac{v}{\mathrm{V}}=\nu$$

$u$, $\lambda$, $\mu$, and $\nu$ must be positive fractions, whence, in addition to the equations

$$u+\lambda=p$$
$$u+\mu=q$$
$$u+\nu=r$$

we shall have the inequations

$$u \geqq 0 \qquad \lambda \geqq 0 \qquad \mu \geqq 0 \qquad \nu \geqq 0$$
$$u+\lambda+\mu+\nu \leqq 1$$

This system is identical with the one obtained in 10, Art. 13, for the determination of the conditions of possible experience in the particular question of Probabilities, in which the above function V presents itself. And a very little attention will show, that if in any case we express as above the relations which must obviously be fulfilled in order that $s$, $t$, &c., may be positive quantities, we shall form a system of equations and inequations precisely agreeing with those which we should have to form in order to obtain the conditions of possible experience, if we sought those conditions, not from the data in their original expression, but from the translated data, as employed in Art. 13.

*Hence, in order that* s, t . . *in the system of Art.* 21, *may be positive, or in the prior system, positive fractions, the problem of which these systems of equations involve the solution must represent a possible experience.*

*Conversely if that problem represent a possible experience, the quantities* s, t . . *will admit of being determined in the system of Art.* 21, *in positive values, or in the prior system, in positive fractional values.*

I have not succeeded in obtaining a perfectly rigorous proof of the latter, or converse proposition in its general form, but I have not met with any individual cases in which it was not trne. I will here only exemplify it in Problem II., Art. 34.

Here the value of V is

$$\mathrm{V}=xyst+xs+yt+xy+x+y+1$$

and the algebraic system employed in the determination of Prob. $xyz$ is

$$\frac{xyst+xs+xy+x}{c}=\frac{xyst+yt+xy+y}{c'}$$
$$=\frac{xyst+xs}{cp}=\frac{xyst+yt}{c'q}=\frac{xyst}{u}$$
$$=xyst+xs+yt+xy+x+y+1 \quad . \quad . \quad . \qquad (16)$$

For the determination of $u$ we hence find the following equation—

$$u\,(u-\overline{c'+cp-1})\,(u-\overline{c+c'q-1})-(cp-u)\,(c'q-u)\,(u-\overline{cp+c'q-1})=0 \qquad (17)$$

the conditions of limitation being

$$u > 0 \qquad u > c' + cp - 1 \qquad u > c + c'q - 1$$

$$u < cp \qquad u < c'q \qquad . \quad . \quad . \quad . \qquad (18)$$

Now, since $u$ is greater then $c + c'q - 1$ it is *à fortiori* greater then $cp + c'q - 1$. Thus within the limits assigned to $u$, all the factors of each term of (17) will be positive.

If then we give to $u$ the value which belongs to the highest of its inferior limits, the first member of (17) will be reduced to its second term, and will be negative. If we give to $u$ the value which belongs to the lowest of its superior limits, the first member of (17) will be reduced to its first term, and will be positive. Moreover, that member is a quadratic function of $u$. Hence there is one root, and only one, within the limits specified.

We must now express $x$, $y$, $s$, and $t$, in terms of $u$. Their values determined from the system (16) are as follows, viz.:—

$$x = \frac{c(1-p)}{u-(+c'q-1)} \qquad y = \frac{c'(1-q)}{u-(c'+cp-1)}$$

$$s = \frac{u-(c+c'q-1)}{c(1-p)} \times \frac{u}{c'q-u}$$

$$t = \frac{u-(c'+cp-1)}{c'(1-q)} \times \frac{u}{cp-u}$$

All these expressions become positive when $u$ is determined in accordance with the conditions (18).

It would seem from the above, as well as from reasonings analogous to those of Proposition I., that when the algebraic system belonging to a problem in the theory of probabilities is placed in the form

$$\frac{V_x}{V} = p, \ \frac{V_y}{V} = q \ \ . \ .$$

the limits of variation of the first member of any equation subject to the condition, that the variables shall all be positive, and shall vary in subjection to all the other equations of the system, will not in general be 0 and 1, as in the case contemplated in Prop. I., but will correspond with the limits of value of the second member of the same equation as determined by the conditions of possible experience.

This conclusion I have in various cases independently verified. The analytical theory still, however, demands a more thorough investigation.

---

## APPENDIX B.

A note to Archbishop WHATELY's Logic, Book III., sec. 14, contains a rule for computing the joint force of two probabilities in favour of a conclusion which, as actually applied, is at variance with the preceding results. For this reason,

and also because the validity of the rule in question has been made the subject of recent controversy, I design to offer a few remarks upon the subject here. The rule is contained in the following extract. "As, in the case of two probable premises, the conclusion is not established except on the supposition of their being *both* true, so, in the case of two (and the like holds good with any number) distinct and independent indications of the truth of some proposition, unless *both* of them *fail*, the proposition must be true: we therefore multiply together the fractions indicating the probability of *failure* of each,—the chances *against* it;—and the result being the total chances against the establishment of the conclusion by these arguments, this fraction being deducted from unity, the remainder gives the probability *for* it. *E.g.*, A certain book is conjectured to be by such and such an author, partly, 1*st*, from its resemblance in style to his known works, partly (2*dly*), from its being attributed to him by some one likely to be pretty well informed: let the probability of the Conclusion, as deduced from one of these arguments by itself, be supposed $\frac{2}{5}$, and, in the other case $\frac{3}{7}$; then the *opposite* probabilities will be, respectively, $\frac{3}{5}$ and $\frac{4}{7}$; which multiplied together give $\frac{12}{35}$, as the probability *against* the Conclusion; *i.e.*, the chance that the work may *not* be his, notwithstanding those reasons for believing that it is: and consequently the probability in *favour* of that Conclusion will be $\frac{23}{35}$, or nearly $\frac{2}{3}$."

A confusion may here be noted between the probability that a conclusion is proved, and the probability in favour of a conclusion furnished by evidence which does not prove it. In the proof and statement of his rule, Archbishop WHATELY adopts the former view of the nature of the probabilities concerned in the data. In the exemplification of it, he adopts the latter. He thus applies the rule to a case for which it was not intended, and to which it is in fact inapplicable.

The rule is given, and the conditions of its just application are assigned in Professor DE MORGAN'S *Formal Logic*, p. 201. Its origin may be thus explained. Let there be two independent causes, A and B, either of which, when present, necessarily produces an effect E. Let $a$ be the probability that A is present, $b$ the probabiliy that B is present; then $1-a$ is the probability that A is absent, $1-b$ the probability that B is absent, $(1-a)(1-b)$ the probability that they are both absent; finally, $1-(1-a)(1-b)$ the probability that they are not both absent. This, then, is the probability that one at least of the causes is present; and therefore it is the probability that the event E, so far as it is dependent upon these causes, will occur. In its special application to arguments viewed as causes of belief or expectation, it would lead to the following theorem. If there are two independent arguments in favour of a conclusion which the premises of either, if granted, are sufficient to establish, the doubt only existing as to the truth of the premises, and if the probability that the premises of the first argument are true is $a$, the probability that the premises of the second argument are true $b$, then the probability that the conclusion is established is $1-(1-a)(1-b)$. Interpreted, this formula gives

Archbishop WHATELY's rule, but the conditions of its valid application are evidently not fulfilled in the example which he has given. To satisfy these conditions, this problem ought to be changed into the following: "There exists a certain quality of style, the possession of which would *prove* the work to be by the author supposed. The probability that the work possesses that quality is $\frac{2}{5}$. There is a person so well informed that his attributing the work to the supposed author would be *conclusive*. The probability that he does attribute it to the author in question is $\frac{3}{7}$. Required the probability, on these grounds, that the supposed author is the real one." But this is evidently not the sense in which the problem was meant to be understood. Thus to take one point, it is not the quality of the style that is a matter of probability, but the mode in which a known and observed quality affects the question of authorship.

Taking the problem in its intended meaning, each of the fractions $\frac{2}{5}$, $\frac{3}{7}$, measuring, not the probability of the truth of certain premises, but the probability drawn from these premises, as conditions, in favour of a certain supposition (I use this word in preference to conclusion), we are no longer permitted to apply the formula above determined. And we are not permitted to do so, because the probabilities with which we are concerned are conditional, and their possession of this character greatly increases the difficulty of the problem. Its rigorous formal solution is given in Art. 34, and shows that the probability sought is, generally speaking, indefinite,—a result which agrees with the conclusions of Bishop TERROT, by whom the error to which attention has been directed was first pointed out, *Transactions of the Royal Society of Edinburgh*, vol. xxi., p. 369.

I trust that I have not in any way misrepresented Archbishop WHATELY's reasoning; and I am the more encouraged to believe that I have not, as a defence of it which appeared in the *United Church Journal* expressly proceeds upon the assumption that the probabilities with which we are concerned are probabilities that the authorship is *proved*. To this view Bishop TERROT justly demurred. Nor was its inconsistency materially diminished by assigning to proof a meaning less absolute than belongs to demonstration. For whatever degree of cogency,—of power to produce conviction,—we suppose to characterize proof, the thing itself belongs to consciousness, and the question whether given evidence is sufficient to convey proof to our minds or not, is a matter of knowledge, not of probability.

# PROCEEDINGS

OF THE

# STATUTORY GENERAL MEETINGS,

AND

## LIST OF MEMBERS ELECTED AT THE ORDINARY MEETINGS,

SINCE NOVEMBER 22, 1852;

WITH

## LIST OF DONATIONS TO THE LIBRARY,

FROM DEC. 5, 1853, TILL APRIL 20, 1857.

# PROCEEDINGS, &c.

*Monday, November* 28, 1853.

At a Statutory General Meeting, Dr CHRISTISON, V.P., in the Chair, the following Office-Bearers were duly elected :—

Sir T. MAKDOUGALL BRISBANE, Bart., G.C.B., G.C.H., President.

Sir D. BREWSTER, K.H., Very Rev. Principal LEE, Right Rev. Bishop TERROT, Dr CHRISTISON, Dr ALISON, Hon. Lord MURRAY, — Vice-Presidents.

Professor FORBES, General Secretary,

Dr GREGORY, Professor SMYTH, — Secretaries to the Ordinary Meetings.

JOHN RUSSELL, Esq., Treasurer.

Dr TRAILL, Curator of Library and Instruments.

JAMES WILSON, Esq., Curator of Museum.

COUNSELLORS.

| | |
|---|---|
| ROBERT CHAMBERS, Esq. | ANDREW COVENTRY, Esq. |
| J. T. GIBSON-CRAIG, Esq. | Rev. Dr JAMES GRANT. |
| WILLIAM SWAN, Esq. | Rev. Professor KELLAND. |
| Prof. WILLIAM THOMSON. | Dr GEORGE WILSON. |
| Dr J. H. BENNETT. | CHARLES MACLAREN, Esq. |
| Dr J. H. BALFOUR. | Rev. Dr ROBERT LEE. |

The following Committee was appointed to audit the Treasurer's accounts :—

J. T. GIBSON-CRAIG, Esq. ANDREW COVENTRY, Esq. JAMES CUNNINGHAM, Esq.

The Meeting then adjourned.

(Signed) JOHN LEE, *V.P.*

*Memorandum.—November* 28, 1853.—At a Statutory General Meeting of this date, Dr CHRISTISON, V.P., made a preliminary statement on the part of the Council to the following effect :—

That, as it appeared from a recent correspondence, that Professor FORBES considered his former resignation of the office of General Secretary to stand good without any formal renewal of it, the Council had taken into consideration the most advisable course of proce-

dure to be followed in consequence of Professor FORBES's continued indisposition ; and having been apprised that such an improvement had taken place in Professor FORBES's health as to warrant strong hopes of his being able to return to Edinburgh, and to business, in the course of next year ; and being also well satisfied with the manner in which the duties of the Secretaryship have been discharged in his absence, the Council had resolved to recommend to the Society that the arrangement of the past Session should be continued for another year.

That the Council having been further informed that Professor FORBES had intimated that he could not accept the usual salary of office for last year, they resolved also to recommend that Mr WILSON be requested to accept the said salary for his services during the preceding Session, and to grant the continuance of these services for the present Session upon the same terms.

The above recommendations were then unanimously adopted and confirmed by the General Meeting of the Society.

*Memorandum.—December* 19, 1853.—At the Ordinary Meeting of this date, the Acting General Secretary, on the part of the Council, stated that Dr GEORGE BUIST of Bombay had been duly elected a Fellow of the Society in the Session of 1845–6, but had not been enrolled in consequence of his absence in India, where he had, until very recently, resided ; that anticipating only a short residence in India after his intended return thither, he was desirous to become a resident Fellow of the Society, and that the Council, having taken into consideration the circumstances of the case, had agreed to recommend that Dr BUIST be enrolled on payment of the usual admission-money, and the annual subscription for the current Session, without the exaction of arrears.

Whereupon it was moved by Dr GREVILLE, seconded by JAMES CUNNINGHAM, Esq., and unanimously agreed to, "That Dr BUIST having been duly elected a Fellow of this Society in the Session of 1845–6, during his absence in India, be now received as an Ordinary Resident Member on payment of his entrance money, and annual contribution for the current Session."

*Monday, November* 27, 1854.

At a Statutory General Meeting, Principal LEE, V.P., in the Chair, the following Office-Bearers were duly elected:—

Sir T. MAKDOUGALL BRISBANE, Bart., G.C.B., G.C.H., President.

Sir D. BREWSTER, K.H.,
Very Rev. Principal LEE,
Right Rev. Bishop TERROT,
Dr CHRISTISON,
Dr ALISON,
Hon. Lord MURRAY,
} Vice-Presidents.

Professor FORBES, General Secretary.

Dr GREGORY,
Dr BALFOUR,
} Secretaries to the Ordinary Meetings.

John Russell, Esq., Treasurer.
Dr Traill, Curator of Library and Instruments.
James Wilson, Esq., Curator of Museum.

COUNSELLORS.

| | |
|---|---|
| Dr J. H. Bennett. | Rev. Dr Robert Lee. |
| Andrew Coventry, Esq. | Professor C. Piazzi Smyth. |
| Rev. Dr James Grant. | Hon. B. F. Primrose. |
| Rev. Professor Kelland. | Sir William Gibson-Craig, Bart. |
| Dr George Wilson. | Major Edward Madden. |
| Charles Maclaren, Esq. | James Cunningham, Esq. |

The following Committee was appointed to audit the Treasurer's accounts:—

Andrew Coventry, Esq. James Walker, Esq., W.S. William Thos. Thomson, Esq.

The Meeting then adjourned.

(Signed) R. Christison, *V.P.*

*Monday, November* 26, 1855.

At a Statutory General Meeting, Dr Christison, V.P., in the Chair, the following Office-Bearers were duly elected:—

Sir T. Makdougall Brisbane, Bart., G.C.B., G.C.H., President.

Sir D. Brewster, K.H.,
Very Rev. Principal Lee,
Right Rev. Bishop Terrot,
Dr Christison,
Dr Alison,
Hon. Lord Murray,
} Vice-Presidents.

Professor Forbes, General Secretary.

Dr Gregory,
Dr Balfour,
} Secretaries to the Ordinary Meetings.

John Russell, Esq., Treasurer.
Dr Traill, Curator of Library and Instruments.
James Wilson, Esq., Curator of Museum.

COUNSELLORS.

| | |
|---|---|
| Dr George Wilson. | Sir William Gibson-Craig, Bart. |
| Charles Maclaren, Esq. | James Cunningham, Esq. |
| Rev. Dr Robert Lee. | Dr Greville. |
| Professor C. Piazzi Smyth. | A. Keith Johnston, Esq. |
| Hon. B. F. Primrose. | Dr Maclagan. |
| Colonel Madden. | William Swan, Esq. |

The following Committee was appointed to audit the Treasurer's accounts:—

J. T. Gibson-Craig, Esq. David Smith, Esq. James Cunningham, Esq.

The Meeting then adjourned.

(Signed) John Lee, *V.P.*

*Memorandum.—November* 26, 1855.—At a Statutory General Meeting of this date, Professor Christison, V.P., in the Chair, the following Resolution was moved by the Treasurer, and unanimously adopted:—

"1. That it having been reported to the Royal Society by the Council, that Sir Thomas M. Brisbane, the venerable and respected President of the Society, has instituted a Prize, to be called "The Makdougall Brisbane Prize," to be awarded by the Council of the Society, in such manner and for such purposes as shall be deemed most expedient for the promotion and advancement of science, the Society takes the opportunity of the first meeting of the session to record the great satisfaction with which it receives this proof of Sir T. M. Brisbane's continued zeal to promote the interests of science, and its grateful sense of the confidence reposed by him in the Society, by conferring on the Council such large and discretionary powers as to the mode of awarding the prize.

"The Society has always viewed with pleasure the connection which has so long subsisted between it and Sir T. M. Brisbane as its President, from which this Society has derived so much benefit and so much honour; and it will henceforth consider the "Makdougall Brisbane Prize" as a permanent and pleasing memorial of that connection.

"2. That a copy of the above resolution be sent to Sir Thomas M. Brisbane."

*Monday, November* 24, 1856.

At a Statutory General Meeting, Principal Lee, V.P., in the Chair, the following Office-Bearers were duly elected:—

Sir T. Makdougall Brisbane, Bart., G.C.B., G.C.H., President.

Sir D. Brewster, K.H.,
Very Rev. Principal Lee,
Right Rev. Bishop Terrot,
Dr Christison,
Dr Alison,
Hon. Lord Murray, } Vice-Presidents.

Professor Forbes, General Secretary.

Dr Gregory,
Dr Balfour, } Secretaries to the Ordinary Meetings.

John Russell, Esq., Treasurer.

Dr Douglas Maclagan, Curator of Library and Museum.

COUNSELLORS.

| | |
|---|---|
| Hon. B. F. Primrose. | Dr Traill. |
| James Cunningham, Esq. | Hon. Lord Neaves. |
| Dr Greville. | Dr Thos. Anderson, Glasgow. |
| A. Keith Johnston, Esq. | Rev. Dr Hodson. |
| Dr Maclagan. | Robert Chambers, Esq. |
| Wm. Swan, Esq. | J. T. Gibson-Craig, Esq. |

The following Committee was appointed to audit the Treasurer's Accounts :—

J. T. GIBSON-CRAIG, Esq. JOHN MACKENZIE, Esq. JAMES CUNNINGHAM, Esq.

The Meeting then adjourned.

*Memorandum.—January* 5, 1857.—At the ordinary Meeting of this date, Dr CHRISTISON, in the absence of Mr RUSSELL, the Treasurer, proposed that the following motion, tabled on the 1st December, be carried :—

That the Meeting do resolve that Laws II., III., and IV., and Law XVI., be altered and stand as follows :—[The alterations are printed within brackets.]

LAW II.

Every Ordinary Fellow, within three months after his election, shall pay [*Two*] Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected—and annually, at the commencement of every Session, the like sum of Three Guineas ; [that this annual contribution shall continue for *ten* years after his admission, and thereafter it shall be limited to *Two* Guineas for fifteen years thereafter.]

LAW III.

All Fellows who shall have paid *twenty-five* years' annual contribution shall be exempt from farther payment.

LAW IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission ; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s. payable by each Resident Fellow, but after payment of such annual contribution for eight years he shall be exempt from any further payment.

[In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.]

LAW XVI.

That the words "and an Assistant Curator," in this Law, shall be omitted.

The motion was seconded by Professor MORE, and was carried, it being recommended to the Council to take means of ensuring that all existing Members shall benefit by the change in the entrance fee and annual contribution after ten years, according to their period of Membership.

*Memorandum.—February* 2, 1857.—At the Ordinary Meeting of this date, it was announced from the chair that Mr RUSSELL desired to be relieved from the office of Treasurer.

The Meeting received Mr RUSSELL'S resignation with the greatest regret ; and resolved that a cordial acknowledgment of his long continued and zealous services to the Society be entered on the Minutes.

*Memorandum.—February* 16, 1857.—At the Ordinary Meeting of this date, Dr CHRISTISON, V.P., the Chairman, stated that the Council had, according to the recommendation made at last Meeting, considered the subject of Mr RUSSELL'S retirement, and had resolved to propose to the Society the following motions :—

"1. That Mr RUSSELL's letter of resignation be recorded in the Minutes."

This motion was unanimously agreed to, and the letter follows:—

SIR,—I have now held the office of Treasurer of the Royal Society for eighteen years, and in the course of next month I shall enter on the 78th year of my age. It seems to me, therefore, full time that the duties of that office should be devolved on some younger and more active Fellow of the Society. I therefore now beg to resign that office, and request you will lay this letter of resignation before the next Meeting of Council.—I am, &c. (Signed) JOHN RUSSELL.

"2. That the following resolution, conveying the thanks of the Society to Mr RUSSELL, be recorded in the Minutes."

This was carried unanimously, and the resolution follows:—

Resolved, That this Meeting do record their deep sense of the zealous and important services rendered to the Royal Society by its late Treasurer, Mr RUSSELL. During the long period of eighteen years for which he has held that office, the funds of the Society have been in a steadily prosperous condition. The benefit of Mr RUSSELL's cordial and effective assistance has been felt not only in this, but in every department of the Society's affairs, and that assistance the Meeting hope may still long continue, notwithstanding Mr RUSSELL's withdrawal from the responsibility of his official position.

"3. That Mr JAMES GIBSON-CRAIG be elected Treasurer, in room of Mr RUSSELL resigned."

This motion was carried unanimously.

*Memorandum.—March* 2, 1857.—At the Ordinary Meeting of this date, Mr D. SMITH made the following motion, which was seconded by Mr NASMYTH, and unanimously adopted:—

"That the Council be requested to consider the propriety of recording in a fitting and permanent manner the feelings of the Society towards their late Treasurer, Mr RUSSELL, for the benefit which the Society has received from his attention and exertions during the time he has acted as Treasurer, and to report their opinion and the manner in which they think such a record should be most appropriately made."

*Memorandum.—April* 20, 1857.—At the Ordinary Meeting of this date, Dr FLEMING moved, and Mr SYME seconded, the following resolutions, of which notice had been given at the previous Ordinary Meeting:—

"1. That the Council has exceeded its powers in reference to the Neill Prize, by determining and announcing the conditions of appropriation before submitting the case for the approval of the Society.

"2. That the conditions announced by the Council are framed with the view of enhancing the importance and promoting the interest of the Society, rather than the recognition of merit, contemplated in a liberal spirit, and so unequivocally indicated by the terms of the bequest.

"3. That the subject be remitted to the Council for reconsideration, with an injunction that the results be laid before the Society for approval, previous to publication."

The Rev Dr GRANT moved, and Mr D. SMITH seconded, the following amendment:—

"That the Society sees no ground for disturbing the arrangements for the appropriation of the Neill Bequest, as announced by the Council, and expresses its approval of said arrangements."

On the vote being taken, 32 voted for the amendment, and 18 for the motion.

The Chairman declared the amendment to be carried.

*Memorandum.—April* 20, 1857.—At the Ordinary Meeting of this date, it was proposed on the part of the Council (in consequence of a remit to them from the Society on the 2d March), that a sum not exceeding Fifty Guineas, should be set apart from the funds of the Society for the purchase of a piece of Plate to be presented to Mr RUSSELL, the late Treasurer, in the name of the Society, and in acknowledgment of his long-continued and valuable services; a notice to this effect having been made at last Ordinary Meeting.

This motion was carried by acclamation.

## MEMBERS ELECTED.

*April* 18, 1853.

Hugh Scott, Esq. of Gala.

*December* 5, 1853.

Græme Reid Mercer, Esq.

*December* 19, 1853.

Dr George Buist, Bombay.

*January* 3, 1854.

Sir John Maxwell, Bart.

*January* 16, 1854.

William Murray, Esq. of Monkland.

*February* 20, 1854.

Dr John Addington Symonds, Clifton, Bristol.

*April* 3, 1854.

Henry Dunlop, Esq. of Craigton.

*April* 17, 1854.

Dr William Bird Herapath, Bristol.

Professor Robert Harkness, Queen's College, Cork.

*December* 4, 1854.

Dr Thomas A. Wise.

Dr James Coxe.

*December* 18, 1854.

Ernest Bonar, Esq.

*January* 2, 1855.

James P. Fraser, Esq.

*February* 5, 1855.

Dr Stevenson Macadam.

*February* 19, 1855.

Robert Etheridge, Esq., Clifton, Bristol.

John Inglis, Esq., Dean of Faculty.

Rev. James S. Hodson, M.A.

*March* 19, 1855.

Dr Wyville T. C. Thomson, Professor of Geology, Belfast.

*April* 2, 1855.

Sir Robert K. Arbuthnot, Bart.

*April* 30, 1855.

Dr Wright, Cheltenham.

*December* 3, 1855.

James Hay, Esq. R. M. Smith, Esq.

*January* 7, 1856.

David Bryce, Esq.

*January* 21, 1856.

William Mitchell Ellis, Esq. Dr George J. Allman.

*February* 4, 1856.

Hon. Lord Neaves. Dr Frederick Penny.

*February* 18, 1856.

Dr Thomas Laycock, Professor of Medicine.

*March* 17, 1856.

Thomas Cleghorn, Esq., Advocate.

*April* 21, 1856.

James Clerk Maxwell, Esq., Professor of Natural Philosophy, Marischal College, Aberdeen.

*January* 5, 1857.

Horatio Ross, Esq. Dr James Black.

*February* 2, 1857.

Dr John Ivor Murray.

*February* 16, 1857.

John Melville, Esq., W.S. John Blackwood, Esq.
Brinsley De Courcy Nixon, Esq.

*March* 2, 1857.

Andrew Murray, Esq. of Conland. Rev. Dr James Macfarlane, Duddingston.
Dr W. M. Buchanan.

*April* 6, 1857.

Thomas Login, Esq., C.E., Pegu.

# LIST OF THE PRESENT ORDINARY MEMBERS,

## IN THE ORDER OF THEIR ELECTION.

---

General Sir THOMAS M. BRISBANE, Bart., G.C.B., &c., F.R.S. Lond.,

**PRESIDENT.**

Date of Election.

1798 Alexander Monro, M.D.
1808 James Wardrop, Esq., London.
Sir David Brewster, K.H., LL.D., F.R.S., Lond., *St Andrews.*
1811 General Sir Thomas Makdougall Brisbane, Bart., G.C.B., G.C.H., F.R.S. Lond.
James Jardine, Esq., *Civil Engineer.*
Alexander Gillespie, Esq., *Surgeon.*
1812 James Pillans, Esq., *Professor of Humanity.*
Sir George Clerk, Bart., F.R.S. Lond.
1813 William Somerville, M.D., F.R.S. Lond.
1814 Right Honourable Viscount Arbuthnot.
John Fleming, D.D., *Professor of Natural Science, New College.*
1815 Henry Home Drummond, Esq., *of Blair-Drummond.*
William Thomas Brande, Esq., F.R.S. Lond., *Professor of Chemistry in the Royal Institution.*
1816 Leonard Horner, Esq., F.R.S. Lond.
1817 Alexander Maconochie Wellwood, Esq., *of Meadowbank.*
William P. Alison, M.D., *Emeritus Professor of the Practice of Physic.*
Robert Bald, Esq., *Civil Engineer.*
1818 Patrick Miller, M.D., *Exeter.*
John Watson, M.D.
Right Honourable John Hope, *Lord Justice-Clerk.*
1819 Patrick Murray, Esq., *of Simprim.*
Thomas Stewart Traill, M.D., *Professor of Medical Jurisprudence.*
Alexander Adie, Esq.

Date of Election.

1819 George Forbes, Esq.
1820 James Keith, M.D., *Surgeon.*
Charles Babbage, Esq., F.R.S. Lond.
Sir John F. W. Herschel, Bart., F.R.S. Lond.
John Shank More, Esq., *Professor of Scots Law.*
Dr William Macdonald, *Professor of Natural History, St Andrews.*
Sir John Hall, Bart., *of Dunglass.*
1821 Sir James M. Riddell, Bart., *Strontian.*
John Lizars, Esq., *Surgeon.*
John Cay, Esq., *Advocate.*
Robert Kaye Greville, LL.D.
Robert Hamilton, M.D.
1822 James Smith, Esq. *of Jordanhill*, F.R.S. Lond.
William Bonar, Esq.
George A. Walker-Arnott, LL.D., *Professor of Botany, Glasgow.*
Very Rev. John Lee, D.D., *Principal of the University of Edinburgh.*
Sir James South, F.R.S. Lond.
Sir W. C. Trevelyan, Bart., *Wallington, Northumberland.*
John Russell, Esq., P.C.S.
1823 Captain Thomas David Stuart, *of the Hon. East India Company's Service.*
Andrew Fyfe, M.D., *Professor of Medicine and Chemistry, King's College, Aberdeen.*
Robert Bell, Esq., *Advocate.*
Admiral Norwich Duff.
Warren Hastings Anderson, Esq.
Alexander Thomson, Esq., *of Banchory.*
Liscombe John Curtis, Esq., *Ingsdon House, Devonshire.*
Robert Christison, M.D., *Professor of Materia Medica.*
John Gordon, Esq., *of Cairnbulg.*
1824 Robert E. Grant, M.D., *Professor of Comparative Anatomy, University College, London.*
Rev. Dr William Muir, *one of the Ministers of Edinburgh.*
James Pillans, Esq.
James Walker, Esq., *Civil Engineer.*
William Wood, Esq., *Surgeon.*
1825 Honourable Lord Wood.
1826 Sir David Hunter Blair, Bart., *Blairquhan, Ayrshire.*
1827 John Gardiner Kinnear, Esq.
James Russell, M.D.
Very Rev. Edward Bannerman Ramsay, A.M., Camb.
1828 Erskine Douglas Sandford, Esq., *Advocate.*
David Maclagan, M.D.
Sir William A. Maxwell, *of Calderwood*, Bart.
John Forster, Esq., *Architect, Liverpool.*
Thomas Graham, A.M., *Professor of Chemistry, London University.*
David Milne Home, Esq., *Advocate.*

Date of Election.
1328 Dr Manson, *Nottingham.*
1829 A. Colyar, Esq.
Sir William Gibson-Craig, Bart., *of Riccarton.*
Right Honourable Duncan M'Neill, *Lord Justice-General.*
Venerable Archdeacon Sinclair, *Kensington.*
Arthur Connell, Esq., *Professor of Chemistry, St Andrews.*
James Walker, Esq., W.S.
1830 J. T. Gibson-Craig, Esq., W.S.
Sir Archibald Alison, Bart., *Sheriff of Lanarkshire.*
Honourable Mountstuart Elphinstone.
James Syme, Esq., *Professor of Clinical Surgery.*
Thomas Barnes, M.D., *Carlisle.*
1831 James D. Forbes, D.C.L., F.R.S. Lond., *Professor of Natural Philosophy.*
Right Honourable Lord Dunfermline.
David Boswell Reid, M.D., *London.*
1832 John Sligo, Esq., *of Carmyle.*
William Gregory, M.D., *Professor of Chemistry.*
Robert Allan, Esq., *Advocate.*
Robert Morrieson, Esq., *Hon. E.I.C. Civil Service.*
Montgomery Robertson, M.D.
1833 Captain Milne, R.N.
His Grace the Duke of Buccleuch, K.G., *Dalkeith Palace.*
David Craigie, M.D.
Sir John Stuart Forbes, Bart., *of Pitsligo.*
Alexander Hamilton, LL.B., W.S.
Right Honourable Earl Cathcart.
1834 Mungo Ponton, Esq., W.S., *Clifton, Bristol.*
Isaac Wilson, M.D., F.R.S. Lond.
Professor Low.
Patrick Boyle Mure Macredie, Esq., *Advocate, of Piercetown.*
John Davies Morries Stirling, Esq.
Thomas Jameson Torrie, Esq.
John Haldane, Esq., *Haddington.*
William Sharpey, M.D., *Professor of Anatomy, University College, London.*
1835 John Hutton Balfour, M.D., A.M., F.R.S. Lond., *Professor of Botany.*
Right Honourable Lord Campbell.
William Brown, Esq., F.R.C.S.
R. Mayne, Esq.
1836 David Rhind, Esq., *Architect.*
Archibald Robertson, M.D., F.R.S. Lond.
1837 John Archibald Campbell, Esq., W.S.
John Scott Russell, Esq., A.M , *London.*
Charles Maclaren, Esq.
Archibald Smith, Esq., M.A., Camb., *Lincoln's Inn, London.*

| Date of Election. | |
|---|---|
| 1837 | Richard Parnell, M.D. |
| | Peter D. Handyside, M.D., F.R.C.S. |
| 1838 | Thomas Mansfield, Esq., *Accountant.* |
| | Alan Stevenson, Esq., *Civil Engineer.* |
| 1839 | David Smith, Esq., W.S. |
| | Adam Hunter, M.D. |
| | Rev. Philip Kelland, A.M., *Professor of Mathematics.* |
| | William Alexander, Esq., W.S. |
| | F. Brown Douglas, Esq., *Advocate.* |
| | Colonel Swinburne, *of Mearns.* |
| 1840 | Alan A. Welwood Maconochie, Esq. |
| | Martyn J. Roberts, Esq., *Fort-William.* |
| | Robert Chambers, Esq. |
| | James Forsyth, Esq., *of Dunach.* |
| | Sir John M'Neill, G.C.B. |
| | John Cockburn, Esq. |
| | Sir William Scott, Bart., *of Ancrum.* |
| | Right Rev. Bishop Terrot. |
| | Edward J. Jackson, Esq. |
| | John Learmonth, Esq., *of Dean.* |
| | John Mackenzie, Esq. |
| | James Anstruther, Esq., W.S. |
| 1841 | John Millar, Esq., *Civil Engineer, Millfield House, Polmont.* |
| | George Smyttan, M.D. |
| | James Dalmahoy, Esq. |
| 1842 | James Thomson, Esq., *Civil Engineer, Milford, Pembrokeshire.* |
| | John Davy, M.D., *Inspector-General of Army Hospitals.* |
| | Robert Nasmyth, Esq., F.R.C.S. |
| | Sir James Forrest, Bart., *of Comiston.* |
| | James Miller, Esq., *Professor of Surgery.* |
| | John Goodsir, Esq.. *Professor of Anatomy.* |
| 1843 | A. D. Maclagan, M.D., F.R.C.S. |
| | John Rose Cormack, M.D., F.R.C.P., *Putney.* |
| | Allen Thomson, M.D., *Professor of Anatomy, Glasgow.* |
| | Joseph Mitchell, Esq., *Civil Engineer, Inverness.* |
| | Andrew Coventry, Esq., *Advocate.* |
| | John Hughes Bennett, M.D., F.R.C.P., *Professor of Physiology.* |
| | D. Balfour, Esq., *of Trenaby.* |
| | Henry Stephens, Esq. |
| 1844 | The Honourable Lord Murray. |
| | J. Burn Murdoch, Esq., *Advocate, of Gartincaber.* |
| | Archibald Campbell Swinton, Esq., *Professor of Civil Law.* |
| | James Begbie, M.D., F.R.C.S. |
| | James Y. Simpson, M.D., *Professor of Midwifery.* |

Date of Election.

1844 David Stevenson, Esq., *Civil Engineer.*
Thomas R. Colledge, M.D., F.R.C.P.E.
1845 James Andrew, M.D.
George Wilson, M.D., *Professor of Technology.*
John G. M. Burt, M.D.
Thomas Anderson, M.D., *Professor of Chemistry, Glasgow.*
1846 A. Taylor, M.D., *Pau.*
S. A. Pagan, M.D.
Rev. Dr James Robertson, *Professor of Divinity and Ecclesiastical History.*
Alexander J. Adie, Esq., *Civil Engineer.*
L. Schmitz, LL.D., Ph.D., *Rector of High School.*
Charles Piazzi Smyth, Esq., *Professor of Practical Astronomy.*
1847 George Makgill, Esq., *of Kemback.*
William Thomson, Esq., M.A. Camb., *Professor of Natural Philosophy, Glasgow.*
J. H. Burton, Esq., *Advocate.*
James Nicol, Esq., *Professor of Natural History, Aberdeen.*
William Macdonald Macdonald, Esq., *of St Martins.*
Honourable Lord Handyside.
Alexander Christie, Esq.
John Wilson, Esq., *Professor of Agriculture.*
Moses Steven, Esq., *of Bellahouston.*
1848 James Tod, Esq., W.S., *Secretary to the Royal Scottish Society of Arts.*
Thomas Stevenson, Esq., C.E.
James Allan, M.D., *Inspector of Hospitals, Portsmouth.*
Henry Davidson, Esq.
Patrick Newbigging, M.D.
William Swan, Esq.
Patrick James Stirling, Esq.
1849 William Stirling, Esq., *of Keir*, M.P.
John Thomson Gordon, Esq., *Sheriff of Mid-Lothian.*
D. R. Hay, Esq.
William Thomas Thomson, Esq.
Honourable Lord Ivory.
William E. Aytoun, D.C.L., *Professor of Rhetoric and Belles Lettres.*
W. H. Lowe, M.D., *Balgreen.*
Honourable B. F. Primrose.
John Stenhouse, M.D., *Islington.*
David Anderson, Esq., *of Moredun.*
W. R. Pirrie, M.D., *Professor of Surgery, Marischal College, Aberdeen.*
Right Honourable The Earl of Minto, G.C.B., *Minto House.*
Right Honourable The Earl of Aberdeen, K.T., *Haddo House.*
Right Honourable The Earl of Haddington, K.T., *Tyninghame.*
His Grace The Duke of Argyll, *Inverary Castle.*
The Most Noble the Marquis of Tweeddale, K.T.

| Date of Election. | |
|---|---|
| 1849 | Edward Sang, Esq. |
| 1850 | William John Macquorn Rankine, Esq., C.E., *Professor of Civil Engineering, Glasgow University.* |
| | Alexander Keith Johnston, Esq. |
| | Sheridan Muspratt, M.D., *Liverpool.* |
| | James Stark, M.D. (Re-admitted.) |
| | Captain W. Driscoll Gossett, R.E. |
| | William Seller, M.D., F.R.C.P.E. |
| | Hugh Blackburn, Esq., *Professor of Mathematics, Glasgow.* |
| | R. D. Thomson, M.D., *London.* |
| | Mortimer Glover, M.D., *Newcastle.* |
| | Beriah Botfield, Esq., *Norton Hall, Northamptonshire.* |
| | J. S. Combe, M.D. |
| 1851 | Sir David Dundas, Bart., *of Dunira.* |
| | Sir George Douglas, Bart., *of Springwood Park.* |
| | John Stewart, Esq., *of Nateby Hall.* |
| | E. W. Dallas, Esq. |
| | Rev. James Grant, D.C.L., D.D., *one of the Ministers of Edinburgh.* |
| | Sir James Ramsay, Bart., *Bamff House, Alyth.* |
| 1852 | Eyre B. Powell, Esq., *Madras.* |
| | Thomas Miller, Esq., A.M., LL.D., *Rector, Perth Academy.* |
| | Allen Dalzell, M.D. |
| | James Cunningham, Esq., W.S. |
| | Alexander James Russell, Esq., C.S. |
| | Andrew Fleming, M.D., *Bengal.* |
| 1853 | James Watson, M.D., *Bath.* |
| | Capt. Robert Maclagan, *Bengal Engineers.* |
| | Rev. Dr Robert Lee, *Professor of Biblical Criticism and Biblical Antiquities.* |
| | James M. Hog, Esq., *of Newliston.* |
| | Rev. John Cumming, D.D., *London.* |
| | Hugh Scott, Esq., *of Gala.* |
| | Græme Reid Mercer, Esq. |
| | Dr George Buist, *Bombay.* |
| 1854 | Sir John Maxwell, Bart., *of Polloc.* |
| | William Murray, Esq., *of Monkland.* |
| | Dr John Addington Symonds, *Clifton, Bristol* |
| | Henry Dunlop, Esq., *of Craigton.* |
| | Dr William Bird Herapath, *Bristol.* |
| | Robert Harkness, Esq., *Professor of Mineralogy and Geology, Queen's College, Cork.* |
| | Thomas A. Wise, M.D. |
| | James Coxe, M.D. |
| | Ernest Bonar, Esq. |
| 1855 | James P. Fraser, Esq. |
| | Stevenson Macadam, Ph.D. |

| Date of Election. | |
|---|---|
| 1855 | Robert Etheridge, Esq., *Clifton, Bristol.* |
| | John Inglis, Esq., *Dean of Faculty.* |
| | Rev. James S. Hodson, D.D., *Oxon., Rector of the Edinburgh Academy.* |
| | Wyville T. C. Thomson, LL.D., *Professor of Geology, Belfast.* |
| | Sir Robert K. Arbuthnot, Bart. |
| | Dr Wright, *Cheltenham.* |
| | James Hay, Esq. |
| | R. M. Smith, Esq. |
| 1856 | David Bryce, Esq. |
| | William Mitchell Ellis, Esq. |
| | George J. Allman, M.D., *Professor of Natural History.* |
| | Honourable Lord Neaves. |
| | Dr Frederick Penny. |
| | Thomas Laycock, M.D., *Professor of the Practice of Medicine.* |
| | Thomas Cleghorn, Esq. |
| | James Clerk Maxwell, Esq., *Professor of Natural Philosophy, Marischal College, Aberdeen.* |
| 1857 | Horatio Ross, Esq. |
| | James Black, M.D. |
| | John Ivor Murray, M.D. |
| | John Melville, Esq., W.S. |
| | John Blackwood, Esq. |
| | Brinsley De Courcy Nixon, Esq. |
| | Andrew Murray, Esq., *of Conland,* W.S. |
| | Reverend Dr James Macfarlane, *Duddingston.* |
| | W. M. Buchanan, M.D. |
| | Thomas Login, Esq., C.E., *Pegu.* |

## LIST OF NON-RESIDENT AND FOREIGN MEMBERS.

ELECTED UNDER THE OLD LAWS.

### NON-RESIDENT.

Richard Griffiths, Esq., *Civil Engineer.*

## LIST OF HONORARY FELLOWS.

His Majesty the King of the Belgians.
His Imperial Highness the Archduke John of Austria.
His Imperial Highness the Archduke Maximilian.
His Royal Highness the Prince Consort.

FOREIGNERS (LIMITED TO THIRTY-SIX.)

| | |
|---|---|
| * M. Biot, | *Paris.* |
| * M. de Hammer, | *Vienna.* |
| * M. de Humboldt, | *Berlin.* |
| M. Agassiz, | *United States.* |
| M. Cousin, | *Paris.* |
| M. Dumas, | *Do.* |
| M. Charles Dupin, | *Do.* |
| M. Ehrenberg, | *Berlin.* |
| M. Elie de Beaumont, | *Paris.* |
| M. Encke, | *Berlin.* |
| M. Flourens, | *Paris.* |
| M. Guizot, | *Do.* |
| M. Haidinger, | *Vienna.* |
| M. Hansteen, | *Christiania.* |
| M. Hausmann, | *Göttingen.* |
| M. Lamont, | *Munich.* |
| M. Leverrier, | *Paris.* |

*N.B.*—The three names marked thus * in the preceding list, were included in the original Honorary List prior to the change of the Law distinguishing British Subjects from Foreigners.

| | |
|---|---|
| M. Liebig, | *Munich.* |
| Dr Von Martius, | *Do.* |
| M. Milne-Edwards, | *Paris.* |
| M. Mitscherlich, | *Berlin.* |
| M. Müller, | *Do.* |
| M. Necker, | *Geneva.* |
| M. Plana, | *Turin.* |
| M. Quetelet, | *Brussels.* |
| M. Regnault, | *Paris.* |
| Prof. Henry D. Rogers, | *Pennsylvania.* |
| M. Gustav Rose, | *Berlin.* |
| M. Studer, | *Berne.* |
| M. Struve, | *Pulkowa.* |
| M. Thenard, | *Paris.* |
| M. Tiedemann, | *Heidelberg.* |

BRITISH SUBJECTS (LIMITED TO TWENTY, BY LAW X.)

| | |
|---|---|
| J. C. Adams, Esq., | *Cambridge.* |
| G. B. Airy, Esq., | *Greenwich.* |
| Robert Brown, Esq., | *London.* |
| Dr Faraday, | *Do.* |
| Thomas Graham, Esq., | *Do.* |
| Henry Hallam, Esq., | *Do.* |
| Sir W. R. Hamilton, | *Dublin.* |
| Sir John F. W. Herschel, Bart., | *Collingwood.* |
| Sir William J. Hooker, | *Kew.* |
| W. Lassell, Esq., | *Liverpool.* |
| Rev. Dr Lloyd, | *Dublin.* |
| Sir Charles Lyell, | *London.* |
| Sir Roderick I. Murchison, | *Do.* |
| Richard Owen, Esq., | *Do.* |
| Sir John Richardson, M.D., | *Lancrig, Westmoreland.* |
| Earl of Rosse, | *Parsonstown.* |
| Robert Stephenson, Esq., | *London.* |
| Rev. Dr Whewell, | *Cambridge.* |

# LIST OF FELLOWS DECEASED, RESIGNED, AND CANCELLED

FROM SEPTEMBER 1853 TO AUGUST 1857.

---

## HONORARY FELLOWS DECEASED.

M. Arago, *Paris.*
M. de Bernstein, *Berlin.*
M. Gauss, *Göttingen.*
M. Melloni, *Naples.*
Sir John Franklin, *London.*
Sir W. E. Parry, *Do.*
M. Cauchy, *Paris.*
M. de Charpentier, *Bex.*
M. Degerando.

## ORDINARY FELLOWS DECEASED.

Robert Jameson, Esq., *Professor of Natural History.*
John Campbell, Esq., *of Carbrook.*
J. G. Children, Esq., F.R.S.
W. A. Cadell, Esq., F.R.S.
Alexander Brunton, D.D.
Honourable Lord Fullarton.
John Wilson, Esq., *late Professor of Moral Philosophy.*
Robert Richardson, M.D., *Harrowgate.*
Richard Philips, Esq., F.R.S.
Rev. Dr William Scoresby, *Exeter.*
Robert Haldane, D.D., *Principal of St Mary's College, St Andrews.*
Sir George Ballingall, M.D., *Professor of Military Surgery.*
Archibald Bell, Esq., *Advocate.*
John Clerk Maxwell, Esq., *Advocate.*
Lieutenant-General Martin White.
Walter Frederick Campbell, Esq.
Sir Robert Abercromby, Bart., *of Birkenbog.*
Dr Wallich, *Calcutta.*
John Dewar, Esq., *Advocate.*
Sir Edward Ffrench Bromhead, Bart., A.M., F.R.S., *Thurlsby Hall.*
Alexander Wilson Philip, M.D., *London.*
W. H. Playfair, Esq., *Architect.*
John Argyle Robertson, Esq., *Surgeon.*

Dr John Macwhirter.

Rev. Dr Robert Gordon, *one of the Ministers of Edinburgh.*

James Wilson, Esq.

George Swinton, Esq.

William Burn Callander, Esq., *of Prestonhall.*

James Ewing, LL.D., *Glasgow.*

Bindon Blood, Esq., M.R.I.A.

William Bald, Esq., M.R.I.A.

James L'Amy, Esq., *Sheriff of Forfarshire.*

Donald Smith, Esq.

O. Tyndal Bruce, Esq., *of Falkland.*

James F. W. Johnston, A.M., *Professor of Chemistry in the University of Durham.*

John Adie, Esq.

William Murray, Esq., *of Henderland.*

George Turnbull, Esq.

David Gray, Esq., *Professor of Natural Philosophy, Marischal College, Aberdeen.*

Right Honourable Lord Rutherfurd.

Honourable Lord Anderson.

Alexander Kemp, Esq.

Colonel Edward Madden.

Dr Marshall Hall.

## RESIGNATIONS.

Arthur Forbes, Esq., *of Culloden.*

Rev. John Hannah, D.C.L., *late Rector of Edinburgh Academy.*

Rev. Francis Garden.

Rev. A. Barry, *Glenalmond.*

James W. Grant, Esq., *of Elchies.*

John S. Blackie, Esq., M.A., *Professor of Greek.*

Right Rev. Bishop Trower, D.D.

## ELECTION CANCELLED.

Duncan Davidson, Esq., *of Tulloch.*

*The following Public Institutions and Individuals are entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh:—*

ENGLAND.

The British Museum.
The Bodleian Library, Oxford.
The University Library, Cambridge.

---

The Royal Society.
The Linnean Society.
The Society for the Encouragement of Arts.
The Geological Society.
The Royal Astronomical Society.
The Royal Asiatic Society.
The Zoological Society.
The Royal Society of Literature.
The Horticultural Society.
The Royal Institution.
The Royal Geographical Society.
The Statistical Society.
The Institution of Civil Engineers.
The Institute of British Architects.
The Ordnance Geological Survey.
The Hydrographical Office, Admiralty.
The Medico-Chirurgical Society.
The Athenæum Club.
The Cambridge Philosophical Society.
The Manchester Literary and Philosophical Society.
The Yorkshire Philosophical Society.
The Chemical Society of London.
The Museum of Economic Geology.
The United Service Institution.
The Royal Observatory, Greenwich.
The Leeds Philosophical and Literary Society.
The Historic Society of Lancashire and Cheshire.
The Royal College of Surgeons of England.

SCOTLAND.

Edinburgh, University Library.
... Advocates' Library.
Edinburgh, College of Physicians.
... Highland and Agricultural Society.
... Royal Medical Society.
... Royal Physical Society.
... Royal Scottish Society of Arts.
Glasgow, University Library.
St Andrews, University Library.
Aberdeen, Library of King's College.

IRELAND.

The Library of Trinity College, Dublin.
The Royal Irish Academy.

COLONIES, &c.

The Asiatic Society of Calcutta.
The Literary and Historical Society of Toronto.

CONTINENT OF EUROPE.

Amsterdam, Royal Institute of Holland.
Berlin, Royal Academy of Sciences.
... Physical Society.
Berne, Society of Swiss Naturalists.
Bologna, Academy of Sciences.
Bonn, Cæsarean Academy of Naturalists.
Brussels, Royal Academy of Sciences.
Buda, Literary Society of Hungary.
Copenhagen, Royal Academy of Sciences.
Frankfort, the Senkenbergian Museum.
Geneva, Natural History Society.
Giessen, University Library.
Göttingen, University Library.
Haarlem, Natural History Society.
Leipzig, Royal Saxon Academy.
Lille, Royal Society of Sciences.
Lisbon, Royal Academy of Sciences.
Lyons, Agricultural Society.
Milan, Royal Institute.

Moscow, Imperial Academy of Naturalists.
Munich, Royal Academy of Sciences of Bavaria (2 copies).
Neufchatel, Museum of Natural History.
Paris, Royal Academy of Sciences.
... Geographical Society.
... Royal Society of Agriculture.
... Society for Encouragement of Industry.
... Geological Society of France.
... Ecole des Mines.
... Marine Depôt.
... Museum of Jardin des Plantes.
Rotterdam, Batavian Society of Experimental Philosophy.
Stockholm, Royal Academy of Sciences.
St Petersburg, Imperial Academy of Sciences.
... M. Kupffer.
... Pulkowa Observatory.
Turin, Royal Academy of Sciences.
... M. Michelotti.
Venice, Royal Institute.

UNITED STATES OF AMERICA.

Boston, the Bowditch Library.
New York State Library.
Philadelphia, American Philosophical Society.
Yale College, Professor Silliman.
Washington, the Smithsonian Institution.

*(All the Honorary and Ordinary Fellows of the Society are entitled to the Transactions and Proceedings.)*

---

## *The following Institutions and Individuals receive the Proceedings only :—*

ENGLAND.

The Scarborough Philosophical Society.
The Whitby Philosophical Society.
The Newcastle Philosophical Society.
The Geological Society of Cornwall.
The Ashmolean Society of Oxford.
The Literary and Philosophical Society of Liverpool.

SCOTLAND.

The Philosophical Society of Glasgow.

COLONIES.

The Literary and Philosophical Society of Quebec.

CONTINENT OF EUROPE.

Utrecht, the Literary and Philosophical Society.

UNITED STATES.

Professor Dana, Connecticut.
Academy of Natural Science, Philadelphia.

# LIST OF DONATIONS.

(Continued from Vol. XX. p. 663.)

---

*December* 5, 1853.

| DONATIONS. | DONORS. |
|---|---|
| Memoirs of the Royal Astronomical Society. Vol. xxi., Parts 1 and 2. 4to. | The Society. |
| Proceedings of the American Association for the Advancement of Science. Sixth Meeting, held at Albany (N. Y.) August 1851. 8vo. | The Association. |
| Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen. V. Band für 1851 & 1852. 4to. | The Society. |
| Mémoires de l'Academie des Sciences de l'Institut de France. Tome xxiii. 4to. | The Institute. |
| Abhandlungen der Philosoph-Philologischen Classe der Königl. Bayerischen Akademie der Wissenschaften. Band xvii., 1[ste] Abtheil. 4to. | The Academy. |
| Nouveaux Mémoires de la Société Hélvetique des Sciences Naturelles. Tome xii. 4to. | The Society. |
| Mittheilungen der Naturforschen Gesellschaft in Bern. 1851. N[r] 195–257. 8vo. | Ditto. |
| Verhandlungen der Schweizerischen Naturforschenden Gesellschaft bei ihrer 36[sten] versammlung in Glarus. 1851. 8vo. | Ditto. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. B[de] 4 & 5. 4to. | The Academy. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. B[de] 9 & 10. 8vo. | Ditto. |
| Abhandlungen der Kaiserlich Geologischen Reichsanstalt. Band. i. 1852. Fol. | The Institute. |
| Astronomical and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1851. 4to. | The Society. |
| The Assurance Magazine and Journal of the Institute of Actuaries. Nos. 12 and 13. 8vo. | The Institute. |
| Journal of the Asiatic Society of Bengal. Edited by the Secretaries. Nos. 230–234. 8vo. | The Society. |
| Journal of the Geological Society of Dublin. Vol. v., Part 3. 8vo. | Ditto. |
| Journal of the Horticultural Society of London. Vol. viii., Parts 2 and 3. 8vo. | Ditto. |
| Journal of the Statistical Society of London. Vol. vi., Parts 1, 2, and 3. 8vo. | Ditto. |
| The Quarterly Journal of the Geological Society. Vol. ix., Parts 2 and 3. 8vo. | Ditto. |
| Journal of the Royal Asiatic Society of Great Britain and Ireland. Vol. xv., Part 1. 8vo. | Ditto. |
| The Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. No 41 (N. S.) 8vo. | Ditto. |
| The Twentieth Annual Report of the Royal Cornwall Polytechnic Society. 1852. 8vo. | Ditto. |
| Silliman's American Journal of Science and Arts. Nos. 44, 45, and 46. 8vo. | The Editors. |

| DONATIONS. | DONORS. |
|---|---|
| Transactions of the Pathological Society of London. Vol. iv. 8vo. | The Society. |
| Memoirs of the Literary and Philosophical Society of Manchester. 2d Series. Vol. x. 8vo. | Ditto. |
| Catalogue of the Birds in the Museum of the Asiatic Society of Bengal. By Edward Blyth. 8vo. | Ditto. |
| Transactions of the American Philosophical Society, held at Philadelphia, for promoting Useful Knowledge. (N. S.) Vol. x., Part 2. 4to. | Ditto. |
| Observations made at the Magnetical and Meteorological Observatory at Hobart Town, in Van Diemen Island. Printed by order of Her Majesty's Government, under the superintendence of Colonel Edward Sabine. Vol. iii. 4to. | Her Majesty's Government. |
| Observations made at the Magnetical and Meteorological Observatory at Toronto, in Canada. Printed by order of Her Majesty's Government, under the superintendence of Colonel Edward Sabine. Vol. ii. 4to. | Ditto. |
| Observations made at the Magnetical and Meteorological Observatory at Bombay. Printed by order of the Honourable East India Company, under the superintendence of Arthur Bedford Orlebar, M.A. 1845, 1846, 1847, and 1848. 4to. | The Hon. E.I.C. |
| Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. 1852. 4to. | The Society. |
| Monatsbericht der Königl. Preuss. Akademie der Wissenschaften zu Berlin. November 1852—Julii 1853. 8vo. | Ditto. |

*January* 3, 1854.

| | |
|---|---|
| Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. No. 43. N. S. 8vo. | The Society. |
| Medico-Chirurgical Transactions. Published by the Royal Medical and Chirurgical Society of London. Vol. xxxvi. 8vo. | Ditto. |
| Mémoires de l'Académie Impériale des Sciences de St Pétersbourg. Sciences. Mathématiques et Physiques. Tome v., 5 & 6 Liv. 4to. | The Academy. |
| Astronomische Beobachtungen auf der Königlichen Universitäts Sternwarte in Königsberg. Angestellt und herausgegeben von Dr A. L. Busche. 25te Abtheilung. Fol. | The Observatory. |

*March* 6, 1854.

| | |
|---|---|
| Lectures on Quaternions. By Sir William R. Hamilton. 8vo. | The Author. |
| Fourth Report of the Council of Management of the Architectural Institute of Scotland. 8vo. | The Institute. |
| Memoirs of the American Academy of Arts and Sciences (N. S.) Vol. v. Part 1. With Map of Toronto. 4to. | The Academy. |
| Proceedings of the American Academy of Arts and Sciences. Vol. ii. From May 1848 to May 1852. 8vo. | Ditto. |
| Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. No. 44. (N. S.) 8vo. | The Society. |
| Journal of the Statistical Society of London. Vol. xvi., Part 4. 8vo. | Ditto. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Second Series. No. 49. 8vo. | The Editors. |
| Journal of the Horticultural Society of London. Vol. ix., Part 1. 8vo. | The Society. |
| Journal of the Asiatic Society of Bengal. Edited by the Secretaries. No. 5. 1853. 8vo. | Ditto. |
| The Assurance Magazine, and Journal of the Institute of Actuaries. No. 14. 8vo. | The Institute. |
| Thirty-third Report of the Council of the Leeds Philosophical and Literary Society. 1852-53. 8vo. | The Society. |
| Jahresbericht über die Fortschritte der reinen, Pharmaceutischen und Technischen Chemie, Physik, Mineralogie und Geologie, &c. Herausgegeben von Justus Liebig et Hermann Kopp. 1847-50. 8vo. | The Editor. |

| DONATIONS. | DONORS. |
| --- | --- |
| Bulletins de l'Academie Royale des Sciences, des Lettres et des Beaux Arts de Belgique. Tome xx. 8vo. | The Academy. |
| Flora Batava. 174 Aflevering. 4to. | King of Holland. |
| Mémoires Couronnées et Mémoires des Savants Etrangers, publiés par l'Académie Royale des Sciences de Belgique. Tome v. 2$^{de}$ Partie. 8vo. Two copies. | The Academy. |
| Memorie della Accademia delle Scienze dell' Istituto di Bologna. Tome iii. 4to. | Ditto. |
| Acta Societatis Scientiarum Fennicæ. Tom. iii., Fasciculus 2. 4to. | The Society. |
| Notiser ur Sällskapets pro Fauna et Flora Fennica Förhandlingar. Pt. 2. 4to. | Ditto. |
| Rendiconto delle Sessioni dell' Accademia delle Scienze dell' Istituto di Bologna. 1851-2. 8vo. | The Academy. |
| Mémoires de l'Académie Royale des Sciences, des Lettres, et des Beaux Arts de Belgique. Tome xxvii. 4to. | Ditto. |
| Mémoires sur les Variations Périodiques et non Périodiques de la Témpérature. Par A. Quételet. 4to. | The Author. |
| Observations de Phénomènes Periodiques. Par A. Quételet. 4to. | Ditto. |
| Mémoires de la Société des Sciences Naturelles de Cherbourg. 1$^{er.}$ Vol. 2$^{e}$ Liv. 8vo. | The Society. |
| A History of the Fishes of Massachusetts. By David Humphreys Storer, M.D., A.A.S. 4to. | The Author. |
| Maritime Conference, held at Brussels, for devising an uniform System of Meteorological Observations at Sea, August and September 1853. 4to. | The Belgian Academy. |

*March* 20, 1854.

| | |
| --- | --- |
| The Quarterly Journal of the Geological Society. Vol. x., Part 1. 8vo. | The Society. |
| Journal of the Statistical Society of London. Vol. xvii., Part 1. 8vo. | Ditto. |
| Journal of the Royal Geographical Society. Vol. xxiv. 8vo. | Ditto. |
| General Index to the Second Ten Volumes of the Journal of the Royal Geographical Society. 8vo. | Ditto. |
| Proceedings of the Literary and Philosophical Society of Liverpool. 1851-3. No. 7. 8vo. | Ditto. |
| A Collection of Tables, Astronomical, Meteorological, and Magnetical. By Lieut.-Colonel J. T. Boileau. 4to. 5 Copies. | The Directors of Hon. E.I.C. |
| Mémoires de l'Academie Nationale des Sciences, Belles Lettres, et Arts de Lyon. Classe des Lettres. Tome 1$^{er}$. 8vo. | The Academy. |
| Mémoires de l'Academie Nationale des Sciences, Belles Lettres, et Arts de Lyon. Classe des Sciences. Tome 1$^{er}$. 8vo. | Ditto. |

*April* 3, 1854.

| | |
| --- | --- |
| Smithsonian Contributions to Knowledge. Vol. v. 4to. | The Institution. |
| Sixth Annual Report of the Board of Regents of the Smithsonian Institution for the year 1851. 8vo. | Ditto. |
| Smithsonian Institution Meteorological Tables. Prepared by Arnold Guyot. 8vo. | Ditto. |
| Portraits of North American Indians. With Sketches of Scenery, &c. Painted by J. M. Stanley. Deposited with the Smithsonian Institution. 8vo. | Ditto. |
| Catalogue of North American Reptiles in the Museum of the Smithsonian Institution. 8vo. | Ditto. |
| Owen's Geological Survey of Wisconsin, Iowa, and Minnesota. With Illustrations. 4to. | The American Government. |
| Schoolcraft's History of the Indian Tribes of the United States. Part 3. 4to. | Ditto. |
| Memoirs and Maps of California. By Ringgold. 8vo. | Ditto. |
| Stanbury's Expedition to the Great Salt Lake. 8vo. With Maps. | Ditto. |
| Report on the Geology of the Lake Superior Land District. By J. W. Foster and J. D. Whitney. Part 2. 8vo. With Maps. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| Official Report of the United States Expedition to explore the Dead Sea and the River Jordan. By Lieut. W. F. Lynch, U.S.N. 4to. | The American Government. |
| Boston Journal of Natural History, containing Papers and Communications read before the Boston Society of Natural History. Vol. vi., Nos. 1 and 2. 8vo. | The Editors. |
| Bulletin de la Société Imperiale des Naturalistes de Moscou. 1851, Nos. 3 & 4; 1852, Nos. 1. 8vo. | The Society. |
| Bulletin de la Société de Géographie. 4ieme Série. Tomes iv. & v. 8vo. | Ditto. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt. 1852, iii. Jahrgang. 1853, iv. Jahrgang. 8vo. | The Institute. |
| Flora Batava. 173 Aflevering. 4to. | King of Holland. |
| Stellarum Fixarum imprimis duplicium et multiplicium positiones mediæ pro epocha 1830,0. Auctore F. G. W. Struve. Fol. | The Russian Government. |
| Mémoire sur les Ouragans de la Mer des Indes, au sud de l'Equateur. Par M. A. Lefebre. 8vo. | Depôt Général de la Marine. |
| Considérations générales sur l'Océan Pacifique pour fair suite à celles sur l'Océan Atlantique et sur l'Océan Indien. Par M. Charles P. de Kerhallet. 8vo. | Ditto. |
| Tableau général des Phares et Fanaux des Cotes de la Méditerranée, de la Mer Noire, et de la Mer d'Azof. 8vo. | Ditto. |
| Abhandlungen der Mathemat. Physikalischen Classe der Koeniglich Bayerischen Akademie der Wissenschaften. Bd. vii., 1 Abth. 4to. | The Academy. |
| Annalen der Königlichen Sternwarte bei München. V. Bd. 8vo. | The Observatory. |
| Jahres Bericht der Münchener Sternwarte für 1852. 8vo. | Ditto. |
| Afrika vor den Entdeekungen der Portugiesen. Von Dr Friedrick Kuntsmann. 4to. | The Author. |
| Studien des Göttingischen Vereins Bergmännischer Freunde. Im namen desselben herausgegeben von J. F. L. Hausmann. Bd. xvi. Heft. 1 & 2. 8vo. | The Society. |
| Nachrichten von der Georg-Augusts Universität und der Königl. Gesellschaft der Wissenschaften zu Göttingen. 1852. 12mo. | Ditto. |

## *May* 1, 1854.

| | |
|---|---|
| Archives du Muséum d'Histoire Naturelle. Publiées par les Professeurs de cet établissement. Tome vii., Liv. 1 et 2. 4to. | The Museum. |
| Actuarial Tables; Carlisle Three-per-Cent. Single Lives and Single Deaths. With Auxiliary Tables. By William Thomas Thomson, F.R.S.E., F.I.A. 4to. | The Author. |
| On the application of Cast and Wrought Iron for Building Purposes. By William Fairbairn, C.E. 8vo. | Ditto. |
| Jahrbuch der Kaiserlich Königlichen Geologischen Reichsanstalt. 1853. iv. Jahrgang. 8vo. | The Institute. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Band vi. 4to. | The Academy. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Band xi. 8vo. | Ditto. |
| Mémoires de l'Académie de Sciences de l'Institut de France. Tome xxiv. 4to. | Ditto. |
| Memoirs of the Geological Survey of the United Kingdom. British Organic Remains. Parts 1, 2, 3, 4, 6, 7. 4to. | H. M. Government. |
| Memoirs of the Geological Survey of Great Britain, and of the Museum of Practical Geology in London. Vol. ii., Parts 1 and 2. 8vo. | Ditto. |
| Museum of Practical Geology and Geological Survey. Records of the School of Mines and of Science applied to the Arts. Parts 1, 2, 3, 4. 8vo. (With various Maps and Pamphlets). | Ditto. |
| Journal of the Geological Society of Dublin. Vol. vi., Part 1. 8vo. | The Society. |
| The Assurance Magazine, and Journal of the Institute of Actuaries. Vol. iv., Part 3. 8vo. | The Institute. |
| Silliman's American Journal of Science and Arts. Vol. xvii., No. 50. 8vo. | The Editors. |
| Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences. Mai 1853–Mai 1854. 4to. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Memorie della Reale Accademia delle Scienze di Torino. Tom. xiii. 4to. | The Academy. |
| Bulletins de la Société de Géographie. Tome v. 8vo. | The Society. |
| Annales de l'Agriculture et de l'Industrie de Lyon. Tome iii. 8vo. | Ditto. |
| Annales de l'Observatoire Physique Central de Russie. 1851. 2 Tomes. 4to. | The Observatory. |
| Journal of the Horticultural Society of London. Vol. viii., Part 4. 8vo. | The Society. |

*December* 4, 1854.

| | |
|---|---|
| Journal of the Royal Asiatic Society of Great Britain and Ireland. Vol. xvi., Part 1. 8vo. | The Society. |
| A Descriptive Catalogue of the Historical Manuscripts in the Arabic and Persian Languages, preserved in the Library of the Royal Asiatic Society of Great Britain and Ireland. By William H. Morley, M.R.A.S. 8vo. | Ditto. |
| Essay on the Architecture of the Hindús. By Rám Ráz. Published for the Royal Asiatic Society of Great Britain and Ireland. 4to. | Ditto. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt. 1853. iv. Jahrgang. Nro 4. October, November, December. 8vo. | The Institute. |
| Mémoires de l'Académie Impériale des Sciences, Belles Lettres et Arts de Lyon. Classe des Lettres. Tome ii. 8vo. | The Society. |
| Mémoires de l'Académie Impériale des Sciences, Belles Lettres et Arts de Lyon. Classe des Sciences. Tome ii. 8vo. | Ditto. |
| Annales des Sciences Physiques et Naturelles d'Agriculture et d'Industrie, publiées par la Société Impériale d'Agriculture, &c., de Lyon. 2me Série. Tome iv. 1852. 8vo. | Ditto. |
| Mémoires présentés par divers savants à l'Académie des Sciences de l'Institut Impérial de France, et imprimés par son ordre. Sciences Mathématiques et Physiques. Tome xii. 4to. | The Institute. |
| Natuurkundige Verhandelingen van de Hollandsche Maatschappij der Wetenschappen te Haarlem. Tweede Versameling. 4e, 9de, 10de, 11de Deel, 1ste Stuk. 4to. | The Society. |
| Philosophical Transactions of the Royal Society of London. 1852, Parts 1 and 2; 1853, Parts 1, 2, 3; 1854, Part 1. 4to. | Ditto. |
| Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam. 1ste Deel. 4to. | The Academy. |
| Det Kongelike Danske Videnskabernes Selskabs Skrifter. Femfte Reeke. Naturvidenskabelig og Mathematisk Afdeling. Bd 3. 4to. | The Society. |
| Abhandlungen, herausgegeben von der Senckenbergischen Naturforschenden Gesellschaft. 1e Bd 1e Lieferung. 4to. | Ditto. |
| Astronomical and Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1852. 4to. | The Royal Society. |
| Natural History of New York. Palæontology of New York. By James Hall. Vols. i. and ii. 4to. | The State of New York. |
| ——— Agriculture of New York. By Ebenezer Emmons, M.D. Vols. i., ii., and iii. 4to. | Ditto. |
| Magnetical and Meteorological Observations made at the Honourable East India Company's Observatory, Bombay, in the year 1850. 4to. | The Hon. East India Company. |
| Astronomical Observations made at the Observatory of Cambridge. Vol. xvii., for 1846, 1847, and 1848. | The Observatory. |
| Mémoires Couronnés et Mémoires des Savants étrangers publiées par l'Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique. Tome xxv. 1851-53. 4to. | The Academy. |
| Annales de l'Observatoire Royal de Bruxelles. Tome x. 4to. | The Observatory. |
| Compte rendu des Travaux du Congrès Général de Statistique, réuni à Bruxelles, les 19, 20, 21 et 22 Septembre 1853. Par A. Quételet. 4to. | The Author. |
| Mémoires de la Société de Physique et d'Histoire Naturelle de Genève. Tome xiii., 2me Partie. 4to. | The Society. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. 7. 4to. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Tables du Soleil exécutées d'après les ordres de la Société Royale des Sciences de Copenhague, par MM. P. A. Hansen et C. F. R. Olufsen. 4to. | The Society. |
| Rendiconto della Societa Reale Borbonica. Accademia delle Scienze. N.S. Nros 4 & 5. 4to. | Ditto. |
| Atti della Reale Accademia delle Scienze, sezione della Societa Reale Borbonica. Vol. vi. 4to. | Ditto. |
| Transactions of the American Philosophical Society, held at Philadelphia. (N.S.) Vol. x., Part 3. 4to. | Ditto. |
| Proceedings of the American Philosophical Society. Vol. v., No. 50. 8vo. | Ditto. |
| Researches upon Nemerteans and Planarians. By Charles Girard. 1. Embryonic development of Planocera elliptica. 4to. | The Author. |
| Notes on new species and localities of Microscopical Organisms. By J. W. Bailey, M.D. 4to. | The Smithsonian Institution. |
| Smithsonian Contributions to Knowledge. Vol. vi. 4to. | Ditto. |
| Catalogue of the described Coleoptera of the United States. By Frederick Ernest Melsheimer, M.D. 8vo. | Ditto. |
| Seventh Annual Report of the Board of Regents of the Smithsonian Institution. 1853. 8vo. | Ditto. |
| The Annular Eclipse of May 26, 1854. Published under the authority of Hon. James C. Dobbin, Secretary of the Navy, by the Smithsonian Institution. | Ditto. |
| Astronomical Observations made during the year 1847 at the National Observatory, Washington. Vol. iii. 4to. | The Observatory. |
| Patent Office Reports, published by the State of Washington. 1851-3. 3 vols. 8vo. | The Government of Washington. |
| Transactions of the Wisconsin State Agricultural Society. 1851 and 1852. 8vo. | The Society. |
| Medico-Chirurgical Transactions. Published by the Royal Medical and Chirurgical Society of London. Vol. xxxvii. 8vo. | Ditto. |
| The Philosophy of Physics, or Process of Creative Development. By Andrew Brown. 8vo. | The Author. |
| Bulletin de la Société Impériale des Naturalistes de Moscou. 1852, Nros 2, 3, & 4; 1853, Nros 1 & 2. 8vo. | The Society. |
| Novorum Actorum Academiæ Cæsareæ Leopoldino-Carolinæ Naturæ Curiosorum. Vol. xxiv. Pars 1. 4to. | The Academy. |
| Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. 1853. 4to. | Ditto. |
| Monatsbericht der Königl. Preuss. Akademie der Wissenschaften zu Berlin. August 1853—Juli 1854. 8vo. | Ditto. |
| Nachrichten von der Georg-Augusts-Universität und der Konigl. Gesellschaft der Wissenschaften zu Göttingen. 1853. 12mo. | The Society. |
| Studien des Göttingischen Vereins Bergmannischer Freunde. In namen desselben herausgegeben von J. F. L. Hausmann. Bd 1, heft 3. 8vo. | The Editor. |
| Siluria. The History of the oldest known Rocks containing Organic Remains, with a brief Sketch of the distribution of Gold over the Earth. By Sir R. I. Murchison. 8vo. | The Author. |
| Museum of Practical Geology and Geological Survey. Records of the School of Mines and of Science applied to the Arts. Vol. i., Part 4. 8vo. | The Museum. |
| The Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. N.S., Nos. 45 and 46. 8vo. | The Society. |
| Proceedings of the Architectural Institute of Scotland. Session 1853-54. 8vo. | The Institute. |
| Twenty-first Annual Report of the Royal Cornwall Polytechnic Society. 1853. 8vo. | The Society. |
| Journal of the Statistical Society of London. Vol. xvii., Part 2. 8vo. | Ditto. |
| The Assurance Magazine, and Journal of the Institute of Actuaries. Vol. v., Part 4; and Vol. v., Part 1. 8vo. | Ditto. |
| List of Members of the Institute of Actuaries of Great Britain and Ireland. 1854-5. 8vo. | The Institute. |
| Athenæum. Rules and Regulations, Lists of Members, and Donations to the Library, 1852, with Supplement for 1853. 12mo. | The Athenæum. |

*December* 18, 1854.

| DONATIONS. | DONORS. |
|---|---|
| Archæologia; or, Miscellaneous Tracts relating to Antiquity, published by the Society of Antiquaries of London. Vols. xxxii., xxxiii., xxxiv., xxxv. 4to. | The Society. |
| Proceedings of the Society of Antiquaries of London. Vols. i., ii,; Vol. iii., Nos. 37–40. 8vo. | Ditto. |
| Catalogue of Roman Coins collected by the late Rev. Thomas Kerrich, M.A., F.S.A., Prebendary of Wells and Lincoln, and presented by his Son, the Rev. Richard Edward Kerrich, M.A., F.S.A., to the Society of Antiquaries of London. 8vo. | Ditto. |
| List of the Society of Antiquaries of London on 23d April 1854. 8vo. | Ditto. |
| Memorie della Academia delle Scienze dell' Istituto di Bologna. Tomo iv. 4to. | The Academy. |
| Neue Denkschriften der Allgemeinen Schweizerischen Gesellschaft für die gesammten Naturwissenschaften. Band xiii. 4to. | The Society. |
| Actes de la Société Helvétique des Sciences Naturelles. Réunie à Sion, les 17, 18, et 19 Août 1852. 8vo. | Ditto. |
| Actes de la Société Helvétique des Sciences Naturelles. Réunie à Porrentray, les 2, 3, et 4 Août 1853. 8vo. | Ditto. |
| Mittheilungen der Naturforschenden Gesellschaft in Bern. N[ros] 258–313. 8vo. | Ditto. |
| Ueber die Symmetrische Verzweigungsweise dichotomer Inflorescenzen. Von H. Wydler. 8vo. | Ditto. |
| Abhandlungen der Historischen Classe der Königlich Bayerischen Akademie der Wissenschaften. B[de] 7. 1[ste] Abtheil. 4to. | The Academy. |
| Gelehrte Anzeigen herausgegeben von Mitgliedern der K. Bayerischen Akademie der Wissenschaften. B[d] 36, 37. 4to. | Ditto. |
| Bulletin de la Société de Géographie. 4[me] Serie. Tome vii., 8vo. | The Society. |
| Bulletins de l'Académie Royale des Sciences, des Lettres, et des Beaux Arts de Belgique. Tome xx., 3[e] Parte; Tome xxi., 1[re] Parte. Annexe aux Bulletins, 1853–4. 8vo. | The Academy. |
| Transactions of the Pathological Society of London. Vol. v. 8vo. | The Society. |
| Proceedings of the Literary and Philosophical Society of Liverpool, during the 43d Session, 1853–54. No. 8. 8vo. | Ditto. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Vol. xviii. Nos. 52, 53, 54. 8vo. | The Editors. |
| The Quarterly Journal of the Geological Society. Vol. ix., Part 1; Vol. x., Parts 2 and 3. 8vo. | The Society. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt. 1854. No. 1, Jan. Feb. Marz. 8vo. | The Institute. |
| Rendiconto della Societa Reale Borbonica. Accademia delle Scienze. N.S. Jan. June 1853. 4to. | The Society. |
| Repertorio Italiano per la Storia Naturale. Repertorium Italicum complectens Zoologiam, Mineralogiam, Geologiam, et Palæontologiam. Cura J. Josephi Bianconi. Vol. i. 8vo. | The Author. |
| Jahresbericht über die Fortschritte der reinen, Pharmaceutischen und Technischen Chemie, Physik, Mineralogie und Geologie, herausgegeben von Justus Liebig & Hermann Kopp. 1853. 8vo. | The Editors. |
| Universalità dei mezzi di previdenzi, difesa, e salvezza per le calamita degl' incendi opera premiata in concorso dalla Accademia delle Scienze dell' Istituto di Bologna. Scritta da Francisco del Guidice. 8vo. | The Author. |
| Bulletins de la Société Vaudoise des Sciences Naturelles. Tome iii. Nos. 25, 28, 30, 31, 32. 8vo. | The Society. |
| Proceedings of the Academy of Natural Science of Philadelphia. Vol. iii., Nos. 3–6. 8vo. | Ditto. |
| Notices of the Meetings of the Members of the Royal Institution of Great Britain. Part 4. Nov. 1853–July 1854. 8vo. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| Report of the Commissioner of Patents for the year 1853. Part i., Manufactures. | Government of Washington, U.S. |
| Seventh Annual Report of the Board of Regents of the Smithsonian Institution for the year 1852. 8vo. | The Institution. |
| Exploration of the Valley of the Amazon, made under the direction of the Navy Department. By William Lewis Herndon and Lardner Gibbon. Part i. By Lieut. Herndon. | The Author. |
| Transactions of the Cambridge Philosophical Society. Vol. i., Part 3. 4to. | The Society. |
| Journal of the Statistical Society of London. Vol. xvi., Part 3. 8vo. | Ditto. |
| General Index to the first fifteen volumes of the Journal of the Statistical Society of London. 8vo. | Ditto. |
| List of Fellows of the Statistical Society of London. Session 1854-1855. 8vo. | Ditto. |
| Memoirs of the Royal Astronomical Society. Vol. xxii. 4to. | Ditto. |
| Journal of the Horticultural Society of London. Vol. ix., Parts 2 and 3. 8vo. | Ditto. |
| Journal of the Asiatic Society of Bengal. Edited by the Secretaries. Nos. 237-242. 8vo. | Ditto. |
| Mémoires de la Société Impériale des Sciences de l'Agriculture et des Arts de Lille. 1853. 8vo. | Ditto. |
| Die Fortschritte der Physik in den Jahren 1850 und 1851. Dargestellt von den Physikalischen Gesellschaft zu Berlin. 6 & 7 Jahrgang. 1^to Abtheil. 8vo. | Ditto. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. B^de. xi. und xii. 8vo. | The Academy. |
| Address to the Boston Society of Natural History. By John C. Warren, M.D. 8vo. | The Author. |
| Monthly Notices of the Royal Astronomical Society. Vol. xiii., 1852-3. 8vo. | The Society. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Vol. xvii., No. 51. 8vo. | The Editors. |

*January* 2, 1855.

| | |
|---|---|
| Flora Batava. 176 Aflevering, 4to. | King of Holland. |
| Transactions of the Architectural Institute of Scotland. Vol. iii., Part 1. 8vo. | The Institute. |
| Annalen der Königlichen Sternwarte bei München. 6 Band. 8vo. | The Observatory. |
| Magnetische Ortsbestimmungen ausgeführt an verschiedenen Puncten des Konigreichs Bayern und an einigen auswartigen Stationen. Von Dr J. Lamont. 1 Theil. 8vo. | Ditto. |
| Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. N.S., No. 47. 8vo. | The Society. |
| Almanaque Nautico para el año 1855. (San Fernando.) 8vo. | The Observatory of San Fernando. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt 1853. No. 3. (Juli, August, September.) 8vo. | The Institute. |
| Transactions of the Royal Scottish Society of Arts. Vol. iv., Part 2. 8vo. | The Society. |
| Proceedings of the Royal Society. Vol. vi., Nos. 91-101. 8vo. | Ditto. |
| Boston Journal of Natural History, containing Papers and Communications read before the Boston Society of Natural History, and published by their direction. Vol. vi., No. 3. 8vo. | Ditto. |
| Proceedings of the Boston Society of Natural History. Jan. 1, 1851—Nov. 16, 1853. 8vo. | Ditto. |
| Proceedings of the American Academy of Arts and Sciences. Vol. iii., pp. 1-104. 8vo. | The Academy. |

*March* 5, 1855.

| | |
|---|---|
| Journal of the Asiatic Society of Bengal. No. 5, 1854. 8vo. | The Society. |
| The Quarterly Journal of the Geological Society. Vol. x., Part 4. 8vo. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| Journal of the Statistical Society of London. Vol. xvii., Part 4. 8vo. | The Society. |
| The Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. N.S. No. 48. 8vo. | Ditto. |
| Catalogue of Stars near the Ecliptic, observed at Markree, during the years 1852, 1853, and 1854, and whose places are supposed to be hitherto unpublished. Vol. iii. 8vo. | Royal Society. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Vol. xix., No. 55. 8vo. | The Editors. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. xii., Heft 5; Bd. xii., Heft 1 & 2. 8vo. | The Academy. |
| Register zu den ersten X. Bänden der Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe. 8vo. | Ditto. |
| Jahrbücher der K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Von Karl Kreil. Herausgegeben durch die Kaiserliche Akademie der Wissenschaften). 2 Bd. 4to. | Ditto. |
| Annalen der Königlichen Sternwarte bei München. VI. Bd. 8vo. | The Academy. |
| Magnetische Ortsbestimmungen an verschiedenen Puncten des Konigreichs Bayern und an einigen auswärtigen Stationen. I. Theil. 8vo. | Ditto. |
| Magnetische Karten von Deutschland und Bayern. Von Dr J. Lamont. Fol. | The Author. |
| Transactions of the Linnæan Society of London. Vol. xxi., Parts 2 and 3. 4to. | The Society. |

*December* 3, 1855.

| | |
|---|---|
| Transactions of the Royal Scottish Society of Arts, Vol iv., Part 3. 8vo. | The Society. |
| The Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. N.S. Nos. 49, 50. 8vo. | Ditto. |
| Transactions of the Architectural Institute of Scotland. Session 1854–5. 8vo. | The Institute. |
| Proceedings of the Royal Society. Vol. vii., No. 14. 8vo. | The Society. |
| Results of Astronomical Observations, made at the Observatory of the University, Durham, from October 1849 to April 1852, under the general direction of the Rev. Temple Chevallier, B.D., F.R.A.S. By R. C. Carrington, Esq., B.A., F.R.A.S. 8vo. | The Observatory. |
| The Assurance Magazine, and Journal of the Institute of Actuaries. Vol. v., Part 4; Vol. v., Part 1. 8vo. | The Institute. |
| Journal of the Statistical Society of London. Vol. xviii., Parts 1, 2, 3. 8vo. | The Society. |
| The Quarterly Journal of the Geological Society. Vol. ii., Parts 1, 2, 3. 8vo. | Ditto. |
| The Journal of the Horticultural Society of London. Vol. ix., Part 4. 8vo. | Ditto. |
| Monthly Notices of the Royal Astronomical Society. Vol. xiv. 8vo. | Ditto. |
| The Journal of the Royal Geographical Society. Vol. xxiv. 8vo. | Ditto. |
| The Journal of the Royal Asiatic Society of Great Britain and Ireland. Vol. xv., Part 2. 8vo. | Ditto. |
| Journal of the Geological Society of Dublin. Vol. vi., Part 2. 8vo. | Ditto. |
| The Quarterly Journal of the Chemical Society. Vol. viii., Part 2. 8vo. | Ditto. |
| Notices of the Meetings of the Members of the Royal Institution of Great Britain. Part 5. | The Institution. |
| Proceedings of the Liverpool Literary and Philosophical Society. Session 1854–5. 8vo. | The Society. |
| Journal of the Asiatic Society of Bengal. Nos. 70, 71, 72. 8vo. | Ditto. |
| Abstracts of the Proceedings of the Ashmolean Society. Vols. i., ii., iii., Part 1. 8vo. | Ditto. |
| Memoirs of the Literary and Philosophical Society of Manchester. 2d Series, Vol. xi., xii. 8vo. | Ditto. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Nos. 57, 58, 59. 8vo. | The Editors. |

| DONATIONS. | DONORS. |
|---|---|
| Collection of Charts published at the Hydrographic Office, London. 8vo. | The Admiralty. |
| Astronomical Observations, made at the Radcliffe Observatory. By Manuel J. Johnstone, M.A., 1850, 1851, 1852. 8vo. | The Observatory. |
| Archæologia, or Miscellaneous Tracts relating to Antiquity, published by the Society of Antiquaries of London. Vol. xxxi. 4to. | The Society. |
| Proceedings of the Society of Antiquaries of London. Vol. iii. No. 52. 8vo. | Ditto. |
| Descriptive and Illustrated Catalogue of the Histological Series contained in the Museum of the Royal College of Surgeons of England. Vol. ii. 4to. | The College. |
| Assault of Sevastopol. Two Topographical and Panoramic Sketches, representing the advanced lines of attack, and the Russian defences, in front of Sevastopol, with a description and remarks. The sketches by Capt. M. A. Biddulph, R.A. | Capt. Younghusband. |
| Transactions of the Zoological Society of London. Vol. iv., Parts 2, 3. 4to. | The Socieety. |
| The Origin and Progress of the Mechanical Inventions of James Watt, illustrated by his correspondence with his friends, and specification of his patents. By James Patrick Muirhead, Esq., M.A., 3 vols. 4to. | The Author. |
| Researches on Colour Blindness. By George Wilson, M.D. 8vo. | Ditto. |
| Magnetical and Meteorological Observations made at the Hon. East India Company's Observatory, Bombay, in the year 1851. 4to. | Hon. East India Company. |
| Astronomical and Magnetical, and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1853. 4to. | The Observatory. |
| Memoirs of the Royal Astronomical Society. Vol. xxiii. 4to. | The Society. |
| Abstracts from the Meteorological Observations taken at the Stations of the Royal Engineers in the year 1853-4. Edited by Lieut. Col. H. James, R.E. 4to. | The Editor. |
| Papers read at the Royal Institute of British Architects. Session 1854-5. 4to. | The Institute. |
| Memoir of Robert Troup Paine. By his Parents. 4to. | The Authors. |
| Materia Medica and Therapeutics. By Martyn Paine, M.D. 12mo. | Ditto. |
| The Institutes of Medicine. By Martyn Paine, M.D. 8vo. | Ditto. |
| Medical and Physiological Commentaries. By Martyn Paine, M.D. 3 Vols. 8vo. | Ditto. |
| A Discourse on the Soul and Instinct. By Martyn Paine, M.D. 18mo. | Ditto. |
| Reports and State Documents published by the Senate of Washington. | The Senate. |
| Documents relating to the Colonial History of the State of New York. Vols. iii. and iv. 4to. | The State of New York. |
| Smithsonian Report. On the Construction of Catalogues of Libraries, and a General Catalogue. 8vo. | The Smithsonian Institution. |
| Smithsonian Contributions to Knowledge. Vol. vii. 4to. | Ditto. |
| Eighth Annual Report of the Board of Regents of the Smithsonian Institution. | Ditto. |
| Bulletins de l'Académie Royale des Sciences, des Lettres, et des Beaux Arts de Belgique. Tome xxi., 2me Partie. Tome xxii., 1re Partie. 8vo. | The Academy. |
| Essai d'une Géographie Physique de la Belgique. Par J. C. Houzean. 8vo. | The Author. |
| Mémoires Couronnés et mémoires des savants étrangers, publiées par l'Académie Royale de Belgique. Tome vi., 2me Partie. 8vo. | The Academy. |
| Annalen der Königlichen Sternwarte bei München. 7 Band. 8vo. | The Observatory. |
| Jahresbericht der Münchener Sternwarte für. 1854. 8vo. | Ditto. |
| Monatsbericht der Königl. Preus, Akademie der Wissenschaften zu Berlin, August, December. 1854. 8vo. | The Academy. |
| Abhandlungen der Mathematisch-physikalischen Classe der Koeniglich Bayerischen Akademie der Wissenschaften. 7 Bd., 2 Abtheil. 4to. | The Academy. |
| Abhandlungen der Historischen Classe der Koeniglich Bayerischen Akademie der Wissenschaften. 6 Bd., 2 Abtheil. 4to. | Ditto. |
| Preisschriften gekrönt und herausgegeben von der Fürstlich Jablonowskischen Gesellschaft zu Leipzig. No. 5. 8vo. | The Society. |
| Nova Acta Academiæ Cæsareæ Leopoldino-Carolinæ Naturæ Curiosorum. Vol. xxiv., Pars 2. 4to. | The Academy. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. viii. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. xiv. und Bd. xv., Heft. 1 und 2. 8vo. | The Academy. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Philosophische Historische Classe. Bd. xiv. and Bd. xv., Heft. 1. 8vo. | Ditto. |
| Almanach der Kaiserlichen Akademie der Wissenschaften. 1855. 12mo. | Ditto. |

### *December* 17, 1855.

| | |
|---|---|
| Jahresbericht über die Fortschritte der reinen, pharmaceutischen und technischen Chemie, &c. Herausg. von Liebig und Kopp. 1854. 8vo. | The Editors. |
| Die Fortschritte der Physik in den Jahren, 1850, 1851, 1852. Dargestellt von der Physikalischen Gessellschaft zu Berlin. 8vo. | The Society. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Bd. xv., Heft. 2 und 3; Bd. xvi., Heft. 1. Philosophisch-Historische Classe. 8vo. | The Academy. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. xv., Heft. 3; Bd. xvi., Heft. 1. 8vo. | Ditto. |
| Abhandlungen der Philosoph.-Philologischen Classe der Koeniglich Bayerischen Akademie der Wissenschaften. Bd. xvii., 2 Abtheil. 4to. | Ditto. |
| Denkrede auf die Akademikir Dr Thaddäus Liber und Dr Georg Simon Ohm, von Dr Lamont. 4to. | Ditto. |
| Almanach der Königlich Bayerischen Akademie der Wissenschaften, für das Jahr 1855. 12mo. | Ditto. |
| Archives du Muséum d'Histoire Naturelle, publiées par les professeurs-administrateurs de cet établissement. Tome vii., Liv. 3 et 4; Tome viii., Liv. 1 et 2. 4to. | The Museum. |
| Aanteekeningen van het verhandelete in de Sectie Vergaderingen van het Provinciaal Utretsch Genootschop van Kunsten en Wetenschappen. 1845-54. 8vo. | The Academy. |
| Verhandeling over de verdiensten van Gijsberet Karel van Hogendorp, als Stautshuishoudkundige ten aanzien van Nederlands, door M. O. Van Rees. 8vo. | Ditto. |
| Description de l'Observatoire météorologique et magnétique à Utrecht. Par P. W. C. Krecke. 8vo. | Ditto. |
| Astronomical and Meteorological Observations made at the Radcliffe Observatory in the year 1853, under the superintendence of Manuel J. Johnson, M.A. Vol. xiv. 8vo. | The Trustees. |
| Memorie della Accademia delle Scienze dell' Istituto di Bologna. Tomo v. 4to. | The Academy. |
| Journal of the Asiatic Society of Bengal. Nos. 3 and 4. 1855. 8vo. | The Society. |
| Almanaque Nautico para el año 1856, calculado de orden de J. M. en el Observatoro de Marina de la Ciudad de San Fernando. 8vo. | The Observatory. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt. 1854. Nos. 2, 3, & 4. 8vo. | The Institute. |
| Bulletin de la Société de Géographie. 4ieme Serie. Tom. 8 & 9. 8vo. | The Society. |
| Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig. 1854-5. 8vo. | Ditto. |
| Bulletin de la Société Impériale des Naturalistes de Moscou. 1853. Nos. 3 & 4. 1854. No. 1. 8vo. | Ditto. |
| Medico-Chirurgical Transactions. Published by the Royal Medical and Chirurgical Society of London. Second Series. Vol. xx. 8vo. | Ditto. |
| Journal of the Ethnological Society of London. Vols. i. ii. iii. 8vo. | Ditto. |
| Transactions of the Pathological Society of London. Vol. vi. 8vo. | Ditto. |
| Journal of the Statistical Society of London. Vol. xviii. Part 4. 8vo. | Ditto. |
| Observations Météorologiques faites à Nijné Tagulsk (Monts Oural). Governement de Perm. 1850-51-52-53. 8vo. | The French Government. |
| Collection of Naval Charts from the *Depôt General de la Marine*. 8vo. | Ditto. |
| Verhandelingen der Koninklijke Akademie van Wetenschappen. 2de Deel. 4to. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Verslagen en Mededeelingen der Koninklijke Akademie van Wetenschappen. 2de Deel. 3de Stuk., 3de Deel. 1ste and 2de Stuk. 8vo. | The Academy. |
| Catalogus der Boekeri van de Koninklijke Akademie van Wetenschappen, gevestigd te Amsterdam, 1ste Afler. 8vo. | Ditto. |
| Analytisch-geometrische Untersuchungen über Allgemeine Verwandtschafts-Verhältnisse von Coordinaten-Systemen. Von J. G. H. Swellengœbel. 4to. | The Author. |
| Kongl. Vetenskaps Akademiens Handlingar. 1853. 8vo. | The Academy. |
| Kongl. Vetenskaps Akademiens Ofversigt. 1854. 8vo. | Ditto. |
| Kongl. Vetenskaps Akademiens Arsberättelser af Wikström. 8vo. | Ditto. |
| Kongl. Vetenskaps Akademiens Arsberättelser af Boheman. 8vo. | Ditto. |
| Memorie della Reale Accademia delle Scienze di Torino. Tome xiv. 4to. | Ditto. |
| Mémoires de l'Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique. Tomes xxviii. xxix. 4to. | Ditto. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd 9. 4to. | The Academy. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften. Philosophisch-Historische Classe. Bd 6. 4to. | Ditto. |
| Jahrbüchr der K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Von Karl Kreil. III. Bd. 1851. 4to. | Ditto. |
| Bulletin der Königl. Akademie der Wissenschaften (München). Nos. 1–52. 4to. | Ditto. |
| Compte Rendu Annuel, par le Directeur de l'Observatoire Physique Central, A. T. Küpffer. 1853. 4to. | The Editor. |
| Della vita e delle opere di Guido Bonatti, Astrologo ed Astronomo del secolo decimoterzo notizie raccolte da B. Boncompagni. 8vo. | The Author. |
| Annuaire de l'Académie Royale des Sciences, &c. de Belgique. 1854–55. 12mo. | The Academy. |
| Académie Royale de Belgique, Bibliographie Académique. 1854. 12mo. | Ditto. |
| Annuaire de l'Observatoire Royal de Bruxelles. 1854–55. 12mo. | Ditto. |
| Almanach Seculaire de l'Observatoire R. de Bruxelles. 1854. 12mo. | Ditto. |

*March* 17, 1856.

| | |
|---|---|
| Exhibition of the Works of Industry of all Nations, 1851. Reports by the Juries on the Subjects in the Thirty Classes into which the Exhibition was divided. 4 vols. fol. | H. F. Talbot, Esq. |
| Journal of the Proceedings of the Linnæan Society. Vol. i., No. 1. 8vo. | The Society. |
| American Journal of Science and Arts. Vol. xxi., No. 61. 8vo. | The Editors. |
| Journal of the Statistical Society of London. Vol. xix., Part 1. 8vo. | The Society. |
| Quarterly Journal of the Geological Society. Vol. xii., Part 1. 8vo. | Ditto. |
| Die Fortschritte der Physik im Jahre 1852. Dargestellt von der Physikalischen Gesellschaft zu Berlin. 2e Abtheil. 8vo. | Ditto. |
| Annalen der Königlichen Sternwarte bei München. Bd 8. 8vo. | The Observatory. |
| Tables showing the Number of Criminal Offenders in England and Wales, in the year 1854. Fol. | The Home Office. |
| A Collection of Charts published at the Hydrographic Office, London. | H. M. Admiralty. |

*April* 7, 1856.

| | |
|---|---|
| Assurance Magazine, and Journal of the Institute of Actuaries. Vol. vi., Part 3. 8vo. | The Institute. |
| Proceedings of the Ashmolean Society, 1855. 8vo. | The Society. |
| The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland. N. S., No. 52. 8vo. | Ditto. |
| Journal of the Asiatic Society of Bengal. N. S., Nos. 5 & 6, 1855. 8vo. | Ditto. |
| Bulletin de la Société de Géographie. 4me Série. Tome x. 8vo. | Ditto. |
| Verhandlungen der Kaiserlichen Leopoldinisch-Carolinischen Akademie der Naturforscher. Bd. xxiv., Supp. Bd. xxv., Heft 1. 4to. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Smithsonian Contributions to Knowledge. Vol. vii. 4to. | Smithsonian Inst. |
| Abhandlungen der Kaiserlich-Königlichen Geologischen Reichsanstalt. Band 2, 1855. Fol. | The Institute. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt, 1855. 8vo. No. 1. | Ditto. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe. Bd. xvii., Heft 3. 8vo. | The Academy. |
| Coup d'œil Géologique sur les Mines de la Monarchie Autrichienne. Par Fr. de Hauer & Fr. de Fœtterle. 8vo. | The Authors. |

*April* 21, 1856.

| | |
|---|---|
| Report of the Yellow Fever Epidemic of British Guiana. By Daniel Blair, M.D. 8vo. | The Author. |
| Physical and Geographical Map of India. By G. B. Greenough. 8vo. | Exrs. of Author. |
| Théorie de l'Antagonisme et de la Pondération. Par M. Alexandre Gérard. 8vo. | The Author. |
| London University Calendar, 1856. 12mo. | The University. |
| Proceedings of the Botanical Society of Edinburgh, 1855. 8vo. | The Society. |
| The Quarterly Journal of the Geological Society. Vol. ii., Part 4. 8vo. | Ditto. |
| Flora Batava. Afleverings 177 & 178. 4to. | King of Holland. |
| Abhandlungen, herausgegeben von der Senckenbergischen Naturforschenden Gesellschaft. 1ster Band, 2te, Lieferung. 4to. | The Society. |
| Memorias de la Real Academia de Ciencias de Madrid. Tome 1 & 2. 8vo. | The Academy. |
| Positions moyennes pour l'époque de 1790,0 des étoiles circompolaires, dont les observations ont été publiées par Jérome Lalande dans les Mémoires de l'Académie de Paris de 1789 et 1790. Par Ivan Frederenko. 4to. | The Editor. |
| Ueber Dr Wichmann's Bestimmung der Parallaxe des Argelanderschen Sterns. Von W. Döllen. 4to. | The Author. |
| Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences. Nov. 1855—Avril 1856. 4to. | The Academy. |

*December* 1, 1856.

| | |
|---|---|
| Collection of Charts, published at the Hydrographic Office, London, with relative Descriptions. | The Admiralty. |
| Results of Meteorological Observations, made at Sundry Academies in the State of New York, from 1826 to 1850 inclusive. Compiled by F. B. Hough, M.D. 4to. | The State of New York. |
| Annual Report of the Trustees of the New York State Library, 1856. 8vo. | Ditto. |
| List of Members and Report of Council, &c., of the Royal Institute of British Architects, 1856. 4to. | The Institute. |
| Map of Boundary between the United States and Mexico. By W. H. Emory, U.S. Commissioner. | Smithsonian Institution. |
| Smithsonian Contributions to Knowledge. Vol. viii. 4to. | Ditto. |
| List of Foreign Correspondents of the Smithsonian Institution, 1856. 8vo. | Ditto. |
| Nova Acta Academiæ Cæsareæ Leopoldino-Carolinæ Naturæ Curiosorum. Vol. xxv., Pars 2. 4to. | The Academy. |
| Mean Zenith Distances. Collection of all the Results of Observations of each star at Heerelogements-Berg, and Deduction of Mean Zenith Distance, 1843. January 0. | The Observatory, Cape of Good Hope. |
| Memoirs of the Royal Astronomical Society. Vol. xxiv. 4to. | The Society. |
| Astronomical, Magnetical, and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1854. 4to. | The Royal Society. |
| Magnetical and Meteorological Observations made at the Hon. East India Company's Observatory, Bombay, in the years 1852-53. 2 vols. 4to. | The Hon. E. I. Company. |
| Memoirs of the American Academy of Arts and Sciences. New Series. Vol. v., Part 2. | The Academy. |

| DONATIONS. | DONORS. |
|---|---|
| Proceedings of the American Association for the Advancement of Science. Sessions 1853, 1854, and 1855. | The Association. |
| Proceedings of the Academy of Natural Sciences of Philadelphia. Vol. vii., Nos. 8-12; and Vol. viii., Nos. 1 and 2. | The Academy. |
| Annales de l'Observatoire Physique Central de Russie. Ann. 1851, 1852, 1853, 3 Tom. 4to. | The Observatory. |
| Comte-Rendu Annuel, Supplement aux Annales de l'Observatoire Physique Central, pour l'année 1853. (2 copies.) | Ditto. |
| Correspondance Météorologique. Publication trimestrielle de l'Administration des Mines de Russie, pour ann. 1853, 1854. 2 Tom. 4to. | The Academy. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften, Mathematisch-Naturwissenschaftliche Classe. Bande x., xi. 4to. (2 copies.) | Ditto. |
| Beschreibung und Lage der Universitäts-Sternwarte in Christiania, von Christopher Hansteen. 4to. | The Observatory. |
| Ofversigt af Finska Ventenskaps-Societetens Forhandlingar, 1838-53. 4to. | The Society. |
| Observations faites à l'Observatoire Magnétique et Météorologique de Helsingfors, sous la direction de Jean Jacques Nervander. Vol. i., ii., iii., et iv. 4to. | The Observatory. |
| Verhandelingen der Koninklijke Akademie van Wetenschappen. Deel. iii. 4to. | The Academy. |
| Oversigt over det danske Videnskabernes Selskabs Forhandlinger i Aaret, 1855. | The Society. |
| Videnskabernes Selskabs Skrifter. 5te Række. Naturvidenskabelig og Mathematisk Afdeling. 4de Bande, 1ste Hefte. | Ditto. |
| Observations Metereologicæ per annos 1832-54 in Grönland factæ. | Ditto. |
| Flora Batava. Part 179. 4to. | King of Holland. |
| Memoire della Accademia della Scienze dell' Istituto di Bologna. Tom. vi. 4to. | The Academy. |
| Rendiconto delle Sessioni 1854-55, dell' Accademia delle Scienze dell' Istituto di Bologna. 8vo. | Ditto. |
| Indices Generales in Novos Commentarios Academiæ Scientiarum instituti Bononiensis. 4to. | Ditto. |
| Mémoires de la Société de Physique et d'Histoire Naturelle de Genève. Tom. xiv. Part 1. 4to. | The Society. |
| Rapport sur les Travaux de la Société Impériale Zoologique, Paris. Par Aug. Duméril. 8vo. | The Author. |
| Ichthyologie Analytique ou Essai d'une Classification Naturelle des Poissons. Par A. M. C. Duméril. 4to. | Ditto. |
| Abhandlungen der Koeniglich Bayerischen Akademie der Wissenschaften. Historischen Classe, Vol. vii., Part 3; Vol. viii., Part 1. Philosoph.-Philologischen Classe, Vol. vii., Part 3. 4to. | The Academy. |
| Gelehrte Anzeigen, herausgegeben von Mitgliedern der K. bayer Akademie der Wissenschaften. Vols. xl. xli. 4to. | Ditto. |
| Archives du Muséum d'Histoire Naturelle. Tome viii., livraisons 3, 4. 4to. | The Museum. |
| Monatsbericht der Koniglichen Preuss. Akademie der Wissenschaften zu Berlin, January—June 1855. 8vo. | The Academy. |
| Observations Météorologiques faites a Nijne-Taguilsk (Monts Oural), Gouvernement de Perm. Année 1854. 8vo. | The Russian Observatory. |
| Bulletin de la Société Vaudoise des Sciences Naturelles. Nos. 34, 37. 8vo. | The Society. |
| Jahrbuch der Kaiserlich-Königlichen Geologischen Reichsanstalt. Vol. vi., Nos. 2, 4; Vol. vii., No. 1. 8vo. | The Institute. |
| Abhandlungen der Kaiserlich-Königlichen Geologischen Reichsanstalt. Vol. iii. 4to. | Ditto. |
| Memoire della Reale Accademia delle Scienze di Torino. 2d Ser. Vol. xv. 4to. | The Academy. |
| Jahrbücher der K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Von Karl Kreil. Vol. iv. 4to. (2 copies.) | Academy of Sciences, Vienna. |
| Abhandlungen der Akademie der Wissenschaften zu Berlin, 1854. 4to. | The Academy. |
| Portrait of Karl Haidinger. | From his Son. |
| Journal of the Royal Asiatic Society of Great Britain and Ireland. Vol. xvi., Part 2. 8vo. | The Society. |

| DONATIONS. | DONORS. |
|---|---|
| Publications of the Dépôt Général de la Marine:—<br>Collection of Charts.<br>Pilote de la Mer Baltique. 8vo.<br>Portulan des Cotes de la Manche. 8vo.<br>Le Pilote Danois. 8vo.<br>Renseignements Hydrographiques sur la Mer d'Azof. 8vo.<br>Description du Golfe de Finlande et de l'entrée du Golfe de Bothnie. 8vo.<br>Observations Chronometriques faites pendant la Campagne de Circumnavigation de la Corvette Capricieuse. 8vo.<br>Manuel de la Navigation dans la Mer Adriatique. 8vo.<br>Exposé du Régimie des Courants Observés dans la Manche et la Mer d'Allemagne. 8vo.<br>Description des Cotes de l'Esthonie, de la Livonie, de la Courland (Russie), de la Prusse, et de la Pomeraine, jusqu'au Cap Darserort. 8vo.<br>Routier de l'Australie Traduit de l'Anglais, &c. 8vo.<br>Annales Hydrographiques, Recueil d'Avis, Instructions, Documents, et Mémoires relatifs a l'Hydrographie et a la Navigation. 8vo.<br>Catalogue Chronologique des Cartes, Plans, Veus de Côtes, Mémoires, Instructions Natiques, &c., qui composent l'Hydrographie Française. 8vo.<br>Annuaire des Marées des Côtes de France, pour l'an 1855. 16mo. | Dépôt-Général de la Marine, Paris. |
| Vierzehn Kupfertafeln zu H. B. Geinitz Darstellung der Flora des Hainichen-Ebersdorfer und des Flöhaer Kohlenbassins. | The Author. |
| Descriptive and Illustrated Catalogue of the Histological Series contained in the Museum of the Royal College of Surgeons of England. Vol. i., Elementary Tissues of Vegetables and Animals. 4to. | The Council of the College. |
| A Monograph on Recent and Fossil Crinoideæ, with Figures and Descriptions of some Recent and Fossil allied Genera. By Thomas Austin, F.G.S. Nos. 1-8. 4to. | The Author. |
| Medico-Chirurgical Transactions, published by the Royal Medical and Chirurgical Society of London. Vol. xxxix. 8vo. | The Society. |
| Report of the U.S. Commissioner of Patents for the year 1854. Agriculture, Arts, and Manufactures. Vol. ii. 8vo. | The Commissioner. |
| Memoirs of the Literary and Philisophical Society of Manchester. Second Series, Vol. xiii. 8vo. | The Society. |
| Proceedings of the Zoological Society of London. Part xx., Nos. 258, 259; Parts xxi., xxii., and Part xxiii., Nos. 301–313. 8vo. | Ditto. |
| The Assurance Magazine and Journal of the Institute of Actuaries, Nos. 25, and 26. 8vo. | The Editor. |
| Proceedings of the American Philosophical Society, 1855. Nos, 53 and 54. 8vo. | The Society. |
| Proceedings of the Boston Society of Natural History, May—December 1855, and January—April 1856. 8vo. | Ditto. |
| Annals of the Minnesota Historical Society, 1856; containing Materials for the History of Minnesota, prepared by E. D. Neill, Secretary. 8vo. | Ditto. |
| Journal of the Royal Dublin Society. No. 1. 8vo. | Ditto. |
| Transactions of the Pathological Society of London. Vol. vii. 8vo. | Ditto. |
| Journal of the Asiatic Society of Bengal. Nos. 77–81 (New Series). 8vo. | Ditto. |
| Astronomical and Meteorological Observations made at the Radcliffe Observatory, Oxford, in the year 1854. Vol. xv. 8vo. | Radcliffe Trustees. |
| Medical Topography of Brazil and Uruguay, with Incidental Remarks. By G. R. B. Horner, M.D. 8vo. | The Author. |
| Description of a New Mollusk, from the Red Sandstone near Pottsville, Pa. By Isaac Lea. | Ditto. |
| Meteorology and Climate of Shanghae; deduced from Observations made during the years 1848–53. By John Ivor Murray, M.D., F.R.C.S.E. | Ditto. |
| The Canadian Journal of Industry, Science, and Art. New Series, Nos. 3 and 5. 8vo. | The Canadian Institute. |

| DONATIONS. | DONORS. |
|---|---|
| Om Dodeligheden i Norge. Af Gilert Sundt. 12mo. | The Author. |
| The Natural History of Deeside and Braemar. By the late William Macgillivray, LL.D. Edited by Edwin Lankester, M.D. 8vo. | From H.R.H. Prince Albert. |
| The American Journal of Science and Arts. Conducted by Professors Silliman and Dana. Second Series. Nos. 62–64. 8vo. | The Editors. |
| Proceedings of the Academy of Natural Sciences of Philadelphia. Vol. vi., Nos. 7–12; Vol. viii., No. 1. 8vo. | The Academy. |
| Report on the Geology of Northern and Southern California. By Dr John B. Trask. 8vo. | The State of California. |
| The Journal of Agriculture, and Transactions of the Highland and Agricultural Society of Scotland. New Series, Nos. 50–53. 8vo. | The Society. |
| Journal of the Statistical Society of London. Vol. ix., Parts 2 and 3. 8vo. | Ditto. |
| The Quarterly Journal of the Geological Society. Vol. xii., Parts 2, 3, and 4. 8vo. | Ditto. |
| Transactions of the Historic Society of Lancashire and Cheshire. Vol. viii. 8vo. | Ditto. |
| Laws of the Historic Society of Lancashire and Cheshire. | Ditto. |
| Journal of the Geological Society of Dublin. Vol. vii., Parts 1, 2, and 3. 8vo. | Ditto. |
| Proceedings of the Royal Geographical Society of London. Nos. 1, 2, 3, 4, and 5. 8vo. | Ditto. |
| Journal of the Royal Geographical Society. Vol. xxv. 8vo. | Ditto. |
| An Account of the Construction of the Britannia and Conway Tubular Bridges. By William Fairbairn, C.E. 8vo. | The Author. |
| On the Application of Cast and Wrought Iron to Building Purposes. By William Fairbairn, C.E. 8vo. | Ditto. |
| Useful Information for Engineers; being a Series of Lectures delivered to the Working Engineers of Yorkshire and Lancashire. By William Fairbairn, F.R.S. 8vo. | Ditto. |
| Journal of the Ethnological Society of London. Vol. iv. 8vo. | The Society. |
| Regulations of the Ethnological Society of London. 8vo. | Ditto. |
| A Treatise on Electricity in Theory and Practice. By Aug. de la Rive. Translated for the Author by Charles V. Walker, F.R.S. Vol. ii. 8vo. | The Author. |
| Report of the Proceedings of the Geological and Polytechnic Society of the West Riding of Yorkshire, 1855. 8vo. | The Society. |
| The Twenty-Third Annual Report of the Royal Cornwall Polytechnic Society, 1855. 8vo. | Ditto. |
| Notices of the Meetings of the Members of the Royal Institution of Great Britain. Part 6. 8vo. | The Institution. |
| Proceedings of the Yorkshire Philosophical Society. Vol. 1. 8vo. | The Society. |
| Proceedings of the Literary and Philosophical Society of Liverpool during the Forty-fifth Session, 1855–56. No. x. | Ditto. |
| Monthly Notices of the Royal Astronomical Society, containing Papers, Abstracts of Papers, and Reports of Proceedings of the Society. Vols. xv. and xvi., Nos. 6–9. 8vo. | Ditto. |
| The Quarterly Journal of the Chemical Society. Nos. 33–35. 8vo. | Ditto. |
| Proceedings of the Royal Society. Vol. viii., Nos. 21, 22. 8vo. | Ditto. |
| List of Fellows of the Royal Astronomical Society, February 1856. 8vo. | Ditto. |
| On the Tree Producing Red Cinchona Bark. By John Eliot Howard. 8vo. | The Author. |
| Papers read at the Royal Institute of British Architects, 1856. 4to. | The Society. |
| Journal of the Proceedings of the Linnean Society. Vol. i., Nos. 2 and 3. 8vo. | Ditto. |
| Narrative of the Origin and Formation of the International Association for obtaining a uniform Decimal System of Measures, Weights, and Coins. By James Yates, M.A. 8vo. | The Author. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, Wien. Philosophisch-Historische Classe, Band xvi., Heft 2; xvii., xviii., xix.; xx. Heft 1. Mathematish-Naturwissenschaftliche Classe, Band xvi., Heft 2; xvii., xviii., xix., and xx., Heft 1. 8vo. | The Academy. |
| Almanach der Kaiserlichen Akademie der Wissenschaften, Jahr. 1856. | Ditto. |
| Jahresbericht über die Fortschritte der reinen, pharmaceutischen und technischen Chemie, Physik, Mineralogie, und Geologie, Herausgegeben von Justus Liebig und Hermann Kopp. Für 1855. 8vo. | The Editors. |

| DONATIONS. | DONORS. |
|---|---|
| Abhandlungen der Koniglichen Gesellschaft der Wissenschaften zu Göttingen. Band. vi. 4to. | The Society. |
| Verslagen en Mededeelingen der Koninklijke Akademie van Wetenschappen, Amsterdam. Afdeeling Natuurkunde, Deel iii., No. 3; iv. Nos. 1–3. Afdeeling Letterkunde, Deel i. ii., No. 1. | The Academy. |
| Berichte über die Verhandlungen der Koniglich Sächsischen Gesellschaft der Wissenschaften zu Leipzig. Philologisch-Historische Classe, 1855, Nos. 3, 4; 1856, Nos. 1, 2. Mathematisch-Physische Classe, 1854, No. 3; 1855, Nos. 1, 2; 1856, No. 1. 8vo. | The Society. |
| Nachtrag zu Stadtrechte der Latinischen-Gemeinden Salpensa und Malaca in der Provinz Bætica. Von Theodor Mommsen. 8vo. | Ditto. |
| Nachtragezur Theorie der Musikalischen Tonverhältnisse. Von M. W. Drobisch, 8vo. | Ditto. |
| Elecktrodynamische Maassbestimmungen insbesondere zurückführung der Strointensitäts-Messungen auf Mechanisches Maas. Von R. Kohlrausch und Wilhelm Weber. 8vo. | Ditto. |
| Resulate aus Beobachtungen der Nebelfleeken und Sternhaufen. Von H. D'Arrest. 8vo. | Ditto. |
| Berechnung der Absoluten Störungen der Kleinen Planeten. Von P. A. Hansen. 8vo. | Ditto. |
| Extrait du Programme de la Société Hollandaise des Sciences à Haarlem, pour l'année 1856. 4to. | Ditto. |
| Natuurkundige Verhandelingen van de Hollandsche Maatschappij der Wetenschappen te Haarlem. 2 Verzameling. Deel xi., Stuk. 2. 4to. | Ditto. |
| Forhandlingar vid de Skandinaviske Naturforskarnes sjette möte i Stockholm, den 14–19 Juli, 1851. 8vo. | Ditto. |
| Kongl. Vetenskaps-Akademiens Handlingar, för ar. 1853, 1854. 8vo. | The Academy. |
| Ars-Berättelse om Botaniska Arbeten och upptäcker för ar. 1851. Af Joh. Em. Wilkström. 8vo. | Ditto. |
| Ofversigt af Kongl. Vetenskaps-Akademiens Forhandlingar. Tolfte argangen, 1855. 8vo. | Ditto. |
| Ofversigt af Finska Vetenskaps, Societetens Forhandlingar.—ii. iii. (1853–56.) 4to. | The Society. |
| Acta Societatis Socientiarum Fennicæ. Tom. iv. and v., Fasc. 1. | Ditto. |
| Recherches Experimentales sur la Végétation. Par Georges Ville. 8vo. | The Author. |
| Medimalcollegiets Skjæbne. 8vo. | The Editor. |
| Syphilisationen Studeret ved Syngesengen, af Wilhelm Bœck. 8vo. | The Author. |
| De Prisca re Monetaria Norvegiæ et de Numis Aliquot et Ornamentis, in Norvegia Repertis. Scripsit C. A. J. Holmboe. 8vo. | Ditto. |
| Kong Christian dens Fejerdes Norske Lovbog, af 1604. Af Fr. Hallager, og Fr. Brandt. 8vo. | Ditto. |
| Om Mundtlig Rettergang og Edsvorne. Af C. Aubert. 8vo. | Ditto. |
| An Historical Summary of the Post-Office in Scotland, compiled from authentic Records and Documents. By T. B. Lang. 8vo. | Ditto. |
| On the Swedish Tabulating Machine of Mr George Schentz. By C. Babbage, F.R.S. 8vo. | Ditto. |
| Annual Report of the Leeds Philosophical and Literary Society. 1855–6. 8vo. | The Society. |
| Universitatis Regiæ Fredericianæ Novæ ædes Descripsit Ch. Holst, Sec. 8vo. | The Author. |
| Histology of the Cholera Evacuations in Man and the Lower Animals. By W. Lauder Lindsay, M.D. 8vo. | Ditto. |
| Almanique Nantico para 1857, Calculado de Orden de S.M., en el Observatorio de Marina de la Ciudad de San Fernando. | The Observatory. |
| Commercium Epistolicum J. Collins et aliorum de Analysi promota, &c., ou Correspondance de J. Collins, et d'autres Savants célebres, du XVII. Siecle, relative à l'analyse supérieure. Publiée par J. B. Biot et F. Lefort. 4to. | Ministre d'Instruction Publique. |
| Det Kongelige Norske Frederiks Universitets Aarsberetning, 1853. 8vo. | The University. |
| Phenomena of the Material World. By D. Vaughan. 8vo. | The Author. |
| On the Practicability of Constructing Cannon of Great Calibre, capable of enduring long-continued use under full Charges. By Daniel Treadwell. 8vo. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| On the Mechanical Properties of Metals as Derived from Repeated Meltings, Exhibiting the Maximum point of Strength, and the Causes of Deterioration. By W. Fairbairn, F.R.S. 8vo. | The Author. |
| On Axes of Elasticity and Crystalline Forms. By W. J. M. Rankine, C.E. 4to. | Ditto. |
| Universal Writing and Printing with Ordinary Letters. By Alexander J. Ellis, B.A. 4to. | Ditto. |
| Experimental Researches in Electricity. Thirtieth Series. By Michael Faraday, D.C.L. 4to. | Ditto. |
| On the Action of Non-Conducting Bodies in Electric induction. By M. Faraday, D.C.L., and P. Riess. 8vo. | The Authors. |
| Expériences sur la Direction des Courants de l'Océan Atlantique Septentrional. Lettre de S. A. I. le Prince Napoléon à M. Elie de Beaumont, Secrétaire perpétual de l'Académie des Sciences. 4to. | The Author. |
| Lois Générales de divers ordres de Phénomènes dont l'analyse dépend d'Equations Linéaires aux Différences Partielles tels que ceux des Vibrations et de la Propagation de la Chaleur. Par L. F. Ménabréa. 4to. | Ditto. |
| Bemærkninger Angaaende Graptolitherne af Christian Boeck. 4to. | Ditto. |
| Report of Committee upon the Experiments conducted at Stormontfield, near Perth, for the Artificial Propagation of Salmon. Read before the British Association by Sir William Jardine, Bart. 8vo. | Sir William Jardine, Bart. |
| Recherches Cliniques sur la Syphilisation. Par D. Wilhelm Boeck. 8vo. | The Author. |
| Du Mouvement imprimé à l'Aiguille Aimantée par l'influence subite de la Lumière du Soleil, avec une Théorie nouvelle fondée sur des Recherches faites par H. W. Jacobæus. 8vo. | Ditto. |
| Schädel abnormer Form in Geometrischen Abbildungen, nebst Darstellung einiger Entwickelungs-Zustände Deckknochen. Von. J. Christ. Gustav. Lucæ, M.D. Folio. | Ditto. |
| Reports of the Surveyor-General, Charles D. Bell, on the Copper Fields of Little Namaqualand, and of Commander M. S. Nolloth, of H.M.S. "Frolic," on the Bays and Harbours of that Coast. Folio. | The Governor, Cape of Good Hope. |
| Uber die durch Molekularbenegungen in starren leblosen Körpern bewirkten Formveränderungen. Von Joh. Friedr. Ludw. Hausmann. 4to. | The Author. |
| Das Chemische Laboratorium der Universität Christiania und die darin Ansgeführten Chemischen Untersuchungen. Herausgegehen von Adolph Strecker. 4to. | The University. |
| Das Christiania-Silurbecken, Chemisch-Geognostisch Untersucht von Theodor Kjerulf. Herausgegeben von Adolfph Strecker. 4to. | Ditto. |
| Akademiske Love for de Studerende ved det Kongelige Norske Frederiks Universtet 8vo. | The University of Christiania. |
| Uber den Syrisch-Ephraimitischen Krieg unter Jotham und Ahas. Von Dr C. P. Caspari. | Ditto. |
| Das Normalverhaltniss der Chemischen und Morphologischen Proportionen. Von Adolf Zeising. 8vo. | The Author. |
| Das Verhältnisk des goldnen Schnitts in seiner Bedeutung für bildende Künstler und Techniker. Von A. Zeising. 8vo. | Ditto. |
| Fagrskinna. Kortfattet Norsk Konge-Saga fra slutningen af det tolfte eller begyndelsen af det trettende aarhundrede. Af P. A. Munch og C. R. Unger. 8vo. | The University of Christiania. |
| Om Sygepleien i Straffeanstalterne i Norge. Ved Frederik Holst, M.D. 8vo. | The Author. |
| Michael Skjelderup. 8vo. | Ditto. |
| Beskrivelse over Skotlands Almueskolevæsen tilligemed forslag til forskjellige foramstalt-ninger til en videre Udvikling of det norske Almueskolevæsen. Af Hartvig Nissen. 8vo. | Ditto. |
| Beretning om fante-eller Landstrygerfolket i Norge. Af Cilert Sundt. 8vo. | Ditto. |
| Beretninger om Sygdomsforholdene i 1842 og 1843 i Danmark, Sverige og Norge, oplæste ved de Skandinaviske Naturforskeres Möde i Christiania 1844. 8vo. | The University of Christiania. |
| Konge—Speilet et Philosophisk-Didactisk Skrift, forfattet i Norge mod Slutningen af det tolfte aarhundrede. 8vo. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| Aslak Bolts Jordebog. Fortegnelse over Jordegods og Andre Herligheder tilhörende erkebiskopsstolen i nidaros, affattet ved erkebiskop aslak bolts Foranstaltning mellem aarene 1432 og 1449. Af P. A. Munch. | The University of Christiania. |
| Kong Olaf Tryggvesöns Saga forfattet paa latin henimod slutningen af det tolfte aarhundrede af Odd Snorreson. Af P. A. Munch. 8vo. | Ditto. |
| Lycidas ecloga et Musæ Invocatio, carmina quorum auctori Johanni van Leeuwen, e vico zegwaart certaminis poetici præmium secundum e legato Jacobi Henrici Hœufft adjudicatum est in concessu publico Academiæ Regiæ Scientiarum, die 13 Maji, anni 1856. 8vo. | The Author. |
| Diem Natalem Augustissimi Regis Caroli Joannis ab Universitate Regia Fredericiana, die 26 Januarii 1838 celebrandum indicit Collegium Academicum. 4to. | The University. |
| Dr Lorenz Hübner's Biographische Charakteristik. Von Joseph Wiskmayr. 4to. | The Author. |
| Ueber die Gliederung der Bevolkerung des Königreichs Bayern. Von Fr. B. W. Von Hermann. 4to. | Ditto. |
| Maps of the Geological Survey of India. | Hon. E.I. Company. |

*December* 15, 1856.

| | |
|---|---|
| The Journal of the Canadian Institute. New Series. No. 6. 8vo. | The Institute. |
| Silliman's American Journal. No. 66. 8vo. | The Editors. |
| Journal of the Statistical Society of London. Vol. xix., Part 4. 8vo. | The Society. |
| Schriften der Universität zu Kiel, 1854, 1855. 4to. | The University. |
| Bulletin de la Société de Geographie. 4ieme Serie, Tome xi. | The Society. |
| Abhandlungen herausgegeben von der Senckenbergischen Naturforschenden Gesellschaft. Band ii., 1ste lieferung. 4to. | Ditto. |

*January* 5, 1857.

| | |
|---|---|
| An Analysis of the Statistics of the Clearing House during the year 1839; with an Appendix on the London and New York Clearing Houses, and on the London Railway Clearing House. By Charles Babbage, F.R.S. 8vo. | The Author. |
| Sanatory Remarks in Connection with Nuisances. By Thomas Williamson, M.D. 8vo. | Ditto. |

*January* 19, 1857.

| | |
|---|---|
| Publications of the Ælfric Society, viz.:— 1. The Homilies of Ælfric, with an English Translation. By Benjamin Thorpe, F.S.A. 2 vols. 8vo. London, 1843–1846. 2. The Poetry of the Codex Vercellensis, with an English translation. By J. M. Kemble, M.A. 8vo. 1844–1856. 3. Anglo-Saxon Dialogues of Salomon and Saturn. By John M. Kemble, M.A. 8vo. 1845–1848. | Wm. Ivory, Esq., W.S. |
| Catalogue of the Law Books in the Library of the Society of Writers to Her Majesty's Signet in Scotland. By William Ivory, W.S. 8vo. Edinburgh, 1856. | The Author. |
| Jahresbericht über die Fortschritte der reinen, pharmaceutischen und technischen Chemie, Physik, Mineralogie und Geologie. Herausgegeben von Justus Liebig und Hermann Kopp, 1855, Zweites Heft. 8vo. | The Editors. |
| Memorias de la Real Academia de Ciencias de Madrid, Tome iii., iv. 4to. Madrid, 1856. | The Academy. |
| Programa para la adjudicacion de premios en el ano 1857. | Ditto. |
| Anuncio del Eclipse anular y Central que tendra lugar el 15 de Marzo de 1858. Por don Antonio Aguilar. 8vo. | The Author. |
| Assurance Magazine, and Journal of the Institute of Actuaries, No. xxvi., January 1857. | The Institute. |

| DONATIONS. | DONORS. |
|---|---|
| Percement de l'Isthme de Suez. Rapport et Projet de la Commission Internationale. 8vo. Paris, 1856. | The Suez Canal Company. |
| Flora Batava. 180 Aflevering. 4to. | King of Holland. |

*February* 2, 1857.

| | |
|---|---|
| Journal of the Asiatic Society of Bengal, No. v. | The Society. |
| Catalogue of Stars near the Ecliptic, observed at Markree during the years 1854–56, and whose places are supposed to be hitherto unpublished. 8vo. Vol. iv. (containing 14,951 stars). | H. M. Government. |
| Supplément aux Comptes Rendus Hebdomadaires de Seances de l'Académie des Sciences. Tom. i. 4to. | The Academy. |
| Mémoires de l'Académie des Sciences de l'Institut Imperial de France. Tom. xxvii. 1re partie. 4to. | Ditto. |
| Mémoires présentés par divers Savants à l'Académie des Sciences de l'Institut Imperial de France, et imprimis par son ordre. Sciences Mathematiques et Physiques. Tom. xiv. 4to. | Ditto. |
| Proceedings of the Zoological Society. 8vo. Nos. 310–313. | The Society. |
| Quarterly Journal of the Chemical Society, No. 36, January 1857. 8vo. | Ditto. |
| Nouveaux Mémoires de la Société Helvetique des Sciences Naturelles. Band xiv. 4to. | Ditto. |
| Mittheilungen der naturforschenden Gesellschaft in Bern, für 1855, 1856. 8vo. | Ditto. |
| Actes de la Société Helvetique des Sciences Naturelles, 1855. 8vo. | Ditto. |
| Verhandlungen der allgemeinen schweizerischen Gesellschaft für die gesammten Naturwissenschaften bei ihrer versammlung in St Gallen am 24, 25, und 26 Juli 1851. 8vo. | Ditto. |
| Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences, May 1856—December 1856. | Ditto. |

*February* 16, 1857.

| | |
|---|---|
| Monograph of the genus Abrothallus. By W. Lauder Lindsay, M.D. 8vo. | The Author. |
| Reports of the Meetings of the Royal Institute of British Architects. | The Institute. |
| Meteorological Observations taken during the years 1829 to 1852, at the Ordinance Survey Office, Phœnix Park, Dublin; to which is added a series of similar observations made at the principal trigonometrical stations, &c., in Ireland. Edited by Captain Cameron R.E. (Lieut.-Col. H. James, R.E., Superintendent of Survey.) | The Right Hon. The Secretary of State for War. |
| Proceedings of the Royal Astronomical Society, xvii., No. 23. 8vo. | The Society. |
| Memoires de la Société Impériale des Sciences Naturelles de Cherbourg. Vol. iii. (1855). 8vo. | Ditto. |
| British Interests in the Canalization of the Isthmus of Suez. 8vo. | Canal Company. |
| Papers read at the Royal Institute of British Architects, January 1857. 4to. | The Institute. |
| Tables showing the number of Criminal Offenders in England and Wales in the year 1855. Folio. | Right Hon. Sec. of State. |
| Instructions for Making Meteorological Observations. | Scottish Meteorological Society. |
| Monthly Returns of Births, Deaths, and Marriages registered in the Eight Principal Towns of Scotland; with the causes of death at four periods of life. | The Registrar-General. |
| Quarterly Returns of Births, Deaths, and Marriages registered in the Divisions, Counties, and Districts of Scotland. | Ditto. |

*March* 16, 1857.

| | |
|---|---|
| First Report of the Committee on Beneficent Institutions (Medical Charities of the Metropolis.) | Statistical Soc. of London. |

| DONATIONS. | DONORS. |
|---|---|
| Journal of the Statistical Society of London. Vol. xx., Part 1. 8vo. | The Society. |
| The Canadian Journal (published by the Canadian Institute), January 1857. | The Institute. |
| Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftlichen Classe, Band xx., Heft 2 und 3; Band xxi., Heft 1 und 2. Philosophisch-Historische Classe, Band xx., Heft 2 und 3; Band xxi., Heft 1 und 2. Register zu den zweiten 10 Banden. 8vo. | Imper. Academy of Vienna. |
| Denkschriften der Kaiserlichen Akademie der Wissenschaften, Wien. Philosophisch-Historische Classe, Siebenter Band. 4to. | Ditto. |
| Tageblatt der 32 versammlung Deutscher Naturforscher und Arzte in Wien im Jahre 1856. Nos. 1–8. 4to. | Ditto. |
| Publications of the Königlisch Sachsische Gesellschaft des Wissenschaften, Leipzig, viz.:— Beitrage zur Kenntniss der Gefässkryptogamen, von Wilhelm Hofmeister 8vo. Die Urkundlichen quellen zur Geschichte der Universität Leipzig in den ersten 150 jahren ihres bestehens, von Friedrich Zarncke. 8vo. Elektrische Untersuchungen von W. G. Hankel. Erste Abhandlung, Uber die Messung der Atmosphärischen Elektricität nach absolutem maasse. 8vo. | The Society. |
| A General Index to the Philosophical Transactions, from the first to the end of the seventeenth volume. By Paul Henry Maty, M.A., F.R.S. 4to. London, 1787. | The University Library, Edinburgh. |
| Supplement to the Quarterly Returns of the Births, Deaths, and Marriages registered in the Divisions and Counties of Scotland. Year 1856. | The Registrar-General. |

*April* 6, 1857.

| | |
|---|---|
| Monthly Return of the Births, Deaths, and Marriages registered in the eight principal towns of Scotland, with the causes of Death at four periods of life, for February 1857; and Supplement to Monthly Returns for year 1856. 8vo. | The Registrar-General. |
| Journal of the Asiatic Society of Bengal. No. 7, 1856. | The Society. |
| Proceedings of the Royal Astronomical Society. Vol. xvii., No. 4. 8vo. | Ditto. |
| Memoirs, &c., of the Geological Survey of the United Kingdom, viz.:— 1. Figures and Descriptions Illustrative of British Organic Remains, 4to. Decades 5 and 8. 2. The Iron Ores of Great Britain. Part I.—The Iron Ores of the North and North-Midland Counties of England. 8vo. 3. Mineral Statistics of the United Kingdom of Great Britain and Ireland, for 1853-55. By R. Hunt, F.R.S. 8vo. 4. Geology of the Country around Cheltenham. By Edward Hull, A.B. 8vo. 5. On the Tertiary Fluvio-Marine Formation of the Isle of Wight. By Edward Forbes, F.R.S. 8vo. 6. Prospectus of the Metropolitan School of Science applied to Mining and the Arts. 6th Session, 1856-57. 7. Collection of Maps and Sections. 8. Annual Report of the Director-General of the Geological Survey. | Sir Roderick Murchison. |
| Almanaque Nautico para 1858, calculado de Orden de S. M. en el observatorio de Marina de la Ciudad de San Fernando. 8vo. | The Observatory. |
| Assurance Magazine, and Journal of the Institute of Actuaries, April 1857. 8vo. | The Institute. |
| Memoir on the Roman Garrison at Mancunium; and its probable influence on the Population and Language of South Lancashire. By James Black, M.D., F.G.S. 8vo. | The Author. |
| Monatsbericht der Koniglichen Preuss. Akademie der Wiesenschaften zu Berlin. Juli 1855—August 1856. 8vo. | The Academy. |
| Abhandlungen der Koniglichen Akademie der Wissenschaften zu Berlin. 1855, 4to. Erster Supplement,—Band. 1854. Folio. | Ditto. |

| DONATIONS. | DONORS. |
|---|---|
| *April* 20, 1857. | |
| A General Catalogue of the Principal Fixed Stars, from Observations made at the Hon. East India Company's Observatory at Madras in the years 1830–43. By T. Glanville Taylor, F.R.S. 4to. | Hon. E. I. Company. |
| Astronomical Observations made at the Hon. East India Company's Observatory at Madras, in the years 1843–47; together with the Recomputation of the Sun and Moon, and Planetary Observations, since 1831. By T. Glanville Taylor, F.R.S. 4to. | Ditto. |
| Astronomical Observations made at the Hon. East India Company's Observatory at Madras. By Capt. W. K. Worster and W. S. Jacob. 1848–52. 4to. | Ditto. |
| Revenue Meteorological Statements of the North-Western Provinces, for the several Official years from 1844–45 to 1849–50. 4to. | Ditto. |
| Meteorological Register kept at the Hon. East India Company's Observatory at Madras. By J. Goldingham, F.R.S., and T. Glanville Taylor, F.R.S., for the years 1822–43. Folio. | Ditto. |
| Transactions of the Royal Society of Literature. Second Series. Vol. v., Part 3. | The Society. |
| Proceedings of the Royal Astronomical Society. Vol. xvii., No. 5. 8vo. | Ditto. |
| Monatsbericht der Königlichen Preuss. Akademie der Wissenschaften zu Berlin. 8vo. November—December 1856. | The Academy. |
| Quarterly Journal of the Geological Society. 8vo. February 1857. | The Society. |
| Journal of the Proceedings of the Linnean Society. 8vo. Vol. i., No. 4. | Ditto. |
| Silliman's American Journal of Science and Arts. 8vo. March 1857. | The Editors. |
| Proceedings of the Berwickshire Naturalists' Club. 8vo. Vol. iii., No. 7. | The Club. |
| Transactions of the Cambridge Philosophical Society. 4to. Vol. ix., Part 4. | The Society. |
| Essays and Heads of Lectures on Anatomy, Physiology, Pathology, and Surgery. By the late Alexander Monro Secundus, M.D., F.R.S.E. With a Memoir of his Life, and Copious Notes, explanatory of Modern Anatomy, Physiology, Pathology, and Practice. By his Son and Successor. 8vo. | Dr Monro. |
| Army Meteorological Register from 1853 to 1854 inclusive, from Observations made by Officers of the Medical Department of the Army, at Military Posts of the United States. Prepared under direction of Brevet-Brigadier General Thomas Lawson. 4to. | Professor Henry D. Rogers. |
| Statistical Report of the Sickness and Mortality in the Army of the United States, from 1839 to 1855. By Richard H. Coolidge, M.D. 4to. | Ditto. |
| Reports of Explorations and Surveys to ascertain the most practicable and economical Route for a Railroad from the Mississippi River to the Pacific Ocean, made in 1853–4. 4to. Vol. i. | Ditto. |
| Narrative of the Expedition of an American Squadron to the China Seas and Japan, in the years 1852-54, under command of Commodore M. C. Perry, U.S. Navy. By Francis L. Hawkins, D.D., LL.D. 4to. Vol. i. Also Vol. iii. of the same work, being Observations on the Zodiacal Light from April 2, 1853, to April 22, 1855. By Rev. George Jones, A.M. 4to. | Ditto. |

# INDEX TO VOL. XXI.


## A

*Alkaloids* (*Vegetable*). On the action of the halogen compounds of ethyl and amyl on some vegetable alkaloids, 27.

*Amides of the Fatty Acids*, researches on, 299.

ANDERSON (THOMAS), M.D. Researches on some of the crystalline constituents of opium. Second Series, 195. On the products of the destructive distillation of animal substances. Part Third, 219. Part Fourth, 571.

*Animal Substances*, products of their destructive distillation, 219, 571.

## B

BALFOUR (JOHN HUTTON), M.D. On certain vegetable organisms found in coal from Fordel, 187.

BENNETT (JOHN HUGHES), M.D. An investigation into the structure of the Torbanehill mineral, and of various kinds of coal, 173.

BOOLE (GEORGE), LL.D. On the application of the theory of probabilities to the question of the combination of testimonies or judgments, 597.

BROWN (JAMES F.). On a general method of substituting iodine for hydrogen in organic compounds, and on the properties of iodopyromeconic acid, 49.

*Buddhist Opinions and Monuments of Asia*, compared with the symbols on the ancient sculptured "Standing Stones" of Scotland, 255.

## C

*Coal.* An investigation into the structure of the Torbanehill mineral, and of various kinds of coal, 173.

*Coal from Fordel*, on certain vegetable organisms found in, 187.

*Chinoline* and its homologues, 377.

*Cinchonine*, on the volatile bases produced by the destructive distillation of, 309.

*Colour.* Experiments on colour as perceived by the eye, with remarks on colour-blindness, 275.

*Combinations*, on a problem in, 359.

CONNELL (A.). On a new hygrometer, or dew-point instrument, 15.

*Crystalline Constituents of Opium*, 195.

## D

DAVY (JOHN), M.D. On the impregnation of the ova of the *Salmonidæ*, 1. Some miscellaneous remarks on the *Salmonidæ*, 245. On the urinary secretion of fishes, with some remarks on this secretion in other classes of animals, 543.

*Diatomaceæ* found in the Firth of Clyde and in Loch Fine, 473.

*Dynamical Top*, for exhibiting the phenomena of the motion of a system of invariable form about a fixed point, with some suggestions as to the earth's motion, 559.

E

*Earth's Crust*, on the laws of structure of the more disturbed zones of, 431.

*English Lake District*, on the meteorology of, including the results of experiments on the fall of rain, the temperature, the dew-point, and the humidity of the atmosphere, at various heights on the mountains, up to 3166 feet above the sea-level, for the years 1851, 1852, and 1853, 81.

*Ethyl and Amyl*, action of halogen compounds of, on some vegetable alkaloids, 27.

F

*Fermat's Theorem*, 403.

FORBES (JAMES D.), D.C.L. Further experiments and remarks on the measurement of heights by the boiling point of water, 235.

G

GREGORY (WILLIAM), M.D. On new forms of marine Diatomaceæ, found in the Firth of Clyde and Loch Fine, 473.

H

*Heat*, dynamical theory of, 123.

*Heights*, measurement of, by the boiling-point, 235.

HOW (HENRY). On the action of halogen compounds of ethyl and amyl on some vegetable alkaloids, 27.

*Hygrometer*. On a new hygrometer, or dew-point instrument, 15.

I

*Involuntary Muscular Fibre*, on the minute structure of, 549.

*Iodine*. A method of substituting hydrogen for iodine in organic compounds, 49.

*Iodopyromeconic Acid*, 49.

K

KELLAND (Rev. PHILIP), M.A. On superposition, 271. On a problem in combinations, 359.

L

LISTER (JOSEPH), F.R.C.S. On the minute structure of involuntary muscular fibre, 549.

M

*Magnetic Declination*. On errors caused by imperfect inversion of the magnet, in observations of magnetic declination, 349.

MAXWELL (JAMES CLERK), B.A. Experiments on colour, as perceived by the eye, with remarks on colour-blindness, 275. On a dynamical top, for exhibiting the phenomena of the motion of a system of invariable form about a fixed point, with some suggestions as to the earth's motion, 557.

*Meteorology of the English Lake District*, 81.

MILLER (JOHN FLETCHER), Ph.D. On the meteorology of the Lake District, including the results of experiments on the fall of rain, the temperature, the dew-point, and the humidity of the atmosphere at various heights on the mountains, up to 3166 feet above the sea level, for the years 1851, 1852, and 1853, 81.

O

*Opium*, crystalline constituents of, 195.

P

*Photometer*, 363.

PONTON (MUNGO). On solar light, and on a simple photometer, 363.

*Prismatic Spectra of the Flames of compounds of Carbon and Hydrogen*, 431.

*Probabilities*, on the possibility of combining two or more probabilities of the same event, so as to form one definite probability, 369. On the application of the theory of probabilities to the question of the combination of testimonies or judgments, 597.

R

ROGERS (H. D.), Hon. F.R.S.E. On the laws of structure of the more disturbed zones of the earth's crust, 431.

ROWNEY (THOMAS H.), Ph.D. Researches on the amides of the fatty acids, 299.

S

*Salmonidæ*, on the impregnation of the ova of, 1.

*Salmonidæ*, miscellaneous remarks on, 245.

*Solar Light*, 363.

*Solar System*, mechanical energies of, 63.

STEWART (BALFOUR). On a proposition in the theory of numbers, 407.

*Sunlight.* Note on the possible density of the luminiferous medium, and on the mechanical value of a cubic mile of sunlight, 57.

*Superposition*, on, 271.

SWAN (WILLIAM). On errors caused by imperfect inversion of the magnet, on observations of magnetic declination, 349. On the prismatic spectra of the flames of compounds of carbon and hydrogen, 431.

T

TALBOT (H. F.), F.R.S. On Fermat's theorem, 403.

TERROT (Bishop). On the possibility of combining two or more probabilities of the same event, so as to form one definite probability, 369.

*Theory of Numbers*, on a proposition in, 407.

*Thermo-Electric Currents*, 123.

THOMSON (WILLIAM). Note on the possible density of the luminiferous medium, and on the mechanical value of a cubic mile of sunlight, 57. On the mechanical energies of the solar system, 63. On the dynamical theory of heat. Part V. Thermo-electric currents, 123.

*Torbanehill Mineral*, 7, 173.

TRAILL (THOMAS S.), M.D. On the Torbanehill mineral, 7.

U

*Urinary Secretion of Fishes*, with some remarks on this secretion in other classes of animals, 543.

V

*Vision*. On the extent to which the received theory of vision requires us to regard the eye as a camera obscura, 327.

W

WILLIAMS (C. GREVILLE). On the volatile bases produced by the destructive distillation of cinchonine, 309. Researches on chinoline and its homologues, 377.

WILSON (GEORGE), M.D. On the extent to which the received theory of vision requires us to regard the eye as a camera obscura, 327.

WISE (THOMAS A.), M.D. Notes on some of the Buddhist opinions and monuments of Asia, compared with the symbols on the ancient sculptured "Standing Stones" of Scotland, 355.

END OF VOLUME TWENTY-FIRST.

NEILL & Co., Printers, Edinburgh.

Page

*Proceedings at Statutory General Meetings, &c.,* . . . . . . . . . . 655

*List of the present Ordinary Members, in the order of their Election,* . . . . . . 665

*List of Non-Resident and Foreign Members, elected under the Old Laws,* . . . . . 672

*Honorary Fellows,* . . . . . . . . . . . . 672

*Fellows Deceased, Resigned, and Cancelled, from* 1853 *to* 1857, . . . . . 674

*Public Institutions, &c., entitled to receive the Transactions and Proceedings of the Society,* . . 676

*List of Donations, continued from Vol. XX., p.* 663, . . . . . . . . 678

*Index,* . . . . . . . . . . . . . . 701

---

The Transactions of the Royal Society of Edinburgh will in future be Sold at the following reduced Prices:—

| Vol. | Price to the Public. | Price to Fellows. | Vol. | Price to the Public. | Price to Fellows. |
|---|---|---|---|---|---|
| I. II. III. | Out of Print. | | XVI. Part 2. | £0 18 0 | £0 14 0 |
| IV. | £0 9 0 | £0 7 0 | „ Part 3. | 0 10 0 | 0 7 6 |
| V. | 0 11 0 | 0 9 0 | „ Part 4. | 0 5 0 | 0 4 0 |
| VI. | 0 11 6 | 0 9 6 | „ Part 5. | 0 7 0 | 0 5 6 |
| VII. | 0 18 0 | 0 15 0 | XVII. | Out of Print. | |
| VIII. | 0 17 0 | 0 14 0 | XVIII. | 2 2 0 | 1 11 0 |
| IX. | 1 0 0 | 0 17 0 | XIX. Part 1. | 2 2 0 | 1 11 0 |
| X. | 0 19 0 | 0 16 0 | „ Part 2. | 0 18 0 | 0 15 0 |
| XI. | 0 14 6 | 0 12 0 | XX. Part 1. | 0 18 0 | 0 14 0 |
| XII. | 0 14 6 | 0 12 0 | „ Part 2. | 0 10 0 | 0 7 6 |
| XIII. | 0 18 0 | 0 15 0 | „ Part 3. | 0 10 0 | 0 7 6 |
| XIV. | 1 5 0 | 1 1 0 | „ Part 4. | 0 10 0 | 0 7 6 |
| XV. | 1 11 0 | 1 6 0 | XXI. Part 1. | 0 15 0 | 0 11 6 |
| XVI. Part 1. | 0 5 0 | 0 4 0 | „ Part 2. | 0 10 0 | 0 7 6 |
| | | | „ Part 3. | 0 7 0 | 0 5 3 |

---

NEILL AND COMPANY, PRINTERS, EDINBURGH.